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ERRATA.

Page 257 :

$$For \quad \frac{10^8 2\pi^2 E^2 e^2 M m}{ch^3 (M+m)} \quad read \quad \frac{2\pi^2 E^2 e^2 M m}{ch^3 (M+m)} .$$

Page 258 :

$$For \quad \frac{16 (M+m)}{4 (M+m)} \quad read \quad \frac{16 (M+m)}{(4M+m)} .$$

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THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions* take this opportunity to acquaint the public that it fully appears, as well from the Council-books and Journals of the Society as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the public that their usual meetings were then continued, for the improvement of knowledge and benefit of mankind: the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them: without pretending to answer for the certainty of the facts, or propriety of the reasonings contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body,

upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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BY

SIR WILLIAM CROOKES, O.M., F.R.S.



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PHILOSOPHICAL TRANSACTIONS.

I. *The Preparation of Eye-preserving Glass for Spectacles.*

By Sir WILLIAM CROOKES, O.M.,^{FRS} F.R.S.

Received October 24,—Read November 13, 1913.

SINCE March, 1909—in connection with the Glass Workers' Cataract Committee of the Royal Society—I have been experimenting on the effect of adding various metallic oxides to the constituents of glass in order to cut off the invisible rays at the ultra-violet and the infra-red ends of the spectrum. The work has been done chiefly in my own laboratory. I have been aided by Mr. Harry Powell, of the Whitefriars Glass Works, who prepared several pots of coloured glass from my formulæ on a much larger scale than could be made outside a glass works. From these glasses cylinders and sheets were made.

The main object of this research is to prepare a glass which will cut off those rays from highly heated molten glass, which damage the eyes of workmen, without obscuring too much light or materially affecting the colours of objects seen through the glass when fashioned into spectacles, but the work necessitated an examination of the screening properties of glass plates for ultra-violet and luminous light, and therefore the research was enlarged so as to embrace the three forms of radiation.

Radiation from Molten Glass.

In order to ascertain what rays are given off from molten glass I spent some time at the Glass Bottle Works of Messrs. Nuttall and Co., St. Helens, and took many photographs of the spectra of the radiations.

Photo-spectrographic and other examinations were made of the radiation emitted from the molten glass under working conditions. Full details of the experiments and results are given in this paper.

At the time I visited Messrs. Nuttall's works light green bottle-glass was being made; the mixture is composed of silica, sodium sulphate, and calcium carbonate or sulphate. The materials are melted in a large fire-brick tank, heated by a flaming mixture of gas and air playing on the surface. The gas is made some distance from the furnace in a "producer." Gas and air are conducted by separate channels to the upper part of the tank, where they mix and burn, the flame reverberating from the arched roof and heating the glass mixture to the requisite degree.

The area of the tank of molten glass is about 82 square yards, and it contains from 300 to 350 tons of the mixture. There are several such tanks in the works. The tank is divided by a fireclay partition into two unequal parts. At the lower part is an opening through which the melted glass can flow. The larger portion of the tank, where the materials are melted together at a high heat, has a surface of about 63 square yards. This is called the "melting end"; when the mixture is well fused and homogeneous the molten glass flows through the opening into the "working end" of about 19 square yards, where the heat is less and the glass is in a viscous state. Fireclay rings of 18 inches internal diameter and a foot deep float on the surface of the viscid glass; any scum on the surface of the tank is thereby kept from contaminating the surface of the glass inside the ring. One ring floats opposite each working opening, and the workmen withdraw the requisite quantity of glass for each operation from the inner surface of the ring.

The light from the melting end of the tank, viewed through a working opening, was brilliant white with a tinge of orange; it was with difficulty the unprotected eye could make out any details. Viewed through dark glasses the surface of the metal in the tank appeared as a seething mass in constant commotion. The surface in the working end was more easy to see. It was of a bright yellow incandescence, and comparatively quiet.

It is not certain what the temperatures are at each end of the tank. So far as one could judge, the temperature at the melting end is about 1500°C ., and at the working end decidedly less—say, 1200°C .

About each opening, especially at the melting end, thin white vapours rose and settled on the surrounding cooler parts. A piece of paper held in this vapour instantly ignited. Examined with a hand spectroscope the yellow line of sodium was seen to be brilliant in this vapour, but the light from the molten glass showed a continuous spectrum in which the sodium line was visible. On one or two occasions a black line was seen in place of the yellow sodium line, showing a reversal. Some of the condensed vapour was collected from the cool sides of the working opening and chemically examined. It was found to consist principally of sodium and calcium sulphates, with a little sodium chloride.

Photo- and Thermo-graphic Experiments.

The spectrograph used for taking photographs of the radiation from the molten glass is the one I described in 'Roy. Soc. Proc.,' vol. lxx., p. 237, May, 1899. It has two quartz prisms, each made up of two halves, one half being right- and the other half left-handed, according to CORNU's plan for neutralising the effect of double refraction. The collimating and camera lenses and the double condensers are also of quartz cut in the same fashion. The slit jaws are made of two acute-angled quartz wedges, edge to edge. The refracting prisms are of 60 degrees angle, and each face

is 35 mm. by 42 mm. The lenses are 52 mm. diameter and 350 mm. focus. The condensers are plano-cylindrical, one being double the focus of the other. In order to ascertain the exact position of any part of the spectrum I might obtain from the radiation from the molten glass, I took photographs on each plate of an alloy of equal molecular weights of zinc, cadmium, tin, and mercury. This alloy gives throughout the photographic region lines, the wave-lengths of which are well known.

The instrument sloped downwards so as to allow the radiation from the surface of the melted glass to enter the condensers, prisms, and slit along the axis. To prevent the great heat injuring the spectrograph Mr. Nuttall allowed the opening to be bricked up, leaving a hole a few inches square in the middle. This was covered with an iron plate with a 2-inch hole in it, and over this a quartz plate was fixed.

Panchromatic films were used. These are sensitive beyond $\lambda 7800$ in the ultra-red, and to the highest ultra-violet rays which will pass through quartz (about $\lambda 2100$). Flexible films had to be used in preference to glass as they had to follow the curvature of the focal plane. Many preliminary experiments were made to ascertain the extent of spectrum to be recorded, its best position on the films, and the exposures needed. The slit of the instrument was generally placed about four feet from the molten surface, and it was found that from ten to fifteen minutes were required to produce a faint image on development. On each film, immediately before the radiation picture was taken, a photograph of the spark spectrum of the quadruple alloy was impressed on the film in such a position that the two spectra would overlap to a very slight degree.

No. 1 photograph was taken at the working end of the tank, where the temperature was lower than at the other end. An exposure of twenty minutes was given, the width of slit being 0.025 mm.

No. 2 photograph was taken in the same conditions as No. 1, but with an exposure of forty-five minutes.

No. 3 photograph was taken at the melting end, where the heat was fiercest. The width of the slit was reduced to 0.1 mm., and an exposure of half-an-hour was given.

No. 4 photograph, at the melting end, was exposed for one hour.

No. 5 photograph, at the melting end, was exposed for two hours.

No. 6 photograph, also at the melting end, was exposed for three hours.

It was not found practicable to give longer exposures.

Whilst these experiments were in progress, other experiments at another opening at the hottest end were tried to see if X-rays could be detected. Sensitive films were wrapped in black paper and then in lead foil in which designs had been cut. These were exposed for varying lengths of time, as near as it was safe to put them to the radiation from the molten glass, bearing in mind that the heat might affect the films. On development, no image of the stencil designs on any of the films could be detected. These results confirm those previously obtained by Dr. BURCH—that X-rays are not emitted by the highly incandescent molten glass.

A careful examination of the six photographs shows a general progressive character, the extent of spectrum photographed extending into the ultra-violet as the length of exposure increases.

The extent of spectrum into the region of the ultra-violet is conveniently shown in the following tabular form* :—

No. 1 photograph, exposed 20 minutes, extends to λ 4520.					
„ 2	„	„	45	„	„ λ 4320.
„ 3	„	„	30	„	„ λ 3790.
„ 4	„	„	60	„	„ λ 3640.
„ 5	„	„	120	„	„ λ 3595.
„ 6	„	„	180	„	„ λ 3345.

Taking the ordinary limit of visibility to lie between λ 3900 and λ 7600, it is seen that with an exposure of three hours to the highest heats the strength of impression does not extend much into the ultra-violet. The heat rays are very strong, and if injury to the eye is caused by exposure to radiation from the molten glass, a protective glass should be opaque to the infra-red rays.

These being present in the radiation from molten glass in far greater abundance than the ultra-violet rays, the inference is that it is to the heat rays rather than to the ultra-violet rays that glass workers' cataract is to be ascribed. It is, however, certain that exposure to excess of ultra-violet light also injuriously affects the eye.

That the ultra-violet rays act on the deeper-seated portions of the eye is shown by the intense fluorescence of the crystalline lens induced by these rays.

Besides the invisible rays at each end of the spectrum, the purely luminous rays, if present in abnormal intensity, are found to damage the eye. It therefore would be an advantage if in addition the obscuring glass for the spectacles were to be of a neutral or grey tint.

Synthetic Preparation of Glasses.

It soon became evident that my best, if not only, chance of solving the problem was to make different glasses in my own laboratory, with the addition of known quantities of pure metallic oxides and earths as colouring or absorbing materials. Lapidary

* In connection with this table the following scale for correlating colours with wave-lengths will be useful :—

Wave-lengths 7230 and below = infra-red.

From λ 7230 to λ 6470 = red.

„ λ 6470 „ λ 5850 = orange.

„ λ 5850 „ λ 5750 = yellow.

„ λ 5750 „ λ 4920 = green.

„ λ 4920 „ λ 4550 = blue.

„ λ 4550 „ λ 4240 = indigo.

„ λ 4240 „ λ 3970 = violet.

„ λ 3970 and above = ultra-violet.

apparatus for cutting, grinding, and polishing was also necessary so that the synthetically made glasses could be cut into plates—polished so as to be tested photographically in the spectrograph already described—and also tested for the percentage of heat rays they obstructed.

Many preliminary experiments were made on the preparation of a clear and colourless glass or flux to serve as a basis for the colouring with the various metallic oxides. Finally, two kinds of soda glass not containing lead were chosen, and Mr. H. Powell of the Whitefriars Glass Works, who had assisted me in the preliminary trials by supplying me with many kinds of glass of different composition and fusibility, made a quantity of these fluxes and supplied them in a crushed condition. In my earlier laboratory experiments the mixture of colouring matter and granulated flux was put into a small “gold pot” of Morgan’s Crucible Co., and gradually heated over a “Meker” gas burner. It is advisable to have one at least of the colouring constituents in the form of nitrate so that its decomposition by heat shall mix and stir the constituents. The decomposition of the nitrate causes a little frothing; therefore it is necessary to add the mixture gradually to the crucible, to avoid frothing over. When all is added and the contents well fritted, the hot crucible is removed to an electric furnace and the temperature slowly raised until the glass is quite fluid. It is stirred at frequent intervals with a stout platinum rod. After an hour the stirring is discontinued, and the temperature kept up for an hour and a-half. The current is then cut off, the openings in the furnace plugged with asbestos to prevent draughts, and the whole allowed slowly to cool to anneal the glass. In some cases the composition of the glass was such that the melting-point had to be raised above 1400°C. , and as this temperature was beyond the safe limit with the platinum strip furnace, a blast-furnace fitted with a “Lennox” electric blower was used; with this arrangement larger quantities of glass could be raised with safety to a much higher temperature.

There are two conditions I have endeavoured to secure of the finished glass—each of great importance. One, the most essential, is the absence of all streaks, striæ, and irregularities of density; the other, the absence of air-bubbles. The first is obtained by repeated stirring and perfect admixture; the freedom from air-bubbles is secured by leaving the glass in perfect repose while the heat is at the highest point. On these and other points I have been much aided by reading an early paper by FARADAY, “On the Manufacture of Glass for Optical Purposes,” the Bakerian Lecture read before the Royal Society in 1829 (*Phil. Trans. Roy. Soc.*, 1830, p. 1). On a small scale it is almost impossible to avoid slight striæ owing to differences of density caused by the long continued heat volatilising some of the soda. FARADAY was much harassed by this dilemma in the manufacture of his optical glass, and tried many experiments to ascertain the cause. To get rid of air-bubbles FARADAY used spongy platinum in powder sprinkled over or added to the bulk of the melted glass. This was found to act pretty well, making the bubbles rise in the same manner as a piece of bread causes bubbles to rise when thrown into a glass of effervescing liquid. To get the full benefit

from this device, however, the glass must be kept perfectly quiet and at the highest temperature for a longer time than in my case was always practicable.

When the crucible has cooled slowly for about twelve hours it is removed from the furnace, and the solid cone of glass removed by breaking the crucible by a few judicious blows with a light hammer. The lump of glass is now taken to the lapidary's table, and slit across the middle and a plate cut from it, which is ground and polished to a determined thickness, usually 2 mm.

Testing Synthetic Glasses for Opacity to Ultra-violet Light.

The plates are now tested for the opacity of the glass to the rays of the ultra-violet end of the spectrum. The spectrograph with complete quartz train, already referred to, was used. By superposing on the radiation from a Nernst lamp the light from a high-tension electric discharge between poles of pure metallic uranium it is possible to produce a practically continuous beam extending from $\lambda 2000$ to $\lambda 8000$, and the absorption of such a beam of radiation, produced by flat plates 2 mm. thick of all my experimental glasses, has been thereby recorded.

To ascertain the amount of heat obstructed by the plates of glass I first used a MELLONI's thermopile as described in his papers,* but I soon found that modification was needed as the pile responds to the orange and red rays as well as to the infra-red.

Plates of dark smoky quartz 2 mm. thick were tested with the thermopile, and it was found that they transmitted nearly all the heat whilst cutting off 80 or 90 per cent. of the light. Biotite (black mica) exerts a similar effect, and is more easily experimented with than smoky quartz. For many years I have used biotite for cutting off light and transmitting heat; I accordingly investigated the properties of many specimens of black and dark brown mica to find out which would be best for this special purpose.

Selection of Black Mica (Biotite) for Diathermancy.

Samples of dark brown and black mica vary considerably in their power of obstructing light and transmitting the infra-red rays. By the kindness of friends connected with the mining and importation of mica, I have been able to examine a large number of samples from different parts of the world.

Some very fine pieces of black biotite were sent by Messrs. Attwater and Sons, who tell me they were mined by them in the extreme north of Norway from a cleft in a mountain at about 2000 feet. This mica is extremely regular in the thickness of the flakes which can be split from it, and the colour of thin pieces is uniform. A piece

* "On the Free Transmission of Radiant Heat through Different Solid and Liquid Bodies," 'Ann. de Chim. et de Phys.,' vol. liii., p. 1; TAYLOR'S 'Scientific Memoirs,' vol. 1, p. 1; "New Researches Relative to the Immediate Transmission of Radiant Heat through Different Solid and Liquid Bodies," 'Ann. de Chim. et de Phys.,' vol. lv., p. 337; TAYLOR'S 'Scientific Memoirs,' vol. i., p. 39.

0.07 mm. thick entirely cuts off the luminous rays, and even the sun's disc is only just seen through a flake 0.06 mm. thick.

In addition to specimens from Norway, Messrs. Attwater and Sons sent me black biotite from German South-East Africa, and some fine pieces of "black amber" mica from Africa. The African mica is uniform in colour, and easily splits into flakes of great regularity. The German mica is difficult to split into uniformly regular flakes, and therefore varies considerably in colour.

Messrs. F. Wiggins and Sons allowed me to select some large sheets of dark brown and black biotite from their stock. The sheets when split are uniform in colour, and in thicknesses below about 0.30 mm. are sufficiently transparent to allow the eye to detect a Nernst glower. Mica differs, however, in transparency, one flake from a sheet being opaque at 0.24 mm., while a flake from another part of the same sheet is slightly transparent in a thickness of 0.34 mm. I also received good black mica from Mr. Henson, who gave as its locality Renfrew, Canada.

Many experiments have been carried on with all these kinds of brown mica to find a quality which would cut off the rays which at Kew were called the "scorching rays" (the infra-red rays), and some of the best results were obtained with the Norwegian mica from Messrs. Attwater and Sons, and the black amber African mica from Messrs. Wiggins and Sons.

There is a certain gradation of transmitted rays according to the thickness of the dark brown micas. All of them cut off rays at the blue end of the spectrum, and as the thickness increases the portion of the spectrum obstructed rapidly tends towards the red end, until a mica is found which affects the photographic plate in a narrowed band round the line B, the exposure being from ten to twenty times as long as would be required for this part of the spectrum to impress itself, were no mica to intervene. To increase the thickness of the dark mica soon obstructs all rays in the red visible to the eye.

Examined by the thermometric apparatus, a thermopile, and in the radiometer balance—described below—the dark micas which allow any trace of visible rays to pass are strongly diathermic. As the thickness of mica increases the deflection of the index spot of light gets less and less, until there is very little action at all. Judging from analogy it is most probable that as the thickness increases the heat rays are cut off in regular gradation from the line B in the red to the longest rays of heat which will affect the radiometer balance.

This is only a hypothesis and does not take into account the possibility of there being dark bands in the infra-red portion of the heat spectrum. Still the hypothesis as a working tool has been of considerable use, and has helped me to select dark mica obscuring media—giving good and concordant results.

Were I to take at random a piece of black mica which would cut off all the rays visible to the eye, and not allow any red and ultra-red rays to pass that would affect the panchromatic plate, I should have no certainty that the piece of mica would not

cut off some of the heat rays which the glass transmitted. If, on the other hand, I select a piece that allows the red rays near the lines B and C to pass (as shown on the photo-spectrograph), some of the action on the radiometer balance would be due to the luminous red rays, and it is not advisable to entirely cut these off as the residual colour thereby might be affected. The plan ultimately adopted was gradually to increase the thickness of the mica until the photo-spectrograph showed that the visible red had just ceased to affect the plate. After experimenting on some hundred kinds and thicknesses of brown and black mica I succeeded in getting a piece which appeared to satisfy all requirements; the examination of the different kinds of glass made in my laboratory during the last four years has been carried on by help of this specimen.

Measurement of Infra-red Radiation.

Before I had tried many experiments with the thermopile and black mica another difficulty cropped up. The heat radiated from the Nernst glower gradually warmed up the apparatus, and unless a long time was allowed between each observation I never could get the index ray of light to return to zero. This and the little sensitiveness of the instrument used induced me to try a mercurial thermometer in place of the thermopile, all other arrangements being as before. The thermometer was specially constructed with a concave bulb coated with carbon from burning camphor. It was divided into tenths of a degree, the scale being about 9 mm. to a degree.

To test, a selected plate of black mica was permanently fixed in the apparatus between the bulb of the thermometer and the slide containing the glass. First of all, upon lighting the Nernst lamp, the rise in temperature was taken, having only the mica plate between the lamp and the thermometer. The thermometer was observed from a distance through a cathetometer. It was allowed to rise about 1°C. , and then with a stop-watch the further rise in sixty seconds was observed. A mean of four observations in this way gave a rise in sixty seconds of—

$$\left. \begin{array}{l} 1.75 \\ 1.60 \\ 1.725 \\ 1.675 \end{array} \right\} \text{mean } 1.683.$$

In making a determination of diathermancy, glass plates 2 mm. thick were prepared and placed close to the thermometer bulb in a holder having a $\frac{1}{2}$ -inch hole; ten observations were made and the mean taken as the value, which was given in percentages of 1.683.

Thus with glass No. 129; mean rise in sixty seconds = 0.26°C. ,

$$\frac{0.26 \times 100}{1.683} = 15.4^{\circ}\text{C.}$$

A mean of ten similar observations gave a value which was taken as the diathermancy of the glass to dark heat. Its athermancy is obtained by subtracting this value from 100. In this way a multitude of glasses were tested, and recently the results have been plotted on a curve which was found to correspond closely with the result subsequently obtained by the radiometer method.

The Radiometer Balance.

In early papers on "Repulsion Resulting from Radiation,"* I showed that the blackened surface of a radiometer was repelled by all the rays of the solar spectrum, from the ultra-violet to a distance at the red end extending far into the ultra-red, the maximum intensity being a little distance below the spectrum line A.

An instrument was accordingly made based on the principle of the radiometer, and somewhat resembling the apparatus described in a paper read before the Royal Society in 1875: "On Repulsion Resulting from Radiation."† It is a torsion balance in which the beam moves in a horizontal plane. Figs. 1 and 2 show the details of the instrument, the letter references being the same in each figure. AB is a thin glass tube, with a bulb at the end, B, and ground flat at the end, A. To the centre of AB is sealed an upright tube, CD, having an arm, E, blown to it for the purpose of attachment to the pump. FG is a very light arm of aluminium carrying at the end, G, a disc of silver-flake mica coated with lamp-black. At the end, F, is a small counterpoise to allow the arm to hang level. H is a glass stopper to the upper part of the tube, CD, which is widened out to form a cup to hold mercury. The stopper in the cup is accurately ground in the tube, as long a surface as practicable being in contact. The horizontal arm is suspended from the stopper by a bifilar suspension of fine quartz fibres, IJ, which are attached to the arm FG by an aluminium stirrup holding at its upper end a silvered glass mirror of one metre focus. The

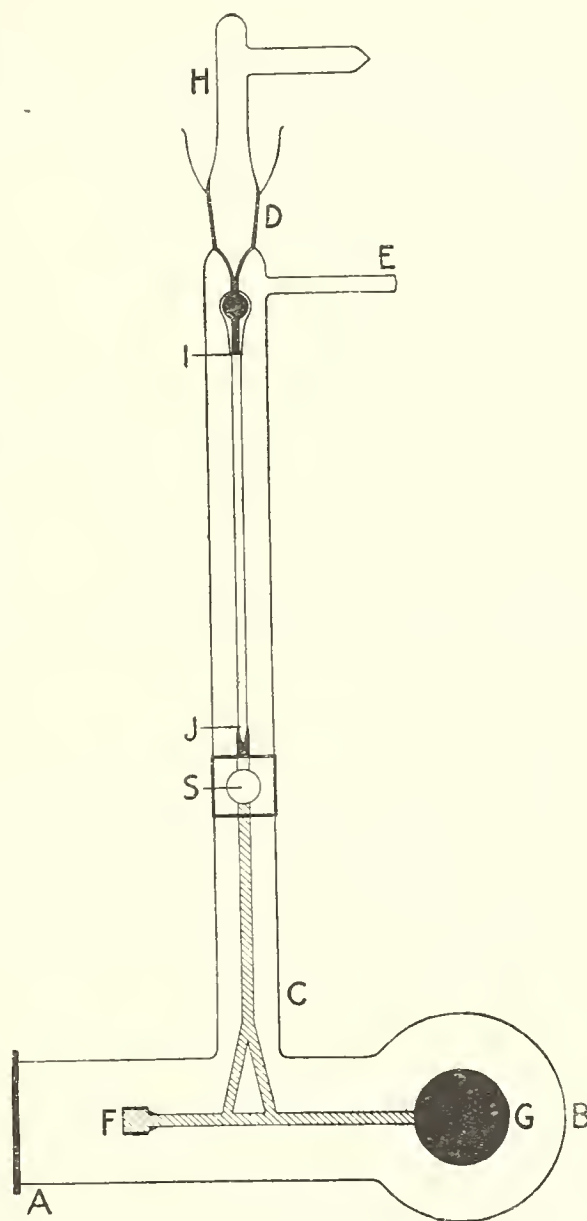


Fig. 1.

* 'Phil. Trans. Roy. Soc.,' 1876, Part II., vol. clxvi., pp. 355-361.

† 'Phil. Trans. Roy. Soc.,' 1875, Part II., vol. clxv., pp. 533-4-5.

vertical tube is blown out and the edges ground flat at the part where the mirror hangs; a flat piece of glass is cemented to it, forming a window through which pass the entering and emerging index beams of light. The end A of the horizontal tube is left open to allow of the adjustment of the arm in its stirrup, and then it is sealed with a flat piece of glass cemented on. The stopper, H, is lubricated with drops of burnt indiarubber so that it can be smoothly rotated to allow the arm to be brought accurately to zero.

Fig. 2 shows the arrangement of the apparatus fitted for testing the samples of glass. The radiometer balance is enclosed in a wooden box having two holes opposite the mirror and the end of the blackened disc at the torsion arm. Great precautions must be taken to avoid all extraneous radiations from acting on the black disc; a slightly conical card tube, as narrow as the angular movement of the ray of light will admit, is attached to the window at K in front of the mirror, S, and another to the bulb at L opposite the black disc.

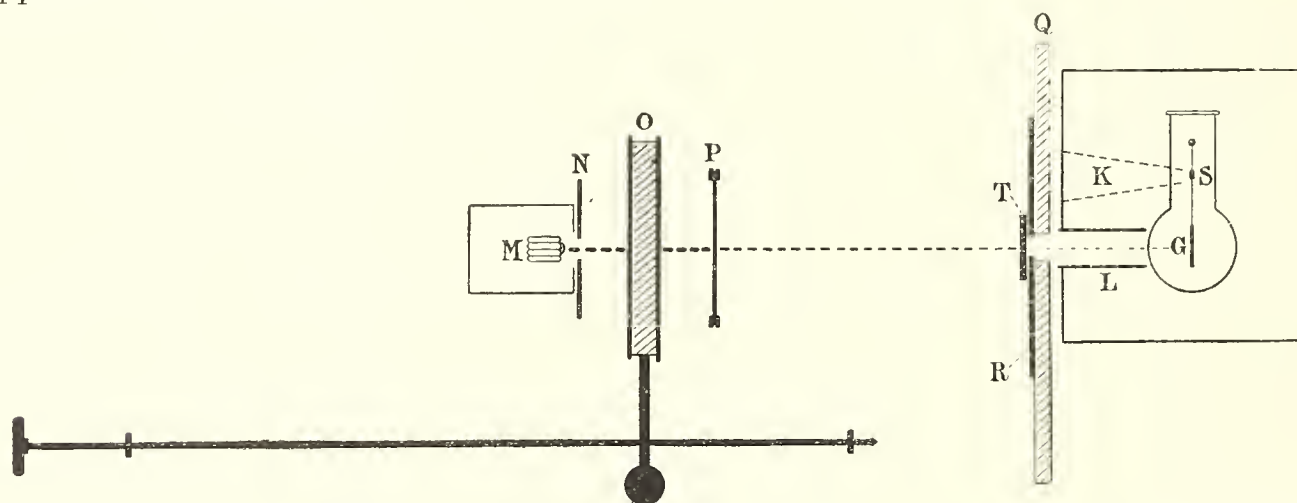


Fig. 2.

- | | | | |
|----------------|--|----|-------------------------------------|
| A, B, C, D, E. | Case of radiometer. (Fig. 1.) | N. | Aluminium screen. |
| F, G. | Radiometer arm with blackened mica disc. | O. | Shutter. |
| H. | Ground glass joint. (Fig. 1.) | P. | Black mica screen. |
| I, J. | Quartz fibres. (Fig. 1.) | Q. | Carrier for holding glass specimen. |
| K. | Guard tube in front of mirror S. | R. | Aluminium screen. |
| L. | Guard tube in front of radiometer disc. | S. | Concave mirror, one metre focus. |
| M. | Nernst glower. | T. | Glass specimen under examination. |

The heat radiation used in these tests is emitted from a Nernst glower, M, enclosed in a metal box with an open end. In front of the glower is an aluminium screen, N, pierced with a centimetre hole. A shutter, O, can be moved up and down by an arm close to the observer. The shutter screen is made of a piece of cork an inch thick, having on each side a plate of polished aluminium. In this way the heating up of the shutter when it is obscuring the ray from the glower is effectually prevented. At P is a frame for supporting the piece of black mica, and at Q is a sliding carrier holding the piece of glass under examination. This is so arranged that it can be drawn out and another piece of glass put in without causing any jar. Behind the glass is an

aluminium screen, R, with a hole in it one centimetre in diameter. The vacuum must be a rather high one, about 40 millionths of an atmosphere.*

The whole apparatus is closely packed with cotton-wool, so that no radiation can get to the black disc but that which comes through the window opposite. The box containing the radiometer balance is firmly attached to the main wall of the house, to avoid as much as possible interference from vibration caused by movements in the room. A spot of light reflected by the mirror, S, from another luminous source is received upon a graduated screen one metre distant in the usual manner.

Testing Synthetic Glasses for Diathermancy.

The mode of procedure is thus :—The mica is put in its place and the lamps started. In about ten minutes the zero is adjusted by means of the rotating stopper. When the spot of light is at zero the shutter, O, is raised, and the extent of the deflection noted. At the end of the first half swing the shutter is lowered, and the whole is left at rest until the light is again at zero. The glass under test, T, is now put in its carrier and slid into place, and the extent of deflection of the spot of light noted when the shutter is raised. The amount of deflection with the same piece of mica interposed is thus obtained with many different kinds of glass, and from the data the order of obstruction to heat rays can be calculated for each. It is a necessary precaution to verify the readings once or twice, and to allow the spot of light to come accurately to zero. It is inclined to shift if observations are repeated too rapidly, owing to the retention of heat by the blackened face of the radiometer disc, and the consequent repulsion between it and the front of the bulb B. This effect soon goes off if a little time elapses between the different observations.

The deflection of the spot of light when the dark mica alone is interposed gives the effect of the total heat ray, and the lessened deflection when the glass under test is also interposed is a measure of the heat it cuts off. By dividing the scale divisions traversed by the luminous index when both glass and mica are in the path of the heat ray by the number of scale divisions traversed when the mica alone is interposed, the result gives the amount of heat obstructed.

Addition of Absorbing Media to the Soda Flux.

The first point to be settled is the effect of dissolving various metallic oxides by fusion in the clear colourless glass. The metal is added in the form of oxide, nitrate, or other salt, according to which is easiest to obtain pure. Unless oxidation of other ingredients is to be avoided the nitrate is preferred, as the copious liberation of gas during the fusion stirs up the fused mixture and assists in making it homogeneous in a much shorter time.

* 'Phil. Trans. Roy. Soc.,' 1876, Part I., p. 301 (the Bakerian Lecture), and 'Roy. Soc. Proc.,' vol. xxv., p. 305.

To be generally useful, it is desirable to obtain a glass which will absorb rays of longer wave-length than about $\lambda 7200$, and so cut off dark heat radiation. It should also be opaque to wave-lengths shorter than about $\lambda 3550$, thus cutting off the most chemically active rays, and also those which give rise to ionisation, *i.e.*, cause the air through which they pass to conduct electricity.

Working in a vacuum and with sensitive plates of emulsion containing no gelatine, Dr. SCHUMANN succeeded in photographing ultra-violet rays as short as $\lambda 1000$. In a paper recently read before the French Physical Society by MM. KARL STOCKHAUSEN and FRITZ SCHANZ, it is stated that the harmful action of light on the eye is due to the ultra-violet rays. It is also shown that the cornea is opaque to rays shorter than $\lambda 3200$.

The crystalline lens is opaque to rays shorter than $\lambda 3500$, and rays of longer wave-lengths than this reach the retina. As the transparency of ordinary spectacle glasses is limited to $\lambda 3000$, it follows that a considerable amount of ultra-violet radiation may reach the cornea through ordinary spectacles.

Method of Testing Glass.

Single metals were at first tried in varying quantities to see if from the colour and properties communicated to the glass they were worth further examination. Each specimen is cut and polished into a plate 2 mm. thick. The plate so prepared is first tested in the spectrum apparatus to ascertain the upper limit of transmission of the ultra-violet rays. It is next put into the radiometer balance to see the percentage of heat cut off, then tested in CHAPMAN JONES's opacity balance* to see the percentage of luminous rays transmitted, and finally the colour is registered in a LOVIBOND's tintometer.†

A large proportion of the known metallic elements were tested in this manner, and a considerable number were proved to be unsuitable. After experiments extending over several months the following elements were selected as likely to be worthy of further experimentation by combining the metals two, three, or four at a time in one glass so as to enable the advantages of one to make up for the shortcomings in another :—

Cerium.	Manganese.
Chromium.	Neodymium.
Cobalt.	Nickel.
Copper.	Praseodymium.
Iron.	Uranium.
Lead.	

* "An Opacity Balance," by CHAPMAN JONES, 'The Photographic Journal,' vol. xxiii., p. 99.

† The tintometer is an instrument devised by Mr. Lovibond (Messrs. Gallenkamp and Co.). Any colour can be matched by a combination of three sets of glasses, coloured respectively red, yellow, and blue, and numbered in order of depth of colour, increasing with the magnitude of the numbers.

I will now take the metals selected for further trials, and give the results of the preliminary test of the glasses so as to ascertain their behaviour in the four instruments above described.

Cerium.

One of the most important additions to soda flux is ceria, which gives a practically colourless glass. Cerium nitrate was generally used, and occasionally cerium borate and ceric oxide. Trial glasses were made, the proportion of metal varying from 1 per cent. to 7.5 per cent. The conclusion arrived at on tabulating and considering the results shown by this series of glasses, is that cerium is of value in cutting off the ultra-violet rays. The glasses are very slightly coloured, and allow nearly all the luminous rays to pass. The heat absorption is about 30 per cent., and does not vary much with the amount of cerium present.

Chromium.

This metal, in quantities of less than 1 per cent. in the glass, exerts a strong action on the ultra-violet rays, cutting them off down to the blue (λ 4550). In larger proportions, either singly or mixed with other metals, the absorption extends as far as λ 5600 (about the middle of the green). Its heat obstructing power is not on a par with that of uranium, being about 30 per cent. for 1 per cent. of chromium metal. The luminous rays transmitted by chromium glass containing 0.85 per cent. of metal are 37 per cent. of the total light, the colour of the glass being green.

Cobalt and Nickel.

Cobalt colours glass a rich blue, and then transmits ultra-violet rays of shorter wave-length than about λ 3200. It cuts off 40 per cent. of the heat, and unless in very small quantity obstructs too much light to be of use. Nickel colours glass brown. In glass its absorption of ultra-violet light is about the same as cobalt. It obstructs a little more heat and is more transparent to light. These two metals separately are of no use for the present purpose, but united they have the valuable property of neutralising each other's colour and giving the glass a neutral grey tint.

It is noteworthy that the colours of nickel and cobalt in aqueous solution are green and pink, whilst in solid solution in glass they are brown and blue, in each case complementary to one another.

Solutions of nickel and cobalt sulphates, containing 5 gr. to 100 c.c. of water, mixed together in the proportion of 2.5 c.c. Ni to 1 c.c. of Co, gave a mixture of a neutral grey colour. The mixture was divided into two parts; one was gradually heated to the boiling-point, while the other was left for comparison at the temperature of the laboratory (16° C.). Compared with the cold solution, the one at the boiling point

was decidedly pink, and it required a further addition of nickel solution to restore the neutrality of colour at the boiling-point, raising the proportion of nickel to cobalt 3·5 to 1 at 100° C. against 2·5 to 1 at 16° C. As the hot solution cooled the neutral tint gradually changed until it became decidedly green.

The same mixed solution, neutral tinted in the cold, was acidified with sulphuric acid. It immediately assumed a very faint tinge of pink, but not so decided a tint as the same neutral coloured solution took when heated to the boiling-point.

A solution was prepared containing nickel and cobalt in the proportion of 2·5 to 1; it was evaporated to dryness and ignited. The mixed oxides were then added to the hard soda flux, and the whole melted together at a high temperature; the glass resulting was cut and polished into a plate 2 mm. thick. It was decidedly blue, although the metals were in the proportion to give neutrality at the ordinary temperature when in aqueous solution. More nickel was added in small proportions at a time, and it was not until the proportion in the glass was 1 cobalt to 5 nickel that a neutral grey glass was obtained.

As a colouring agent cobalt is stronger than nickel. It is not easy to get an exact proportion, as the neutral point is difficult to hit with accuracy. If 4 of nickel instead of 5 are used with 1 of cobalt the glass is of a decided bluish tint, while if 6 of nickel are added the colour is brown.

Copper.

Copper by itself as a constituent of glass is not of much advantage. It colours the glass blue, has not much action on the ultra-violet rays, but cuts off three-fourths of the heat rays. It forms a useful addition to other colouring agents in tending to neutralise those of the orange-yellow colour.

Iron (FeO).

Iron is introduced into the glass as ferrous sulphate, care being taken to avoid all oxidising agents. The fused mass is stirred with a carbon rod, and a little powdered charcoal added to the melt in the crucible. In quantities from 1 to 2·5 per cent. the obstruction to heat radiation is great, and increases with the quantity of metal present. One per cent. of iron cuts off about 65 per cent. of the heat, and 2·3 per cent. cuts off about 89 per cent. The action on the ultra-violet end is but slight, the photographs extending to λ 3467 with the lowest amount of iron, and to λ 3560 with the largest amount experimented with. The light transmitted by the 2 mm. plate is 71 per cent. with the least amount of iron and 50 per cent. with the largest amount. The colour of the glass is greenish blue. Iron in the ferrous state, therefore, will prove useful on account of its communicating adiabatic property to the glass.

Iron (Fe₂O₃).

In this state of oxidation iron glass cuts off ultra-violet light to a limited extent. When small proportions only are present, such as 0·25 per cent., the rays are transmitted from λ 3500 (far in the ultra-violet), and it is only when the amount of iron in the per state rises to about 2 per cent. that the glass becomes opaque to the rays near λ 4000 (about the limit of visibility in the violet). A glass of this composition cuts off about 63 per cent. of the total heat. The colour is almost pure yellow. The glass transmits about 75 per cent. of the incident light. Iron in the per state, therefore, is another metal useful in combination.

Lead.

A plate 2 mm. thick was cut from a block of FARADAY'S "heavy glass" (boro-silicate of lead) prepared by himself, and tested as a sample of lead glass. It is practically colourless and transparent, and is opaque to the ultra-violet above λ 3800. Its action on the heat rays is slight, only cutting off about 38·5 per cent.

Manganese.

Glass containing manganese is of a reddish purple colour. In respect to obstruction to the ultra-violet and heat rays manganese has no special action. It has, however, been experimented with to obtain a neutral coloured glass by adding it to glass containing a greenish colouring agent.

Neodymium and Praseodymium.

These two bodies would be useful in the quest for a suitable glass were they to be obtained at a price which would not be prohibitive. In aqueous solutions praseodymium salts are greenish yellow, and those of neodymium are of a violet-rose colour. Mixed together in the proportion of five parts of praseodymium to one of neodymium the mixture is of a neutral grey. In solid solution in glass the colours are—praseodymium greenish yellow, and neodymium lilac. Melted together in glass in the same proportion as in aqueous solution, the resulting colour is also neutral grey. In this respect these two elements differ from nickel and cobalt, inasmuch as the colours remain constant either in aqueous solution or melted in glass, and the proportion required to obtain neutrality of tint appears to be the same in each case.

Uranium.

Glasses were prepared containing from half a per cent. of uranium to over 4 per cent. The colour of the glasses with smallest quantity of metal is very faint

brown, and with the highest proportion it is yellowish brown. The opacity for ultra-violet light increases as the glass is richer in metal, the one with about 4 per cent. uranium being opaque to the indigo and violet down to the blue. The heat absorbed at most is about 55 per cent. These results show that uranium is a metal likely to be useful in combination.

Composition of Glasses Specially Selected for Practical Use.

Whilst bearing in mind that the chief object of this research is to find a glass that will cut off as much as possible of the heat radiation, I have also attacked the problem from the ultra-violet and the transparency points of view. Taking each of these desiderata by itself I have succeeded in preparing glasses which cut off over 90 per cent. of heat radiation, which are opaque to the invisible ultra-violet rays, and are sufficiently free from colour to be scarcely noticeable when used as spectacles. But I have not been able to combine in one specimen of glass these three desiderata in the highest degree. The ideal glass which will transmit all the colours of the spectrum cutting off the invisible rays at each end, is still to be discovered.

As far as transparency, however, is concerned it will not be an unmixed advantage for the sought-for glass to be quite clear and colourless. The glare of a strong light on white cliffs, expanses of snow, electric light, &c., is known to be injurious to the eye, and therefore a tinted glass combining good obstruction to the heat radiation and ultra-violet rays is the best to aim for.

Grey or neutral tints are the most pleasant to wear. They do not appreciably alter the natural colours of objects, and are a great relief to the eye. Many glasses are met with in commerce of different colours which are found by experience to suit the public demand for tinted spectacles. They are of various tints of yellow, green, blue, and neutral, and therefore I do not think it will be wrong to select the tints of my glasses, suitable in other respects, no darker than those which appear to suit the public taste.

As a basis for the preparation of the coloured glasses, Mr. Powell prepared for me a quantity of hard soda-flux mixture of the following composition :—

Sand	61·00
Sodium carbonate, anhydrous	25·50
Sodium nitrate, recrystallised	5·00
Calcium carbonate, precipitated	7·20
Borax	0·75
Arsenic trioxide	0·55
	100·00

This mixture whilst melting loses about 25 per cent. in weight. When first melted, it is colourless; on second melting it acquires the usual faint greenish tint of soda-

lime glass. The contents of a large potful of the melted flux was poured into cold water. The broken-up mass was sent to my laboratory and has been used in the preparation of the test glasses. The object of pouring the molten flux into water is to break it up and render it easy to powder for convenience of adding other ingredients. In this state I call it "Fused Soda Flux."

Sometimes it is found advisable to use the flux in its raw state without previous melting, and when working upon the large scale this is the best plan. I then call it "Raw Soda Flux."

Without counting numerous preliminary experiments, I have made and fully tested over 300 tinted glasses, the quantitative composition of each being known. From these glasses I have selected a certain number which possess valuable qualities in respect of athermancy, adiactinity, and transparency.

I will now give the composition of these glasses, and the special properties in respect to the desired results.

Glass 150.

Fused soda flux	90.00
Cerium borate	8.13
Nickel sulphate, crystallised	0.07
Uranoso-uranic oxide.	1.80
	100.00

In Glass 150 a small amount of nickel has been added to the cerium and uranium. The colour is pale yellow, and it is opaque to ultra-violet radiation, the limit being λ 3613. It cuts off 37 per cent. of the heat radiation and transmits 73 per cent. of the incident light. The tintometer numbers are :—Red, 4.0; yellow, 3.5; blue, 0.5.

Glass 158.

Fused soda flux.	89.75
Cerium borate	8.18
Ferroso-ferric oxide	2.03
Chromic oxide	0.09
	100.00

This glass is pale greenish yellow. It is quite opaque to all the ultra-violet rays, the limit being λ 3700. It cuts off 63 per cent. of heat radiation, and transmits 54 per cent. of the light.

The glass has a pale greenish yellow colour. Its tintometer numbers are :—Yellow, 2.75; blue, 3.5.

Glass 165.

Raw soda flux	87·56
Cerium borate	8·00
Ferrous sulphate, crystallised	3·00
Uranic oxide	0·55
Nickel oxide.	0·09
Chromic oxide	0·80
	100·00

This glass cuts off practically all the ultra-violet rays shorter than λ 3680, and 38 per cent. of the heat radiation. It transmits 48 per cent. of the light. Its colour is a pale yellowish green. Its tintometer numbers are :—Yellow, 4·0 ; blue, 2·50.

Glass 187.

This glass and the next are both cerium glasses. No. 187 is composed of:—

Fused soda flux.	83·0
Cerium nitrate, crystallised.	17·0
	100·0

The heat rays cut off by this glass are only 27 per cent., but its other qualifications are important. It is practically opaque to ultra-violet radiation, the limit being λ 3650, and it transmits 99 per cent. of the light.

In the tintometer and opacity meter no colour or want of transparency can be detected, although against white paper in a good light a faint tinge of yellow is perceptible.

Glass 197.

Fused soda flux	79·00
Cerium nitrate, crystallised.	20·50
Nickel sulphate, crystallised	0·30
Cobalt sulphate, crystallised	0·05
Uranoso-uranic oxide	0·15
	100·00

Nickel and cobalt are here mixed in the proportion to make a neutral tinted glass, the other ingredients only slightly modifying the colour. The colour is a pale neutral. It is opaque to ultra-violet rays of shorter wave-length than λ 3799, and cuts off 41 per cent. of the heat rays. It is transparent to 45 per cent. of the incident light. The numbers in the tintometer are :—Red, 2·00 ; yellow, 3·00 ; blue, 5·50.

Glass 202.

I have obtained a neutral tinted glass by neutralising the orange-yellow colour communicated to glass by iron in the ferric state, by adding to it a little cobalt, which colours it blue. The composition is:—

Soda flux.	95·15
Fe ₂ O ₃	4·75
CoSO ₄ , 7H ₂ O	0·10
	100·00

The resulting glass is opaque to ultra-violet of shorter wave-length than λ 3830, it cuts off 83 per cent. of the heat radiation, and transmits 25 per cent. of light. The tintometer numbers are :—Yellow, 2·5 ; blue, 5·5.

Glass 210.

Fused soda flux	89·0
Ferrous sulphate, crystallised	8·9
Chromic oxide	1·3
Carbon, in fine powder.	0·8
	100·0

Glass 210 is of a bluish-green colour, the blue colour communicated by the ferrous oxide making the colour of the chromium not so pure a green. It cuts off all ultra-violet rays of shorter wave-length than λ 3620. It obstructs 87 per cent. of the heat radiation, and transmits 30 per cent. of light. The tintometer numbers are :—Yellow, 2·0 ; blue, 16·0.

Glass 217.

A further attempt was now made to get the iron in the ferrous state, and a glass was prepared of the following composition :—

Fused soda flux	96·80
Ferroso-ferric oxide	2·85
Carbon	0·35
	100·00

A few percentages of finely powdered carbon are added to this mixture before fusion to keep the iron in the proto-state. Any excess of carbon rises to the top of the fused mass and prevents oxidation of the iron. This glass is of a pale blue colour, and cuts off the ultra-violet rays shorter than λ 3550. It cuts off 96 per cent. of the heat

radiation, and transmits 40 per cent. of light. Spectacles made of this glass are very pleasant. The glass is bluish with a tinge of green. In the tintometer the numbers are :—Yellow, 2·0 ; blue, 8·0.

Glass 221.

Fused soda flux	80·0
Cerium nitrate, crystallised	13·4
Uranoso-uranic oxide	6·6
	100·0

In this glass the action of uranium on the ultra-violet rays is added to that of cerium. The colour is faint yellow. It practically cuts off all the ultra-violet radiations, the limit being λ 3685, and it also cuts off 39 per cent. of the heat rays. Its transparency is 60 per cent. In the tintometer the number is :—Yellow, 8·0.

Glass 238.

Raw soda flux	77·0
Cerium nitrate, crystallised	23·0
	100·0

Glass 238 is similar in composition to Glass 187, but it contains more cerium. Like 187 it is practically opaque to ultra-violet light, the limit being about λ 3610, the injurious rays being beyond this wave-length. It cuts off 34 per cent. of the heat, and transmits 71 per cent. of the luminous rays. In the tintometer the numbers are :—Red, 0·375 ; yellow, 0·375 ; blue, 1·500.

Glass 240.

Further experiments with biotite showed that its special property was due to the iron protoxide it contained, and another glass was therefore made in which the iron could be introduced in the form of a definite salt of known composition. Ferrous oxalate was chosen, the proportions being as follows :—

Raw soda flux	90
Ferrous oxalate, $\text{FeC}_2\text{O}_4 \cdot 2\text{H}_2\text{O}$	10
	100

The glass made from this material does not cut off quite so much of the heat rays as the one prepared from biotite, the rays obstructed being 88 as against 94 per cent. But the ultra-violet is entirely intercepted, the limit being λ 3950. It transmits 36 per cent. of the luminous rays. It is of a smoky green colour. The tintometer numbers are :—Yellow, 4·0 ; blue, 4·0.

Glass 246.

This is a glass made on a larger scale at the Whitefriars Glass Works, and of the same composition to start with as Glass 240, namely, 10 per cent. of ferrous oxalate with raw soda flux. At the suggestion of Mr. Harry Powell a small quantity of red tartar and powdered wood charcoal was added to prevent oxidation; the resulting glass is sage-green in colour, a plate 2 mm. in thickness cuts off ultra-violet rays down to λ 3800, its opacity to heat radiation is 98 per cent., and it transmits 27·6 of the incident light. The tintometer numbers are :—Yellow, 5·5 ; blue, 11·0.

A plate of this glass 1 mm. thick cuts off the ultra-violet rays down to λ 3550, its opacity to heat radiation is 83 per cent., and it transmits 47·9 per cent. of the incident light. The tintometer numbers are :—Yellow, 2·25 ; blue, 5·0.

Glass 247.

Raw soda flux	92·00
Cerium borate	6·30
Nickel oxide	0·04
Ferric oxide	1·60
Chromic oxide	0·06
	100·00

Glass 247 has a faint green tint and is practically opaque to the ultra-violet rays, the limit being λ 3620. It obstructs 29 per cent. of the heat radiation, and transmits 71 per cent. of the luminous rays. The tintometer numbers are :—Yellow, 2·0 ; blue, 2·0.

Glass 248.

Fused soda flux	94·60
Cerium nitrate, crystallised	4·72
Uranic oxide	0·30
Nickel oxide	0·30
Cobalt sulphate, crystallised	0·08
	100·00

Glass 248 is similar in composition to the one described in (197), p. 18, but is of a darker neutral tint. It is opaque to ultra-violet rays shorter than λ 3550, and cuts off 47 per cent. of the heat radiation. It transmits 30 per cent. of the incident light. Its tintometer numbers are :—Red, 2·0 ; yellow, 2·5 ; blue, 7·5.

<i>Glass 249.</i>	
Fused soda flux	88·47
Ferric oxide	1·50
Cobalt sulphate, crystallised	0·03
Cerium nitrate, crystallised	10·00
	100·00

Glass 249 is of a pale blue tint almost opaque to the ultra-violet rays, the limit of wave-length being λ 3550 ; to all rays of shorter wave-length it is opaque. It cuts off 51 per cent. of the heat radiation, and transmits 63 per cent. of the luminous rays. Its tintometer numbers are :—Yellow, 0·5 ; blue, 2·0.

<i>Glass 250.</i>	
Raw soda flux	88·00
Cerium borate	5·00
Ferrous sulphate, crystallised	4·15
Uranoso-uranic oxide	2·75
Chromic oxide	0·10
	100·00

Glass 250 has a yellow colour with a faint tinge of green. It cuts off ultra-violet light, being opaque to radiation shorter than λ 3685. It only obstructs 25 per cent. of the heat rays, and is transparent to the extent of 74 per cent. In the tintometer the numbers are :—Yellow, 7·00 ; blue, 1·25.

<i>Glass 251.</i>	
Raw soda flux	92·0
Ferrous sulphate, crystallised	8·0
	100·0

During the fusion the iron is partially peroxidised. Glass 251 is of a faint yellow colour ; it is opaque to ultra-violet rays shorter than λ 3550, it obstructs 37 per cent. of the heat radiation, and it transmits 89 per cent. of the incident light. Its tintometer number is :—Yellow, 1·25.

<i>Glass 252.</i>	
Raw soda flux	72·60
Cerium nitrate, crystallised	24·90
Copper sulphate, crystallised	2·10
Nickel oxide	0·40
	100·00

The colour of Glass 252 is faint bluish green. In it copper is introduced to counter-act the yellow-brown tint given by the cerium and nickel, and to obtain a tint approaching neutral. It is opaque to ultra-violet rays of shorter wave-length than λ 3680, cuts off 47 per cent. of the heat radiation, and transmits 45 per cent. of light. The tintometer numbers are :—Yellow, 2·0 ; blue, 4·5.

Glass 253.

I have already explained that black mica (biotite) is almost perfectly transparent to heat radiation, and at the same time opaque to the luminous rays. I thought it would be interesting to see what would be the effect of melting up some pieces of highly diathermanous biotite with soda flux. After many experiments it was found that a glass of a neutral tint could be made by melting biotite at a high temperature and fusing the result with flux. The proportions are—

Raw soda flux	88·5
Black biotite, fused	11·5
	100·0

On testing the resulting glass it was found that it had remarkable properties in respect to the heat rays, but in exactly the opposite way to what was expected. It offers an almost complete obstruction to the invisible heat rays, and it cuts off 94 per cent. of the heat radiation, and allows 30 per cent. of the incident light to pass through. It is opaque to ultra-violet rays of shorter wave-length than λ 3610. It is of a sage-green colour, and its tintometer numbers are :—Yellow, 4·0 ; and blue, 7·0.

Discussion of the Foregoing Results.

I have already said that the progress of this research has widened since I commenced investigations three years ago. Then my object was to find a glass which would cut off the heat so as to preserve the eyes of those engaged in glass works. I soon found it difficult, even if it were advisable, to confine the research to the action on heat rays alone. Thus the glasses now described include specimens suitable for spectacles adapted to all requirements—from Eyes of Youth to Eyes of Age.

The first necessity therefore is to find a glass which will cut off as much as possible of the heat radiation. Glass 246—sage-green in colour—is almost perfect in this respect, as it cuts off 98 per cent. of the heat. Glass 217—of a pale blue tint—is opaque to 96 per cent. of heat radiation. Next comes Glass 253—of a neutral tint—which cuts off 94 per cent., and then comes Glass 240—of a neutral tint—cutting off 88 per cent., Glass 210—of a green tint—cutting off 87 per cent. heat radiation, and Glass 158—of a pale greenish yellow—cutting off 63 per cent. of the heat radiation.

If more light is desired to enable the workers to see better at the expense of a little athermaney, Glass 249 is commendable. It is a very pale blue and transmits 63 per cent. of luminous rays while cutting off 51 per cent. of the heat rays. Spectacles of this glass scarcely appear to obstruct light at all, and the colours of objects are practically unchanged.

For ordinary use, when no special protection against heat radiation is needed, the choice will rest on whether the ultra-violet or the luminous are most to be guarded against; or whether the two together are to be toned down. To begin with I will take into consideration glasses most effective in cutting off ultra-violet rays. Ordinarily the visible spectrum is assumed to end at the Fraunhofer line K, λ 3933, but light can easily be distinguished some distance beyond by the naked eye. For instance, about fifty years ago, with a quartz spectroscope with solar rays, I could see L, λ 3820, and M, λ 3727, though now I cannot see above K. It may therefore be considered that the ultra-violet rays which are to be cut off on account of their probable injurious action are those of shorter wave-lengths than, say, λ 3700. The most effective glasses for this purpose are 158, 150, 240, 246, 202, and 197, all of which are opaque to rays shorter than 3700. The colours are pale green, yellow and neutral, they transmit ample light so that a choice of tints is available to suit individual taste.

If much transparency is required there is a choice between glasses 187, 150, 251, 250, 247, and 238, which transmit from 99.5 to 70 per cent. of the incident light. The choice between this range of glasses will depend on the conditions required, *i.e.*, on the absolute transparency or the colour. The colours are pale tint of yellow, green, and neutral.

If only a moderate degree of transparency is desired Glasses 158, 249, and 221 may be selected; the light transmitted ranges from 70 to 60 per cent., and the tints are of pleasant green, blue, and orange.

When glasses are required which are restful to the eyes in the glare of the sun on chalk cliffs, expanses of snow, or reflected from the sea, Glasses 249, 197, 252, 165, 210, and 248 are most suitable, the tints being yellow, green, and neutral. Moreover, they have the advantage of cutting off practically all the ultra-violet rays and also a considerable amount of the heat radiation.

For convenience of reference all the glasses above described are arranged in the following four tables:—

- I. Absorption of Heat Rays.
- II. Absorption of Ultra-violet Rays.
- III. Transmission of Luminous Rays.
- IV. Reduction of Glare.

This latter property is the most valuable for general use in brilliant light.

TABLE I.—Absorption of Heat Rays.

Glass No. 246 cuts off 98 per cent.

„	217	„	96	„
„	253	„	94	„
„	240	„	88	„
„	210	„	87	„
„	158	„	63	„

TABLE II.—Absorption of Ultra-violet Rays.

Glass No. 240 opaque to rays of shorter wave-length than 3950.

„	202	„	„	„	3830.
„	246	„	„	„	3800.
„	197	„	„	„	3800.
„	158	„	„	„	3700.
„	221	„	„	„	3685.
„	150	„	„	„	3620.

TABLE III.—Transmission of Luminous Rays.

Glass No. 187 transmits 99 per cent.

„	251	„	89	„
„	250	„	74	„
„	150	„	73	„
„	247	„	71	„
„	238	„	71	„

TABLE IV.—Reduction of Glare.

Glass No.	Absorption of heat.	Absorption of ultra-violet.	Transmission of light.	Colour.
	per cent.		per cent.	
249	51	3550	63	Pale blue.
197	41	3800	45	Pale neutral.
252	47	3680	45	Faint blue-green.
165	38	3680	42	Pale yellow-green.
210	87	3620	30	Blue-green.
248	47	3550	30	Dark neutral.

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By F. E. SMITH, *A.R.C.Sc.*

(From the National Physical Laboratory.)

Communicated by R. T. GLAZEBROOK, *M.A., F.R.S.*

Received October 27,—Read December 11, 1913.

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HISTORICAL INTRODUCTION.

THERE are many methods by which a resistance can be measured absolutely in the electromagnetic system of units, and all of these necessarily involve absolute measurements of a length and of a time. The length may be the axial length or radius of a coil, or the radius of a disc, or it may involve all of these, and the time may be the time of vibration of a magnet, or of a rotation of a coil, or of the period of an alternating current. In any case, the precision secured in the measurement of a resistance depends primarily on the accuracy obtained in these measurements of length and of time.

The first absolute measurements* of a resistance were made by KIRCHHOFF† in 1849, but it is to W. WEBER‡ that we owe the first distinct proposal (in 1851) of a definite system of electrical measurements according to which resistance can be measured in terms of an absolute velocity.

WEBER devised three methods by which the resistance of a wire can be determined absolutely, and he published the results of experiments by two of these. The first was by means of an earth inductor, and the second by observing the damping of a swinging magnet, the results obtained differing among themselves by 5 parts in 1900. In 1853§ he made a determination of the specific resistance of copper, but the experiments were made more to develop the methods than for exact measurements.

In 1862 WEBER|| made a more exact determination of resistance using a method compounded of his first two methods and eliminating the constant of the galvanometer. The results of these experiments were embodied in a determination of the value of the Siemens unit and of a standard coil sent to him by Sir WILLIAM THOMSON, but the unit obtained was about 8 per cent. less than the 1863 unit of the British Association Committee on Electrical Standards.¶

The measurements made by MAXWELL, FLEEMING JENKIN, and BALFOUR STEWART for the British Association Committee, although subsequently found to be incorrect by nearly 1·5 per cent., are the first with any claim to precision. The method adopted by these experimenters is that of the rotating coil and was devised by Prof. THOMSON (later Lord KELVIN) independently, we believe, of a prior suggestion by W. WEBER. The apparatus consists of a short-circuited coil rotating about a

* Admirable summaries and criticisms of various methods will be found in WIEDEMANN'S 'Electricität,' vol. IV., p. 910; MASCART and JOUBERT'S 'Leçons sur l'Electricité,' II., p. 581; Lord RAYLEIGH, 'Phil. Mag.,' 1882; WIEDEMANN, 'Phil. Mag.,' 1882; GLAZEBROOK, 'Electrician,' 1890; DORN, 'Wissenschaftliche Abhand. der Phys. Techn. Reichsanstalt,' vol. II., p. 357, 1895.

† "Bestimmung der Constanten von welcher die Intensität inducirter elektrischer Ströme abhängt," 'Pogg. Ann.,' Bd. 76, S. 412.

‡ 'Elektrodynamische Maassbestimmungen,' or 'Pogg. Ann.,' Bd. 82, S. 337.

§ 'Abh. d. Kön. Ges. d. Wissenschaften zu Göttingen,' Bd. 5.

|| 'Zur Galvanometrie,' Göttingen, 1862.

¶ 'Reports of B.A. Electrical Standards Committee,' 1863.

vertical axis in the earth's magnetic field. Currents are induced in the coil and these produce a deflection of a small magnet suspended at the centre. The dimensions of the coil and the time of its rotation being known, the resistance of the wire of the coil can be calculated. This method was also used by Lord RAYLEIGH and Prof. SCHUSTER* in 1881, by Lord RAYLEIGH† in 1882, and by W. WEBER‡ also in 1882. The results are given in Table I.

In the method of the earth inductor due to W. WEBER a coil is mounted on a vertical axis with the mean diametral plane in the magnetic meridian. In circuit with the coil is a ballistic galvanometer through which a quantity of electricity flows when the coil is quickly turned through half a revolution. The resistance of the whole circuit can be calculated from the dimensions of the coils, the constants of the galvanometer, and the deflection produced by the half rotation of the coil. Methods based on this inductor principle have been used by F. KOHLRAUSCH and G. WIEDEMANN.§

In a third method used by KIRCHHOFF, two coils, between which there is a mutual inductance, are joined up in series with a battery and galvanometer, and a resistance, R , joins the junction of the two coils to a point on the circuit between the galvanometer and battery. The steady current deflection of the galvanometer is first observed and then the throw due to one of the coils being removed to a position in which the mutual inductance is zero. The dimensions of the coils and the constants of the galvanometer enable the resistance R to be calculated. Methods based on this inductive principle have been used by ROWLAND,|| GLAZEBROOK,¶ and MASCART, DE NERVILLE, and BENOIT.** Many of the developments of the method are of extreme importance, and, except for the essential principle being the same, there is little in common. Similar remarks apply also to the developments of other methods.

The method adopted by RÔITI†† and HIMSTEDT‡‡ is somewhat similar to that of KIRCHHOFF, but, instead of having to observe the deflection due to a single impulse, a constant deflection due to a series of impulses is obtained. The current through one of the fixed coils is made and broken n times per second, and the galvanometer circuit is made only on the make or break of the current. The method adopted by GUILLÉT§§ belongs in part to this class and in part to the method suggested by LIPPMANN.

* RAYLEIGH and SCHUSTER, 'Roy. Soc. Proc.,' vol. 32, 1881.

† RAYLEIGH, 'Phil. Trans.,' vol. 173, 1882.

‡ WEBER, 'Der Rotationsinduktor,' 1882.

§ G. WIEDEMANN, 'Abh. der Berl. Ak.,' 1884.

|| ROWLAND, 1876, 'Physical Papers,' pp. 145-239; The Johns Hopkins Press, 1902.

¶ GLAZEBROOK, 'Phil. Trans.,' vol. 174, p. 223, 1883.

** MASCART, DE NERVILLE, and BENOIT, 'Ann. de Chemie et de phys.,' VI., p. 1, 1885.

†† RÔITI, 'Nuovo Cimento,' III., 15, 1884.

‡‡ HIMSTEDT, 'Berichte der Naturforschenden Ges. zu Freiburg i. B.,' Heft 1, 1886.

§§ GUILLÉT, 'Journ. de Physique,' 8, pp. 471-477, 1899.

Another method used by WEBER is commonly called the method of damping. A magnet is suspended within a coil and set in oscillation (*a*) when the circuit is open, and (*b*) when the circuit is closed. The periods and logarithmic decrements are observed, and from a comparison of the results the resistance of the coil can be calculated. This method was used by WEBER and with important modifications by DORN,* WILD,† and KOHLRAUSCH.‡

In the method due to LORENZ§ a metallic disc is rotated at a constant rate in a magnetic field produced by a current which circulates through a coil co-axial with the disc. The disc is touched at its circumference and centre by two wires, and the difference of potential is balanced against that at the extremities of a resistance, *R*, the current through which is the same as that circulating through the coil. When the mutual inductance of the coil and disc circumference, and the rate of rotation of the disc are known, the resistance *R* can be calculated. Methods based on this principle have been used by LORENZ,|| Lord RAYLEIGH and Mrs. SIDGWICK,¶ ROWLAND and KIMBALL,** DUNCAN, WILKES, and HUTCHINSON,†† JONES,‡‡ and AYRTON and JONES.§§

FOSTER,|||| and afterwards LIPPMANN,¶¶ suggested the use of a rotating coil, but contact with the extremities of the coil was made only at the moment when the induced voltage was a maximum. The induced voltage is balanced by that due to an external current (which may produce the field in which the coil rotates) through a known resistance. Observations by a method based on this principle have been carried out by LIPPMANN and WUILLEUMIER.***

The most recent determination of a resistance in absolute measure is due to A. CAMPBELL.††† In CAMPBELL's experiments two very nearly equal alternating currents in quadrature, taken from a two-phase alternator of sine-wave voltage, are passed through a resistance *R* and the primary circuit of a variable mutual inductance respectively. The ratio of these two currents is measured by passing them through equal resistances and comparing the deflections on an electrostatic

* DORN, 'WIED. Ann.,' 17, 1882, and 36, 1889.

† WILD, 'Mém. de l'Ac. des Sc. St. Petersburg,' tome 32, Nro. 2, 1884.

‡ KOHLRAUSCH, 'Abh. der bayr. Ak. d. W.,' Bd. 16, 1888.

§ LORENZ, 'POGG. Ann.,' 149, p. 251, 1873.

|| LORENZ, 'WIED. Ann.,' 25, p. 1, 1885.

¶ Lord RAYLEIGH and Mrs. SIDGWICK, 'Phil. Trans.,' vol. 174, p. 295, 1883.

** ROWLAND and KIMBALL, 'La Lumière Electrique,' vol. 26, pp. 188, 189, 477, 1887.

†† DUNCAN, WILKES, and HUTCHINSON, 'Phil. Mag.,' p. 98, 1889.

‡‡ JONES, 'Electrician,' p. 552, 1890. Also 'B.A. Electrical Standards Reports,' 1893, 1894.

§§ AYRTON and JONES, 'B.A. Electrical Standards Reports,' 1897.

|||| G. CAREY FOSTER, 'B.A. Electrical Standards Reports,' 1870.

¶¶ LIPPMANN, 'Comptes Rendus,' 95, p. 1348, 1882.

*** WUILLEUMIER, 'Journal de Physique,' 11, 9, p. 220, 1890.

††† CAMPBELL, 'Roy. Soc. Proc.,' 87, 1912.

voltmeter placed across either resistance. The voltage across the resistance R is balanced (by the help of a tuned vibration galvanometer) against the voltage induced in the secondary circuit of the mutual inductance. The value of the variable mutual inductance is found by comparison with a fixed mutual inductance whose value is calculated from its dimensions, and the resistance R is determined in terms of this inductance and the frequency of the alternating current.

Table I. gives the principal results, the values given in columns 5, 6, and 7 being those given by the experimenters. It will be seen that in some cases mercury standards of resistance were available, and in other cases the results are given in terms of the British Association unit (B.A. unit) or the Siemens unit. When mercury standards of resistance were available, the results (previous to 1892) state the length at 0° C. of a column of mercury having a uniform cross-section of 1 sq. mm. and a resistance of 1 ohm. The Siemens unit of resistance is the resistance at 0° C. of a column of mercury 100 cm. in length and 1 sq. mm. in cross-section; results which give the absolute value of the Siemens unit may therefore be reduced to give the length representing 1 ohm by taking the reciprocal of the absolute value and multiplying by 100.

In 1892, and again in 1908, the international ohm was defined as the resistance of a specified column of mercury. In 1892 Dr. VON HELMHOLTZ pointed out that a difficulty arose in determining the cross-section of a column of mercury owing to there being some doubt as to the correct value for its density. He suggested that the difficulty should be avoided by stating the mass of a mercury column of a given length which has a resistance of 1 ohm. This was agreed to and the international ohm was defined as the resistance at 0° C. of a column of mercury 14.4521 gr. in mass and having a length of 106.3 cm. The number 14.4521 is the product of 1.063 and 13.5956, the latter number representing at that time the mean of the best determinations of the density of mercury at 0° C. The cross-section of the specified column is therefore equal to 1 sq. mm. or nearly so. The ratio of the international ohm to the Siemens unit may therefore be taken as 1.063.

The B.A. unit is so much referred to in the earlier determinations that it may be useful to state clearly what is meant by the unit. In 1864 Messrs. MATTHIESSEN and HOCKIN constructed a number of coils of various materials to represent at certain specified temperatures resistances of 10^9 cm./sec. units of resistance as determined by the 1862-3 British Association Committee on Electrical Standards. The resistances of these coils did not keep absolutely constant, and in after years the B.A. unit was taken as the mean of the values of six of these coils at the temperature at which they were stated by HOCKIN to be correct. The B.A. unit of one period is not, therefore, necessarily the same as that of another period. Every precaution was, of course, taken to ensure constancy, but with wire standards of resistance great difficulty is experienced. In after years* it proved possible to trace the changes in these coils

* 'B.A. Elec. Stands. Committee Report,' 1908.

with what appears to be a fair measure of success, and the corrections due to changes in the coils can in certain cases be calculated.

It follows, therefore, that if the B.A. unit of any particular period is known in terms of the resistance of a column of mercury, and if the coils used for the absolute measurements remained constant in resistance from the time of their measurements in terms of a mercury column to the time of their absolute measurement, the results given in Table I. can in all cases be reduced to give the length of the column of mercury having a resistance of 1 ohm.

But it is very probable that the resistance of many of the coils did not keep constant, and it is not possible for us to reduce the results except in a few cases. The instances referred to are the determinations by Lord RAYLEIGH, by Dr. GLAZEBROOK, by VIRIAMU JONES, and by AYRTON and JONES. In all of these cases comparisons were made with the B.A. standard coils, and the details of these comparisons have been preserved and published. In the Report of the B.A. Electrical Standards Committee for 1908, the changes in resistance of the coils used by Lord RAYLEIGH and Dr. GLAZEBROOK have been traced, and a comparison of the mercury standards of resistance made by Lord RAYLEIGH, Dr. GLAZEBROOK, and F. E. SMITH is given in Table VIII. of the same report. This comparison, together with the notes on the standards used, enables us to express Lord RAYLEIGH's and Dr. GLAZEBROOK's results in terms of the present mercury standards of the National Physical Laboratory. This we have done, the results being marked (S), while (A) indicates the results given by the author.

Referring first to Lord RAYLEIGH's determination in 1882, we find that comparisons were made with certain B.A. unit coils and with mercury standards of resistance. However, the terminals of the latter were not at 0° C. but between 5° C. and 6° C., and it was shown by Dr. GLAZEBROOK in 1888 that an error of 24 parts in 100,000 was introduced because of this. If we apply a correction of this amount, Lord RAYLEIGH's 1882 value of the ohm in centimetres of mercury becomes $106\cdot24(1+0\cdot00024) = 106\cdot26_5$, and the 1883 value becomes $106\cdot214(1+0\cdot00024) = 106\cdot239$. These values are given in Table I. (within 1 part in 10,000) as $106\cdot26$ and $106\cdot24$.

Dr. GLAZEBROOK's determination of the ohm was made in 1882, and he constructed mercury standards of resistance in 1888. The principal resistance coil employed in 1882 was a platinum-silver coil known as "flat," and this also was used in 1888. In the interval it was assumed to have kept constant—there was at that time no certain evidence to the contrary. A careful survey of the history of the coils, which is published in the B.A. Report for 1908, shows, however, that "flat" increased in resistance in the interval 1882–1888 by 41 parts in 100,000. Dr. GLAZEBROOK's value for the ohm in centimetres of mercury is $106\cdot29$, and this, when corrected for the change in the resistance coils, becomes $106\cdot29(1-0\cdot00041) = 106\cdot25$.

The coils used by VIRIAMU JONES in 1894 were compared with the B.A. standards

by Dr. GLAZEBROOK. One of these coils was No. 3715, and its value in 1894 was stated by Dr. GLAZEBROOK to be 1·00026₀ ohms (international) at 14°·95 C., the relation between the B.A. unit and the ohm (international) being taken as 1 ohm = 1·01358 B.A. unit.* From the results obtained at the N.P.L. in 1908 we conclude that this coil increased in resistance in the interval 1894–1908 by 7 parts in 100,000. Its value in 1908 was measured to be 1·00066 international ohms at 16°·0 C. or 1·00034 international ohms at 14°·95 C. When allowance is made for the rise in resistance of 7 parts in 100,000, it will be seen that the difference from Dr. GLAZEBROOK'S value is 1 part in 100,000. We conclude, therefore, that the value given by VIRIAMU JONES in 1894 is not in error because of any uncertainty in the values of the resistance coils used.

Similarly we have investigated the coils used by AYRTON and JONES in 1897, and we find the values in Board of Trade ohms agree with the values in international ohms within 1 or 2 parts in 100,000. We conclude, therefore, that the length of the column of mercury representing the ohm is (from their experiments) $\frac{106\cdot300}{1\cdot00026} = 106\cdot274$ cm.

Unfortunately, we are not sufficiently acquainted with the standards used by other investigators to reduce their results, and in the last column of the table the results given, except for the cases already dealt with, are those only in which mercury standards were available.

SECTION 1.—INTRODUCTORY.

The instrument described in this paper is the outcome of a desire expressed by the late Prof. J. VIRIAMU JONES at a meeting of the British Association Committee on Practical Electrical Units and Standards, in 1893. Prof. JONES expressed the hope that in the near future there might be constructed an apparatus based on the method used by LORENZ, which would be kept in constant use in a national laboratory and embody in concrete form a proper ultimate standard of electrical resistance.

In 1900 the Drapers Company of London promised to Prof. JONES the funds for the construction of such an instrument, and after Prof. JONES'S death in 1901 the Company placed £700 at the disposal of the Executive Committee of the National Physical Laboratory in order that the instrument might be made.

The apparatus was to be in memory of Prof. JONES, and to be constructed under the superintendence of the late Prof. AYRTON and of Dr. GLAZEBROOK. Delay in proceeding with the work arose owing to the construction of the Ayrton-Jones current balance, and it was not until after completion of the balance in 1907 that a start was made. Unfortunately, Prof. AYRTON was in very poor health, and although keenly interested in the work he did not live to take any part in it.

The form of apparatus eventually decided on was considerably larger than anticipated in 1893. The metal work was much too heavy for machining in the

* 'B.A. Elec. Stands. Committee Reports,' 1892 and 1894.

TABLE I.—Results of Absolute Measurements of a Resistance.

Date.	Observer.	Principle of method.	Resistance standards used.	Results given by author.			Value of ohm in cm. of mercury.
				B.A. unit in ohms.	SIEMEN'S unit (100·0 cm. of mercury) in ohms.	Ohm in cm. of mercury.	
1863	{ MAXWELL, JENKIN, and BALFOUR (B.A. Committee) }	Rotating coil	—	1·000	—	—	—
1873	LORENZ	Induced currents	?	—	0·9337	—	—
1876	ROWLAND	Rotating coil	Coils in B.A.U.	0·9878	—	—	—
1881	RAYLEIGH and SCHUSTER	"	"	0·9893	—	—	—
1882	RAYLEIGH	"	"	0·98651	—	{	106·24 (A) 106·26 (S)
1882	H. WEBER	"	"	0·9877	—	—	—
1882	DORN	Damping of magnet	Coils (SIEMENS)	—	0·9482	—	—
1882	GLAZEBROOK	Induced currents	Coils in B.A.U.	0·98665	—	{	106·29 (A) 106·25 (S)
1883	RAYLEIGH and SIDGWICK	LORENZ	B.A.U. and mercury	0·98677	—	106·214	106·214 (A) 106·24 (S)
1884	ROWLAND	Induced currents	Coils in B.A.U.	0·98627 ± 40	—	—	—
1884	ROWLAND and KIMEALL	LORENZ	B.A.U. and mercury	0·98642	—	106·32	106·32 (A)
1884	{ MASCART, DE NERVILLE, and BENOIT }	Induced currents	"	—	—	106·30	106·30 (A)
1884	RÔTI	"	{ B.A.U. and coils of SIEMENS and STRECKER } { Coils (SIEMENS and HALSKE) } Mercury	—	—	105·896	—
1884	WILD	Damping of magnet	"	—	0·94315	106·027	—
1885	LORENZ	Earth inductor	{ Coils (SIEMENS and HALSKE) } Mercury	—	—	105·93	105·93 (A)
1885	WIEDEMANN	Induced currents	{ Coils (SIEMENS and HALSKE) } Mercury	—	—	106·265	106·265 (A)
1886	HIMSTEDT	Damping of magnet	"	—	{	{	{
1888	F. KOHLRAUSCH	"	"	—	Series 1	> 106·01	—
1889	DORN	LORENZ	"	—	Series 2	< 106·16	—
1889	{ DUNCAN, WILKES, and HUTCHINSON }	Foster and Lippmann	{ Mercury and B.A.U. } Mercury	—	—	106·405	106·34 (A)
1890	WUILLEUMIER	Induced currents	"	—	—	106·274	106·243 (A)
1890	V. JONES	LORENZ	"	—	—	106·34	106·34 (A)
1892	HIMSTEDT	Induced currents	"	—	—	106·267	106·267 (A)
1894	V. JONES	LORENZ	{ Coils from GLAZEBROOK } { Coils (Board of Trade) } { Coils (CARPENTIER) } N.P.L. mercury	—	—	106·259	106·259 (A)
1897	AYRTON and JONES	"	(1 B.O.T. ohm = 1·00026 ohm)	—	—	106·326	106·326 (S)
1899	GUILLET	Induced currents	—	—	—	106·214	106·274 (S)
1912	CAMPBELL	Alternating currents	—	—	—	106·27	106·27 (A)

The results given by the experimenters are indicated by (A). In a few cases it has been possible to express the results in terms of the mercury standards of resistance of the National Physical Laboratory. These results are indicated by (S). The method of reducing the results is explained on pp. 31–33.

Laboratory workshops, and Sir ANDREW NOBLE, F.R.S., was approached with a view to this part of the instrument being made in the workshops of Sir W. G. Armstrong, Whitworth & Co. at Elswick. Sir ANDREW NOBLE not only undertook that this should be done, but took such a keen interest in the work that he generously provided it at very much less than the cost price.

In preparing the plans we had full access to the working drawings of the McGill-Lorenz apparatus made for Prof. CALLENDAR by Messrs. Nalder Bros. & Co. Although we were unable to take much advantage of this opportunity owing to the big difference in the designs of the two instruments, we desire to express our thanks to Messrs. Nalder Bros. for their kindness.

The instrument herein described possesses many new features of importance. It is permanent, but determinations of resistance made from time to time will not assume constancy of any dimensions; it is comparatively easy to use, and the results obtained are believed to be of high precision.

SECTION 2.—THE METHOD OF LORENZ.

In this method, which was first employed by LORENZ* in 1873, a rotating disc is placed in a magnetic field produced by a current which circulates through a coil coaxial with the disc. In the apparatus used by LORENZ wire brushes made contact with the disc at its circumference and centre, and the circuit was closed by connecting the brushes, through a galvanometer, with the extremities of a resistance R . The voltage induced by the uniform rotation of the disc at n revolutions per second is Min , where M is the mutual inductance of the coil and a circle coincident with the edge of the disc, and i is the current through the coil. This induced voltage is balanced against the difference of potential Ri between the extremities of the resistance R through which the same current flows as through the coil. When equality of voltage is indicated by the galvanometer $R = nM$. M is calculated from the dimensions of the coil and disc and thus R is found. The coil used was of a large number of turns in order to make M as great as possible.

In the 'Philosophical Magazine' for November, 1882, Lord RAYLEIGH compares the method of LORENZ with other methods. After stating that he is disposed to consider LORENZ's method the best, he proceeds to deal with some of the difficulties. The first of these is the smallness of the resistance R which can be directly measured, and this led Lord RAYLEIGH to adopt a system of shunted resistances which, for part of our work, we also have used. Lord RAYLEIGH remarks that the influence of terrestrial magnetism and the thermo-electric effects at the sliding contacts are both very appreciable and give rise to trouble during the observations, but they can be eliminated by observing only the effect of reversing the battery current.

The more important portion of Lord RAYLEIGH's comments deals with the ratio of

* LORENZ, 'POGG. Ann.,' 149, p. 251, 1873.

the dimensions of the coil and disc. The results are given of some calculations of the mutual inductance M of a coil of radius A and circle of radius a . The rates of variation of M with change in the diameter of the coil are calculated for various values of a/A , and it is shown that when a/A increases in value the rate of variation of M with change in a and also of A increases. Further, it is shown that, by using two coils separated to a considerable distance, it is possible so to proportion the radii and the distance apart of the coils that the error of mean radius of the coil does not affect the result; the diameter of the disc and the distance apart of the two coils are then the important quantities. Lord RAYLEIGH remarks: "It is clear that M vanishes both when A is very small and when it is very large; from which it follows that there must be some value of A for which the effect is a maximum and therefore independent of small variations of A ." The same is, of course, true for the disc; by suitably proportioning the dimensions, the error of mean radius of the disc may be rendered negligibly small. This fact led Mr. A. CAMPBELL* to design a standard of mutual inductance in which the radius of the secondary is not required to be known with great precision; it also guided us in fixing the dimensions of our coils and discs so that the diameters of the latter need not be accurately known.

In the first and second series of experiments carried out by Lord RAYLEIGH the inductance coils were situated nearly in the plane of the revolving disc as in LORENZ's original use of the method. In the third series the coils were separated from the disc to such a distance as to render the accuracy of the results practically independent of the mean radius of the coils. It is right to say here that in the design of the apparatus described in this paper we were largely influenced by Lord RAYLEIGH's investigations and by Mr. CAMPBELL's work.

Of the experimental difficulties noted by Lord RAYLEIGH the more important are: (1) troublesome thermo-electric effects at the sliding contacts notwithstanding that the edge of the disc was amalgamated with mercury, and (2) effects due to terrestrial magnetism. As before stated, both of these effects are eliminated by taking a sufficient number of readings with reversals of the current, but it is evident that good readings cannot be taken if the magnitude of the effects is subject to sudden fluctuation.

In 1890 Prof. J. V. JONES† made a number of suggestions towards a determination of the ohm. He had made a number of experiments in his laboratory at the University College, Cardiff, and stated that he was of opinion that, if apparatus were constructed on a large scale and with a certain perfecting of detail, a single set of observations would give a result accurate within 1 part in 10,000. In the electrical observations the principal difficulties which he had contended with were: (1) variations in the thermo-electric effects at the brush contacts, and (2) variations in the rate of rotation of the disc. A considerable reduction of the first difficulty was

* CAMPBELL, 'Roy. Soc. Proc.,' A, vol. 79, p. 428.

† Paper read before the British Association, 1890.

brought about by using a brush consisting of a single wire perforated by a channel through which a constant flow of mercury was maintained. With regard to the variation of speed, Prof. JONES was of opinion that "no time or trouble spent in securing a constant speed will be lost for the purpose in view." It was suggested that the standard coil should consist of a single layer of wire, as the mutual inductance of the coils and disc could then be calculated with great accuracy.

The apparatus made by Messrs. Nalder Bros.* for Prof. CALLENDAR, who was then at the McGill University, Montreal, was, in general arrangement and dimensions, similar to Prof. JONES's Cardiff apparatus. Contact with the edge of the disc was made by three small tangential phosphor-bronze tubes lightly pressed on it, at points separated by angular distances of 120 degrees, and through these tubes mercury flowed on to the edge of the disc. The employment of three brushes was suggested by ROWLAND to eliminate small errors due to imperfect centering of the coil and disc, and was a distinct improvement. The possible sources of error were considered, and in a paper by Prof. AYRTON and Prof. JONES* an equation is given showing the rates of variation of the mutual inductance of the coil and disc with changes (1) in the radius A of the coil, (2) in the radius a of the disc, and (3) in the axial length $2x$ of the helix. The equation given is

$$\frac{dM}{M} = 1.246 \frac{dA}{A} + 2.346 \frac{da}{a} - 0.0997 \frac{dx}{x}.$$

The value of M was 45814.5 cm., of A 26.7039 cm., and of a 16.5354 cm., from which it is readily calculated that an error of 0.002 cm. in the measurement of the diameter of the disc introduced an error of nearly 5 parts in 100,000 in the value of M . An error in the measurement of the diameter of the coil of 0.002 cm. introduced an error of 14 parts in 100,000 in the value of M .

It will be seen that the experimental difficulties experienced by observers using the Lorenz method were mainly due to thermo-electric troubles at the brush contacts, want of uniformity of speed, and the effect of terrestrial magnetism. There are no difficulties attendant on the accurate evaluation of the mutual inductance of a coil and disc if the dimensions are accurately known. If a single layer coil is used, its dimensions may be determined with great accuracy; if a coil of many layers is employed, it appears best to follow Lord RAYLEIGH's practice and use two coils at a considerable distance apart in order that errors of measurement of the radius of a coil may be rendered negligibly small.

The Lorenz apparatus described in this paper was designed in 1908, our object being to determine a resistance in absolute measure with a precision within a few parts in a hundred thousand. The apparatus described eliminates the effect of terrestrial magnetism and largely reduces thermo-electric troubles at the brush contacts by the employment of two discs of equal diameters. The coils are of one layer only and

* 'British Association Reports,' 1897.

their dimensions, together with those of the discs, may be determined with ease and with precision; in addition, any possible error in the measurement of the diameters of the discs is made negligibly small. Instead of employing one rotating conductor, a number are used, and the induced voltages may be placed in parallel or in series. The machine is of the best construction, great attention being paid to lubrication in order to avoid irregularities in the speed. As already stated, the heavy metal work was kindly undertaken by Sir W. G. Armstrong, Whitworth & Co., but the smaller work such as the chronograph, commutators, brushes, coil fittings, &c., many of which demanded great skill in their construction, were made by Mr. F. H. MURFITT, who has charge of the instrument shop of the Laboratory, and whom we also thank for many suggestions.

SECTION 3.—GENERAL DESCRIPTION.

The instrument consists of two metallic discs which support ten conducting wires rotating in a magnetic field produced by a current in four coils; an electric motor is used as the source of power. Phosphor-bronze wire brushes make contact with segments made of the same alloy attached to the ends of the rotating wires, and the difference of potential between the brush contacts at the ends of a single wire, or that between five wires, is balanced against the difference of potential between two points on a standard resistance, the current through which is the same as that flowing through the four coils. A diagrammatic sketch of the arrangement is shown in fig. 1.

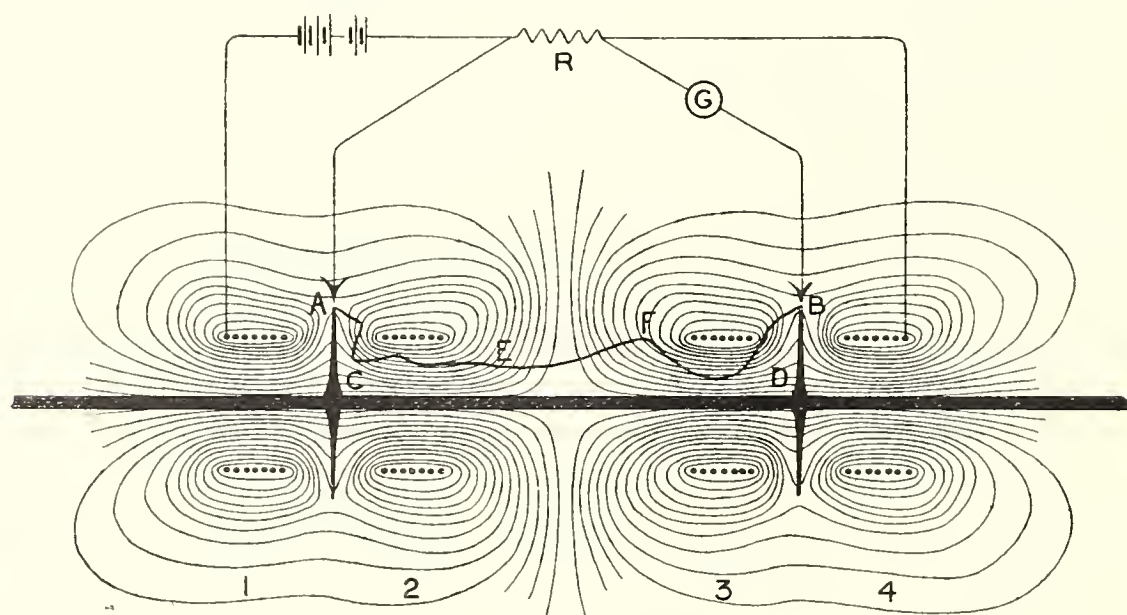


Fig. 1.

It will be seen that the current in the coils 1 and 2 is in the opposite direction to that in the coils 3 and 4. The resulting magnetic fields are opposed in direction, and the value of the field at points in the neighbourhood of the edge of a disc is zero or nearly so. A difference of potential is produced between the ends of a rotating conductor, and its value is dependent on the position only of the ends of the

conductor and not upon its shape, conditionally that the conductor passes through the coils carrying the current. Thus the difference of potential at the extremities of a conductor ACDB is not altered if its shape is changed to AEFB. The segments which form the ends of the conductors are insulated from each other and from the discs.

The brushes consist of phosphor-bronze wires stretched by two spiral springs, and resemble violin bows. Each brush makes contact with one or two segments over a length varying from 5 to 6 cm., and leaves a segment at a tangent thus making the pressure greatest at the mid-point of contact. Petrol is employed as a lubricant.

There are two principal ways of using the apparatus. In the first the ten brushes are included in a circuit so as to be in series. When each brush is in contact with a single segment, the differences of potential due to five rotating conductors are added together, the remaining five conductors being ineffective. When each brush connects two neighbouring segments the ten rotating conductors are arranged in five sets of two in parallel and the total potential difference is the same as before. It is easily seen that by having a comparatively large arc of contact between each brush and a segment (or segments) and twice as many segments as brushes, the circuit made through the brush contacts is never broken.

In the second method, the brushes are divided into two sets of five in parallel and the total potential difference is the same as that of a single rotating conductor.

All the coils are wound with bare copper wire on hollow marble cylinders having double-threaded screw grooves cut on the surfaces. The two wires on any one cylinder form two adjacent helices which may be connected in series or in parallel. In the general use of the instrument they were connected in series, but they may at any time be disconnected from each other and an insulation test made between them. There are eight helices in all and these are connected by means of small concentric cables to a plug board and commutators which enable the direction of the current in any coil to be changed at will.

Each of the cylinders is mounted on a strong metal support and its position with regard to a disc may be altered with ease by screw adjustments. The distance between the mid-planes of two coils is measured by means of microscopes.

The electric motor used for driving is situated at a considerable distance from any one of the coils, and its influence on the result was calculated and also experimentally proved to be negligibly small. A commutator is fixed to the axle of the motor, and this serves to charge and discharge a condenser placed in one arm of a Wheatstone bridge. By keeping the bridge permanently balanced, the speed of the Lorenz apparatus is maintained constant. A directly driven chronograph enables the speed to be calculated.

The whole instrument is supported on gun-metal rails embedded in a solid block made with Keene's cement, sand, and ballast. With the exception of the motor, no magnetic material is used in the construction of the apparatus or supports. The

complete apparatus is diagrammatically shown in fig. 2; fig. 3 is a photograph of the instrument taken from the end farther from the motor. The overall length is seven metres.

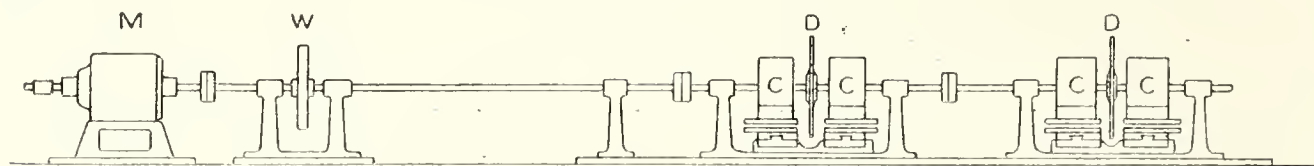


Fig. 2.

M, motor; W, fly-wheel; DD, discs; CCCC, coils on cylinders.

SECTION 4.—THE MOTOR AND FLY-WHEEL.

The electric motor M, fig. 2, is coupled to a shaft supporting a fly-wheel W; the fly-wheel shaft is in turn coupled to another supporting one of the discs, and the latter shaft is coupled to a similar one supporting the other disc. The coupling between the fly-wheel shaft and that supporting one of the discs is an insulating one, washers and tubes of stabilit being used to secure good insulation.

The electric motor is a shunt-wound one supplied by Messrs Crompton & Co., Ltd. The machine has four poles and has a commutator of the radial type. The shaft is of phosphor bronze and the motor is mounted on a base of the same alloy in order to reduce the quantity of magnetic material to a minimum. The stray field of the motor in an axial direction is surprisingly small, its intensity at a distance of 400 cm. amounting to 0.0006 C.G.S. units only. Our original intention was to enclose the motor in a double shell of soft iron, and the base of such a shell is interposed between the motor and its support; the results of our observations on the stray field showed this to be unnecessary and the idea was abandoned. The effect of the motor on the mutual inductance of the coils and discs is discussed in Section 19.

The fly-wheel is of phosphor bronze. The outer diameter is 50 cm. and the weight of metal in the rim is about 80 kgr.

SECTION 5.—THE ROTATING DISCS AND THEIR SUPPORTS.

The portion to the right of the instrument (fig. 2) consists of two similar parts, and only one of these will be described. Fig. 4 shows one of the parts.

The disc D is of rolled phosphor bronze fitted on a shaft S made of an alloy of copper and aluminium (10 per cent. aluminium and 90 per cent. copper). The original intention was to have the shaft of phosphor bronze, and two such shafts were made but were rejected on account of their appreciable magnetic susceptibility. Phosphor-bronze billets having the requisite magnetic properties could readily be obtained, but in such cases they failed to give satisfactory results with the mechanical tests and were therefore unsuited for shafting. The diameter of a disc is about 53 cm., and that of the shaft is 5 cm.

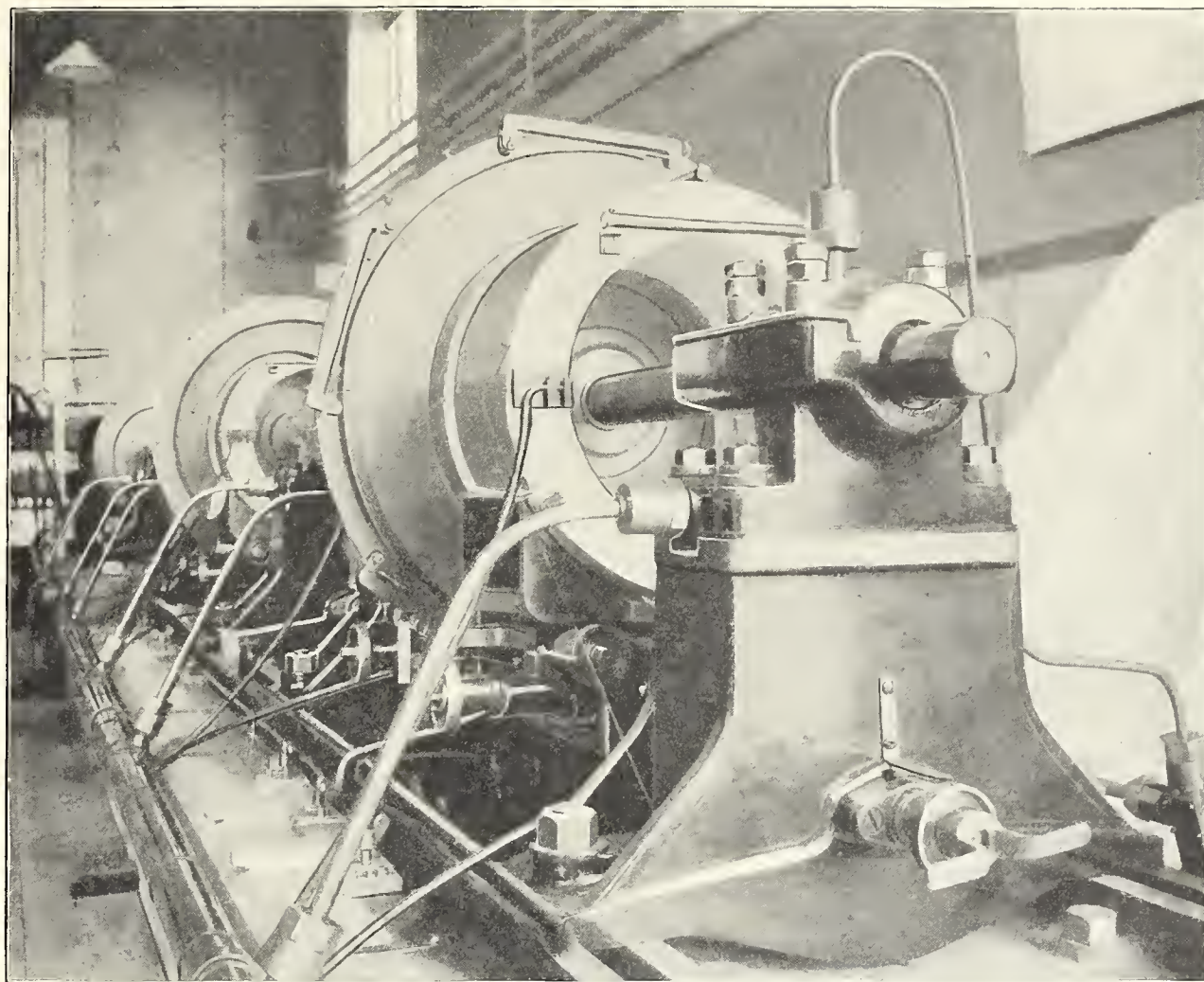


Fig. 3.

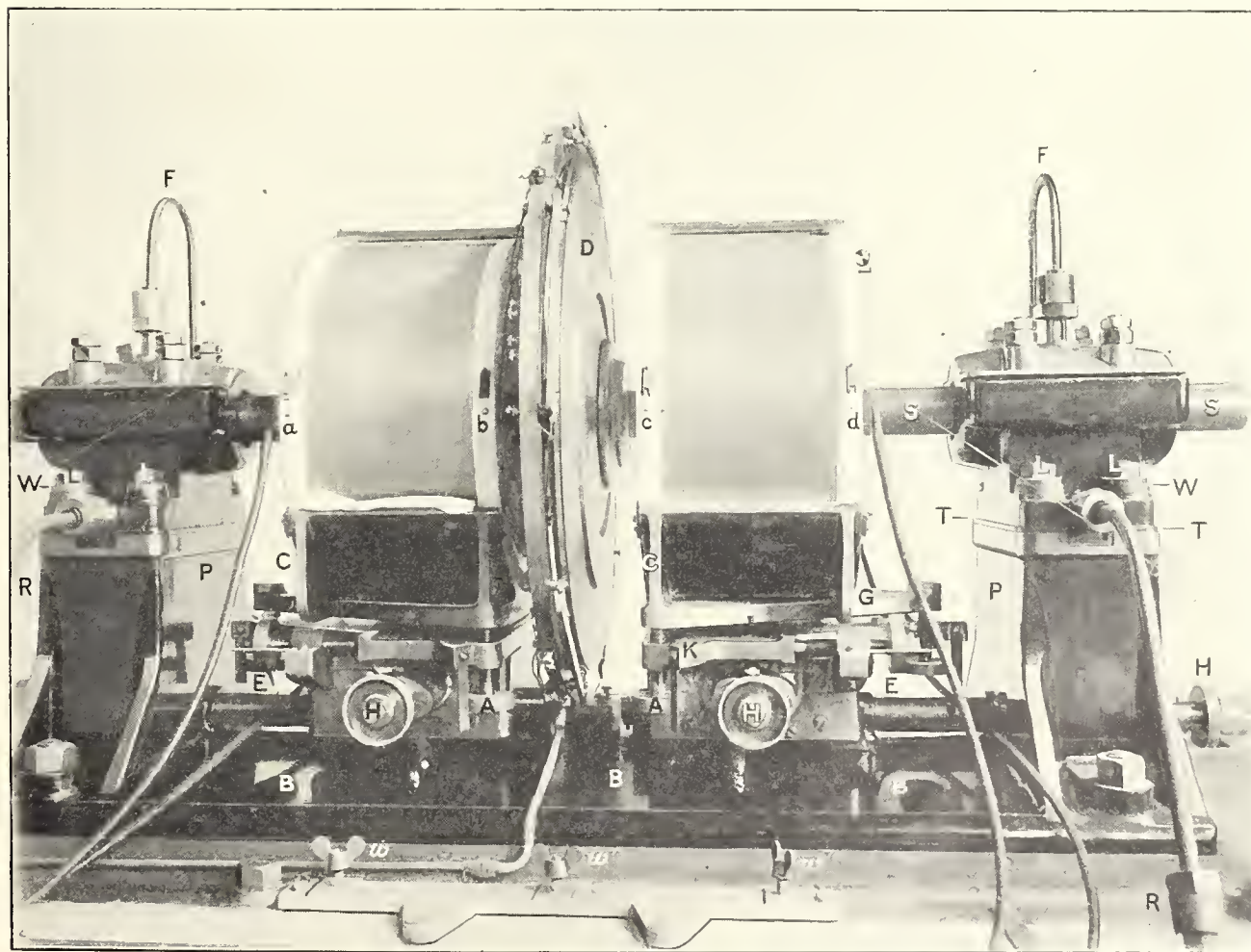


Fig. 4.



The shaft runs in bearings made of a special alloy of tin (69 per cent.), zinc (29·6 per cent.), and copper (1·4 per cent.), this being the best non-magnetic alloy we know of in which a copper aluminium shaft will run without trouble. The pedestals PP and bed B, supporting the bearings, are of phosphor bronze, except for slabs of stabilit T, 6 mm. in thickness, which insulate the bearings from the bed. The upper part of a pedestal is bolted to the lower part by phosphor-bronze bolts L, stabilit washers W, and stabilit tubes being used to insulate the upper and lower parts. The insulation resistance between the shaft and the bed has been measured on many occasions, but has never been less than a thousand megohms. If two metallic discs with uninsulated rims and conductors are employed, such insulation is essential, but it is not necessary with insulated conductors and segments such as we have used.

The oil feed (F) and return pipes (R) are of copper and are insulated from the pedestal by glass and ebonite tubes and oil-resisting rubber washers. Oil throwers are fitted to the shaft and effectually prevent the oil travelling outwards from the bearings.

SECTION 6.—THE ROTATING CONDUCTORS WHICH PASS FROM THE EDGE OF ONE DISC TO THE EDGE OF THE OTHER DISC.

We have already briefly described the system of ten conducting wires passing from disc to disc, and it will be realised that the discs in the present apparatus serve only to support the radial conductors and are employed for no other purpose. The wires used consist of No. 26 double silk-covered copper wire, shellacked, and covered with silk tube.

In order to place the ten wires in position, the shaft was drilled centrally and parallel to its length from the coupling between the two discs to points within 20 cm. of the discs, and radial holes were drilled in the shaft in these latter positions. On the coupling between the discs blocks of ebonite are screwed, and these support terminals to which the ends of the wires can be attached. Ten wires pass from the segments attached to the edge of one disc, through a channel milled in the side of the disc, and again through a brass tube screwed on the shaft, into a radial hole; after passing through the central hole drilled in the shaft they emerge at the coupling, and the free ends of the wires are attached to terminals on the ebonite blocks. Similarly, ten wires pass from the edge of the other disc to corresponding terminals, and by making suitable connections between the terminals we obtain a system of ten conductors which pass between the segments on the two discs. As already explained, the path of the conductors is unimportant, conditionally that they pass through the coils.

On the edge of a disc ten segments of stabilit are screwed. Some of these can be seen in the photograph (fig. 3), and in the sketch (fig. 5); a sectional view of a segment attached to a disc is shown in fig. 5c. On emerging from the channel milled

in the disc, the wires pass along a groove *G*, milled in the stabilizer, and are soldered to ten phosphor-bronze wire segments, of square section, which are screwed to the stabilizer in the manner shown in the illustration. This method was adopted because of the strain to which the segments are subjected when the machine is running at full speed and which results in an increase in the effective diameter of a disc, measurements of which are given in Section 16. The wire for the phosphor-bronze segments is 25 sq. mm. in cross section and was kindly drawn for us by the London Electric Wire Company. Some of it was further drawn down into circular section wire of 0.12 mm. diameter, and this latter was employed for the brushes. Phosphor bronze was chosen for the segments and brushes because of the negligible tendency to bind and tear when this alloy is employed for moving parts in contact.

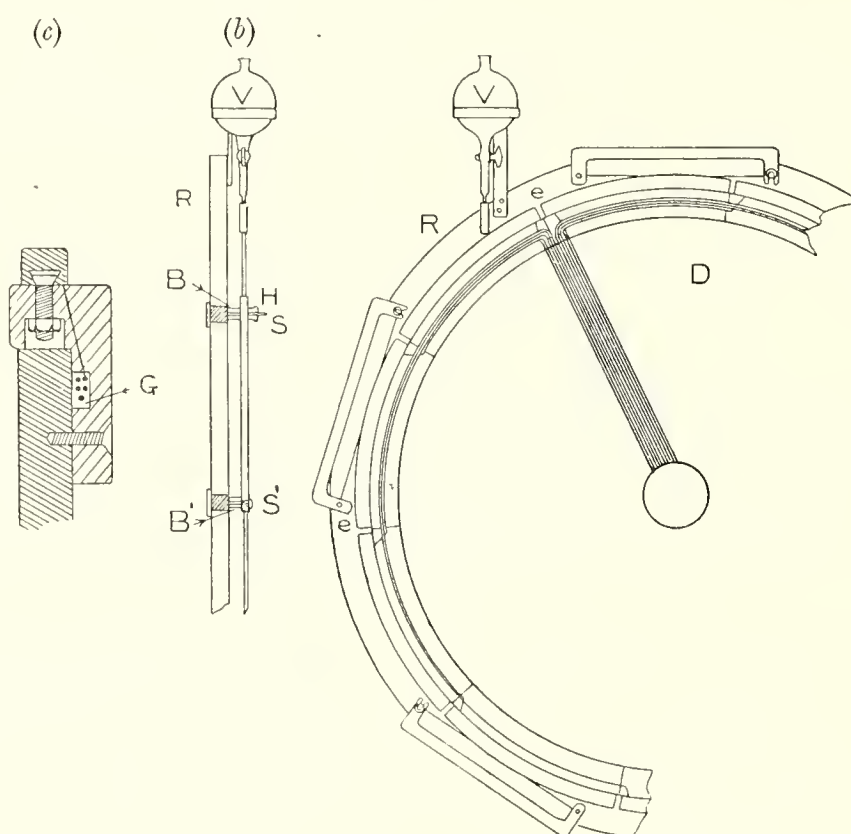


Fig. 5.

Fig. 5 shows five segments in position on a disc and illustrates the manner in which the wires pass from the segments to the main shaft. Before making any resistance measurements the shaft was rotated slowly, and with the aid of a small portable milling machine a smooth surface was turned on the segments. The sharp edges which resulted at *e, e* (fig. 5) were rounded in order to avoid a cutting action on the wire brushes.

SECTION 7.—THE BRUSHES AND THEIR LUBRICATION.

To obtain experience regarding the best form and number of brushes, preliminary apparatus consisting of two rotating discs of phosphor bronze was constructed in 1908.

The apparatus was astatic and our experiments were directed to eliminating trouble due to thermo-electric effects at the brush contacts.

The first brushes experimented with were of fine phosphor-bronze wire, each brush consisting of about 50 wires bound together after the manner of a common pencil brush. A light spiral spring ensured contact between a brush and the edge of a disc. A circuit through brushes, discs, and a galvanometer was completed, and the discs (25 cm. in diameter) were run at a speed of about 1500 revolutions per minute. Whatever thermo-electric effects existed at the points of contact of the brushes on one disc must have been in opposition to those at the brush contacts on the other disc, but the galvanometer deflection was far from steady, the variations in the thermo-electric effects amounting to about 0.0001 volt. Although these preliminary experiments of ours were so unsatisfactory, the results were much better than those obtained when the circuit was completed through brush contacts at the edge of one disc and others on the shaft of the apparatus.

An even more disturbing feature of our experiments was the inconstancy of the resistance of the galvanometer circuit. With one brush on each disc the resistance of the contacts was measured as something less than 0.1 ohm when the discs were stationary, but when the discs were rotating the contact resistance increased to 5, 10 and 20 ohms, and at times the circuit appeared to be broken. This effect was undoubtedly due to a vibration of the brushes brought about by the motion of the air at the edges of the discs. In practice it was not possible to prevent this by increasing the pressure applied to a brush, as by doing so the discs were worn away very rapidly and the points of contact became very hot. Shielding devices were tried with some small success, but the most favourable results were obtained by employing a number of brushes in parallel and placing in series with each brush a resistance of 10 ohms, thus ensuring that any increase or diminution of the resistance of one brush contact would have but little effect on the circuit as a whole. Our experimental results showed clearly that the employment of a large number of brushes was beneficial, but an increase in the number did not greatly improve the results, unless a resistance was placed in series with each before the brushes associated with any one disc were placed in parallel.

A further advance was made by lubricating the edges of the discs. Fatty oils and greases are impossible for such a purpose as the contact resistances are enormously increased and the general result is far worse than when no lubricant at all is used. Acheson graphite, aquadag and oildag were tried, but the results were not satisfactory. It is not very well known that paraffin oil improves all ordinary contacts such as those associated with slide wires, plugs, &c., but throughout the National Physical Laboratory paraffin oil is largely used for such purposes. We tried it therefore as a lubricant for the brushes. There was a marked improvement in the results, and we continued to use it in this preliminary work. Many kinds of brushes were tried: some were wires of phosphor bronze, some of phosphor-bronze gauze, some of

copper wire, and solid slipper brushes of phosphor bronze were also used. The most satisfactory results were obtained with brushes of fine wire.

After the erection of the Lorenz apparatus we continued our experiments with brushes. The discs of the new apparatus are about 53 cm. in diameter and they revolve about 1050 times per minute, so that the velocity at the edge of a disc is about 1750 m. or 1 mile per minute. This velocity is greater than the velocity at the edges of the discs in our preliminary apparatus, and we found the brush difficulties correspondingly increased. The variation in the resistance of the galvanometer circuit due to the varying contact resistances rendered accurate work impossible, and we were led to design a brush which would not be set in vibration except to a very small extent and by the use of which thermo-electric effects would produce comparatively small disturbances. A short description of these brushes appeared in the 'Annual Report of the National Physical Laboratory for 1911,' and we are aware that the short description there published has led several investigators to try similar brushes, and the results have been reported as satisfactory.

We were guided in the design by the following considerations: (1) a fine cylindrical wire if stretched offers little resistance to a stream of air, and whatever vibration is set up will probably be regular and can be controlled by adjustment of the tension on the wire; (2) if a wire such as AB (fig. 6) is in tension and makes contact with the

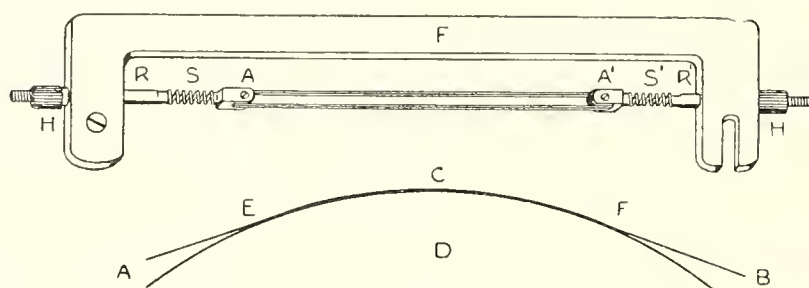


Fig. 6.

edge of a rotating disc D over the arc ECF, the pressure will be greatest at a point close to C the mid-point, and will gradually fall in value until at E and F it is zero. The maximum rise of temperature and the maximum thermo-electric effect will probably be at C, and the thermo-electric force will gradually diminish as we approach E and F and be zero at these points. Without the completion of any other circuit it is clear that electric currents will be produced which will flow from the wire to the disc in some parts and from the disc to the wire in other parts. If, therefore, a galvanometer is included in the circuit containing the junction of wire and disc, the resultant deflection will be very much less than that which would be produced by the *maximum* thermo-electric effect. A number of stretched wires in parallel should, of course, give better results than one. The form of brush actually used in our experiments is depicted in fig. 6. The size was largely governed by the arc of contact desired and the number of segments on the discs.

The brushes are of phosphor-bronze wire 0.12 mm. in diameter, and to obtain as small a thermo-electric effect at the contacts as possible the wire was drawn from other wire of square section similar to that employed for the segments on the discs. To make a brush, the fine wire is wound in screw cut grooves of 0.25 mm. pitch, cut on small brass cylinders which can rotate about, or be clamped to, the axles AA'. The ends of the wire are soldered to the cylinders and the wire brush thus formed is put in tension by operating the milled heads HH. The spiral springs SS' are of stout phosphor-bronze wire and are soldered to brass rods RR', of square section, which slide in square cut grooves cut in the frame F. The overall length of a brush is 20 cm. In practice we found the tension required for good working to be very small; a tension corresponding to the pull produced by the suspension of a 200 gr. weight was found to be most satisfactory. Some of the brushes were made with eight turns of wire, some with three turns, and a few with one turn; the majority were of three turns.

From the moment we commenced to use this style of brush with petrol as a lubricant, the thermo-electric variations produced very much less trouble. The variations are not more than one-fiftieth of those found with any other form of brush we have experimented with, and the variation in the resistance of the contacts when ten brushes are placed in series is so small that the sensitiveness of the galvanometer remains constant within the limits of error of our measurements. However, in the case of ten brushes in series, the tension on the wires has been somewhat greater than that recorded above.

Without a lubricant the brushes are not satisfactory. The wire is gripped by the disc and under certain conditions the brushes vibrate longitudinally and produce at the same time a chattering noise. Also the wire and disc become rugged and the temperature at the point of contact is very much greater than when petrol is used as a lubricant. We believe the petrol to have three beneficial influences: (1) as a lubricant; (2) as a cooling agent; (3) as a cleanser getting rid of all traces of grease and dirt. The amount of petrol to be supplied and the manner of supply was the subject of many experiments, but it is sufficient to state here the manner eventually adopted.

Fig. 5 shows a brush in position and a petrol supply vessel V. The brush is secured to a massive phosphor-bronze ring R (see figs. 3 and 5) by the screws SS', and it can be placed in contact with a segment or removed from such contact by loosening the terminal head H and giving the brush frame a circular motion about the pivot screw S'. The screws SS' and the brass distance pieces BB' are insulated from the ring R by means of ebonite sleeves, the latter allowing of some adjustment of the screws SS'. The ring R is supported by the bed B (fig. 4) but is insulated from it. Stretched phosphor-bronze wires support the ring laterally and give it the necessary rigidity.

The petrol supply vessel is of glass, the tube being drawn down to a capillary about 0.5 mm. in diameter at its lower end. On the narrow glass tube thus formed, a

piece of circular lamp wick about 3 cm. long is slipped; the lower end of this wick just touches the rotating segments and thus feeds petrol directly on to the surface of the segments. The usual rate of supply of petrol was about 500 c.c. in twenty minutes. When the apparatus is running well, an observer may place the end of a finger on the rotating segments and find it well flooded with petrol removed from the rim. The wicks are renewed at least every day and the brushes wiped with clean chamois leather. In our experiments a set of ten brushes lasted usually for six or nine complete sets of observations; after that number the surfaces of the wires became somewhat rough and the results were not quite so satisfactory. The magnitude of the changes of the thermo-electric effects with two sets of five brushes in parallel will be realized when it is said that often for intervals of twenty minutes the rapid variations in the total thermo-electric voltage did not exceed 0.1 microvolt. A slow progressive variation was commonly observed, but this was not a source of trouble.

SECTION 8.—THE COIL SUPPORTS.

Each marble cylinder weighs about 50 kgr. and is supported on a phosphor-bronze cradle C (fig. 4) so that its axis is coincident with the axis of the shaft. The base of the support is a triangular casting G, which is supported in turn by three levelling screws A on the "hole, slot, and plane" principle. The pitch of the levelling screws is one millimetre and the heads of the screws are divided into one hundred equal divisions, thus enabling any particular vertical motion to be repeated within one-hundredth of a millimetre.

A second triangular casting K carries the levelling screws and is fitted over a large central stud attached to a slide, so that the cylinder may be rotated about a vertical axis if necessary. The maximum angular motion is 12 degrees and the magnitude of any motion can be directly read on the engraved head of the horizontal screw E to half a minute of arc. Backlash is avoided by the use of strong phosphor-bronze springs.

A cylinder and the cradle supporting it can be moved 5 cm. in two horizontal directions at right angles by means of two slides, the motions being controlled by screws, the heads H of which are divided to read hundredths of a millimetre as in the case of the screws for the vertical motion. For these movements also strong phosphor-bronze springs are employed to prevent backlash.

SECTION 9.—LUBRICATION OF BEARINGS.

The bearings of the machine are nine in number, each of those of the motor being about 9 cm. long and each of the remaining seven about 14.5 cm. long. The lubricant used is best turbine oil which is fed to the bearings under a pressure of about 15 lbs. per square inch, the rate of supply being a cubic foot of oil every five minutes. The oil supply tank and pump are of phosphor bronze and are situated under the

floor, $1\frac{1}{4}$ m. below the level of the bearings of the main shaft, and 4 m. distant from the nearest point of the machine. The pump is driven by a small motor of one-eighth horse-power and forces the oil through copper pipes to the bearings of the machine. After passing through the bearings the oil returns through copper drain pipes to the tank and is strained through fine copper gauze before again entering the pump. The system works excellently, no trouble whatever having been experienced.

SECTION 10.—REGULATION OF SPEED.

On the axle of the motor a commutator is fitted which serves to charge and discharge a condenser four times for each revolution of the shaft. The condenser is placed in one arm of a Wheatstone bridge, the other arms of which are platinum-silver resistance coils. Balance of the bridge results for a particular frequency only of charge and discharge, and to maintain a balance over a considerable length of time the frequency and therefore the speed of the Lorenz apparatus must be kept constant. To ensure constancy of the arms of the bridge, the condenser and the platinum-silver resistances were kept in a constant temperature room; a small variable resistance in series with one of the arms was in general adjusted to secure a balance when the speed was that best suited for the resistance measurements, but after the latter measurements had commenced the speed only was controlled to maintain the balance. The galvanometer used was a suspended coil instrument, the spot from which was received on a ground glass scale mounted over the fly-wheel; an assistant observer, S. WATTS, applied a variable pressure to the fly-wheel and so maintained the balance of the bridge. A change in the speed of the Lorenz apparatus of 1 part in 10,000 produced a deflection of the galvanometer spot of 4 mm., and in general, a balance was maintained for twenty minutes or more with a maximum deflection not greater than 2 mm.; occasionally, better results than this were obtained. An adjustable resistance is in series with the field coils of the motor, and before attempting to govern the speed this resistance is altered until a speed results which is very slightly greater than that desired.

The motor was run from a battery of large storage cells and very good results were obtained with no regulation at all, if the current flowed sufficiently long (generally from one to two hours) to raise the field coils to an approximately constant temperature. However, the method finally adopted enabled resistance measurements to be made a few minutes after starting the motor. A somewhat similar method of controlling speed for measurements of capacity has been in use for several years both at the National Physical Laboratory and at the Bureau of Standards.

SECTION 11.—METHOD OF RECORDING THE SPEED.

The chronograph described is very similar to one designed at the Bureau of Standards,* and made by the Société Genevoise, for details of which the writer

* 'Bull. Bureau of Standards,' vol. 3, p. 561.

is indebted to Dr. ROSA and other members of the Bureau of Standards. The instrument described below, which is of brass, was made by Mr. MURFITT of the Instrument Department of the Laboratory.

The method of recording the speed is a direct one. A chronograph drum is geared to the main shaft of the apparatus through the medium of two worm wheels and two spur wheels, the gear ratio being 174. The usual speed of the main apparatus is about 1044 per minute, and under such conditions the drum of the chronograph makes one revolution in ten seconds. An electro-magnet is supported on a carriage which is connected to a split-nut engaging with a screw of 1 mm. pitch. The direction of travel of the carriage is parallel to the axis of the drum and it advances 1 mm. for each revolution of the drum. Every second a current passes through the electro-magnet and the latter operates a small punch, which, striking through a typewriting ribbon, prints a dot on a sheet of paper carried by the drum. The circumference of the drum is exactly 500 mm., so that when six revolutions per minute are made, successively recorded dots are 50 mm. apart. The split-nut fixed to the carriage carrying the electro-magnet can be disengaged from the driving screw and the carriage rapidly run along the rails supporting it. When the printing punch is pressed forward during this operation a line parallel to the direction of travel of the electro-magnet is printed on the paper. This line is hereafter called the base line and by measuring the angle between it and a row of dots recorded during a run, the speed can be calculated with great accuracy.

The method of calculating the speed is as follows: At a speed of exactly 1044 revolutions per minute there are $1044/174 = 6.0000$ revolutions of the chronograph drum per minute. The resulting record will therefore be ten rows of dots, the dots being 1 mm. apart, and the rows 50 mm. apart and parallel to the base line. If the speed is slightly diminished or increased, the rows of dots will slope upwards or downwards. In practice, a set of observations for the measurement of a resistance is made to last for at least 1000 seconds (*i.e.*, about 17 minutes), and the resulting rows of dots are therefore more than 10 cm. long. From a portion of the base line lying beneath (or above) a row of dots a length of 10 cm. is marked off, and from the extremities of the 10 cm. line ordinates are drawn to the nearest row of dots. If these ordinates are equal in length the rows are parallel to the base line, and the distance between two dots recording an interval of 1000 seconds (10 cm. run) is 50,000 mm. if the distance is measured along the trail of the recorded seconds. If the ordinates are not equal in length the machine was running either faster or slower than 1044 revolutions per minute; which of these holds good is decided by the direction of the slope. Suppose the difference between the ordinates is 14.5 mm. and that the machine was running faster than 1044 per minute. The mean speed is calculated to be

$$1044 \times \frac{50,000}{50,000 - 14.5} = 1044.30_3 \text{ revolutions per minute.}$$

It is clear that a difference between the ordinates of half a millimetre corresponds to 1 part in 100,000 of the

speed. The drum is 50 cm. long and a record lasting 80 minutes can therefore be made.

Fig. 7 is a full-size reproduction of a portion of a record, taken June 26, 1913. The recorded speed is $1043\cdot96_2$ revolutions per minute. The rows of dots enable the mean speed to be calculated with an error certainly not greater than 1 part in 100,000, and the speed throughout (as illustrated by the rows of dots lying in a straight line and not a wavy one) is wonderfully uniform. The general fluctuations of speed cannot be detected on the record, but their magnitude has been estimated in another way as explained in Section 23.

A mercury contact is used on the pendulum of the standard clock and a relay is operated continuously by the current which passes. The rate of the clock is of course allowed for.

SECTION 12.—MAGNETIC TESTS.

Magnetic tests were made on all the materials employed. With the exception of the motor, we are satisfied that the permeability of no part of the machine and the concrete bed on which it rests differs from unity by more than 2 parts in 100,000.

Every part of the machine bears a distinctive number. When each part was cast, a lug about 5 cm. long and 5 cm. in diameter was cast with it, and after being stamped with the same number as the casting it was forwarded by Sir W. G. Armstrong, Whitworth & Co. to be tested for magnetic quality. In the case of springs, rods, tubes, &c., pieces were cut from these and used for the tests.

The method of testing was similar to that employed for the parts of the current balance.* Soft iron wire and ferrous sulphate were used to calibrate the apparatus and sufficient sensitiveness was obtained to detect a difference from unit permeability of about 1 part in 100,000. Thus, when powdered ferrous sulphate having a permeability of about $1\cdot00044$ was contained in a glass tube having the same cross section as most of the test pieces, the resulting galvanometer deflection was 5·2 mm. The test pieces of marble were of much larger cross section than the glass tube and the sensitiveness was correspondingly increased.

Trouble was experienced with a number of brass rods and tubes and with the shaft of the apparatus. The first shaft was made of phosphor bronze and had a magnetic permeability of $1\cdot006$; in consequence this shaft was rejected. The material obtained for the second shaft was of an alloy of copper and aluminium (copper 90 per cent., aluminium 10 per cent.) and was also rejected as its permeability was about $1\cdot002$. The third shaft was made with specially pure aluminium and copper in the same proportions as before, and its permeability differs from that of air by an amount too small to be detected. The billet for the shaft was prepared by The Broughton Copper Company, Limited, of Manchester, and we thank them for the care taken in its preparation.

* 'Phil. Trans,' A, vol. 207, p. 475, 1908.

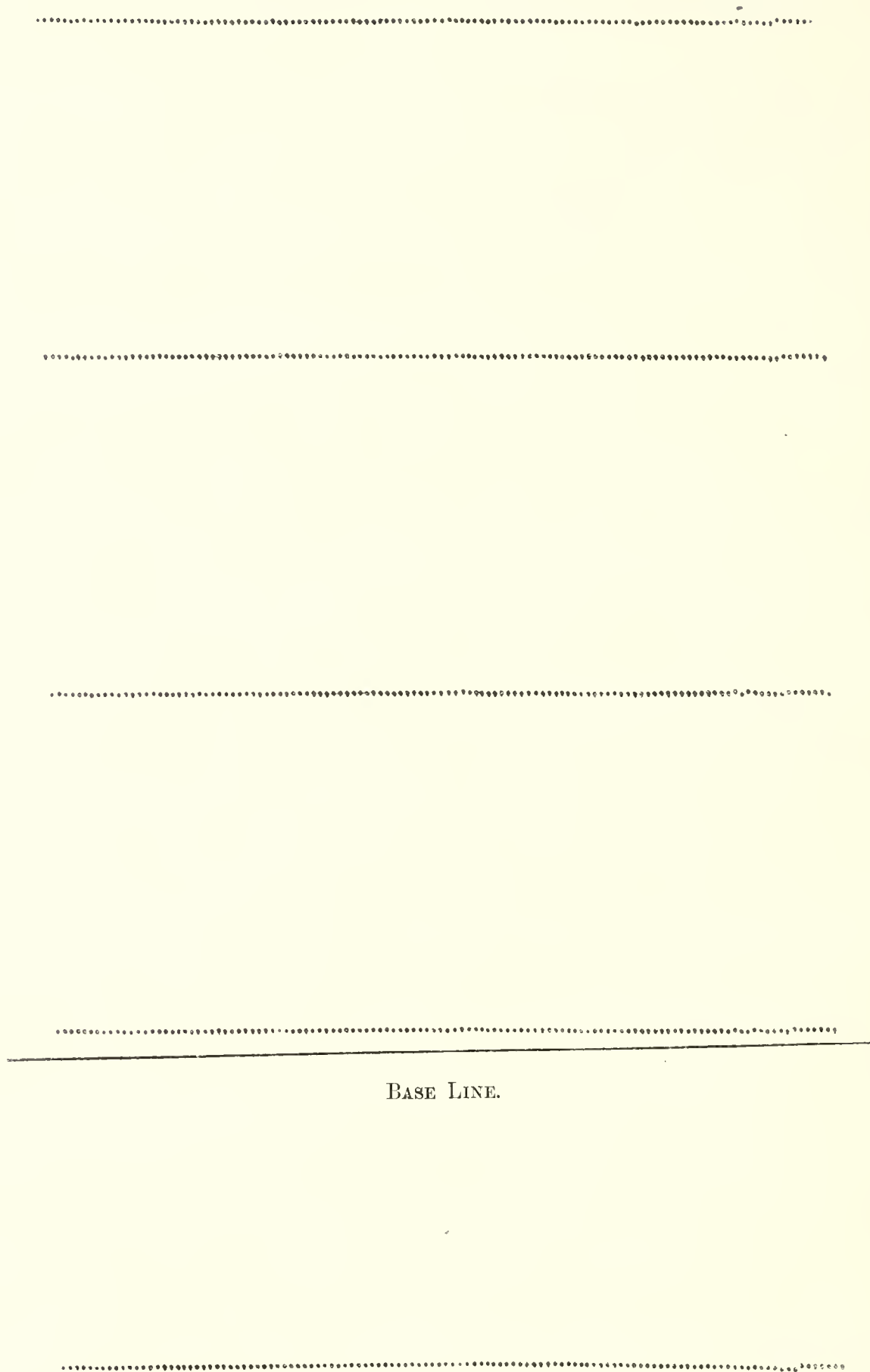


Fig. 7.

The samples of Portland cement tested varied in magnetic permeability from 1.0005 to 1.0020. It is possible that metallic iron finds its way into the cement from the steel grinding machines, and the Associated Portland Cement Manufacturers very kindly offered to exclude such contamination by grinding some cement through mill-stones. At the same time the manufacturers pointed out that a certain amount of oxide of iron in combination with lime and silica is invariably present in Portland cement, and Keene's cement was suggested as being suitable for our purpose. This is a white cement absolutely free from iron compounds of any sort. We tested several samples for magnetic quality, and finding it quite satisfactory decided to use it for the block on which the apparatus rests. No trouble was experienced in obtaining sand and ballast free from magnetic substances.

SECTION 13.—CONSTRUCTION AND MEASUREMENT OF THE COILS.

“First Statuary” Carrara marble was chosen for the material of the cylinders. We were guided in our choice by our experience with the cylinders for the Ayrton-Jones current balance, the tests then made showing marble to be an excellent electrical insulator and of negligible magnetic susceptibility.

The cylinders were prepared in the rough by Messrs Walton, Gooddy & Cripps, whom we thank for the trouble they took in choosing masses of marble practically free from veins. The small shiny specks which are often present in Carrara marble consist of iron pyrites which has a magnetic susceptibility of about 0.0005. The conductivity of iron pyrites is much greater than that of marble and any small specks on the surface of a cylinder should therefore be removed; we found that a small crystal of pyrites pressed between two plates of copper reduced the insulation between the plates from a value which was practically infinity to 1000 ohms. Clearly a large number of crystals at the surface of a marble cylinder would introduce a serious source of error.

The coefficient of expansion of the marble was determined by direct measurements on the cylinders and found to be 5.0×10^{-6} for 1°C. , the temperature range being from 11°C. to 20°C. These measurements are described in the section dealing with the measurements of the mean diameters of the coils.

The cylinders chosen are free from flaws and cavities. Of six cylinders which were submitted, two exhibited “ground flaws” and were in consequence rejected. These “ground flaws” appear to have been produced by a rupture in the marble many thousands of years ago and subsequent re-union by pressure or equivalent agency. In all, Messrs Walton, Gooddy & Cripps prepared 16 cylinders in the rough and the four best of them were chosen for our work.

The marble cylinders for the Ayrton-Jones current balance were baked in an oven at 140°C. for 30 hours and afterwards immersed in hot paraffin wax. At the same time a marble rod was subjected to similar treatment, and since that time this rod has

been kept under observation in the Metrology Department. Possibly there is a flaw in the rod, but it is certain its length has not kept so constant as other rods which were not baked and immersed in wax. We decided, therefore, to omit the treatment in the case of the four cylinders for the Lorenz apparatus.

The cylinders were turned in a manner very similar to that employed for the suspended coils of the current balance. The inner and end surfaces of a cylinder were turned with the cylinder fixed to the face plate of the lathe, but the outer surfaces and double spiral grooves were cut when the cylinder was mounted on a specially constructed mandrel supported between dead centres. The inner and outer surfaces of a cylinder are practically concentric and the ends at right angles to the axis.

The turning of each cylinder occupied about four weeks. The cylinders referred to as Nos. 1 and 2 were turned by the late Mr. TAYLERSON of the Engineering Department of the Laboratory. Mr. TAYLERSON had marked ability for accurate work of this kind and made many useful suggestions during the progress of the work. The turning of cylinders Nos. 3 and 4 was very ably done by Mr. TRIBE of the Engineering Department.

The winding of the coils was carried out in a manner identical with that employed for the coils of the current balance. It is only necessary to state here that the coils were wound with the wire in tension, the effective load on the wire during winding being 4 kgr. The cylinders were rotated very slowly and after each revolution a stoppage was made for measurements to be taken of the diameter of the wire.

The leads of all the coils must lie in a plane containing the axis of the cylinder, for otherwise the mutual inductance of the leads and the discs will not be zero. To

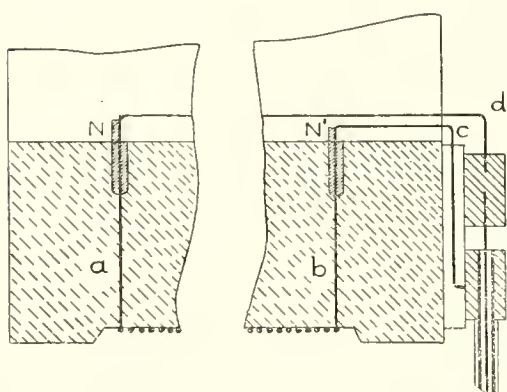


Fig. 8.

ensure the absence of any correction due to such a cause, the connections to the coils were made in the manner shown in fig. 8. At the points where the coils terminate, two radial cylindrical holes *a* and *b* are drilled and enlarged to admit of the slotted brass nipples *NN'* being screwed into them. Soldered connections are made between the nipples *NN'*, the leads *c* and *d*, and the leads through *a* and *b*. The leads *c* and *d* pass to two brass blocks mounted on an ebonite piece screwed to one end of the cylinder, and to these same blocks a small concentric cable is secured in the manner illustrated in the figure. The whole of the leads shown in the figure lie in an axial plane of the coil and this same plane contains the leads connected to the other coil on the cylinder.

An estimate was made of the accuracy with which the number of turns is known. On each cylinder there are two coils each having 96 turns and a diameter of about 36 cm.; the length of wire to each coil is, therefore, about 108 m. From observations

on the radial holes and a consideration of the methods adopted for drilling them, the number of turns is considered to be correct within 2 parts in 1,000,000.

The copper wire with which the eight coils are wound was supplied by The London Electric Wire Company on bobbins of the same diameter as the cylinders. It is hard drawn No. 24 S.W.G. and its mean diameter, obtained from about 800 measurements taken when winding the coils, is 0.557₅ mm. The diameter was also measured in the machine employed to determine the diameter of the coils, the mean of eighty measurements being identical with that already given.

Measurement of the Mean Diameters of the Coils.

Twelve axial planes at angular distances of 15 degrees apart are marked on the end faces and on the ungrooved portions of the outer cylindrical surfaces of each cylinder. These planes are numbered 1 to 12 and are the reference planes in the diametral measurements. Each cylinder was mounted in turn on the mandrel employed for turning the outer surfaces and supported between dead centres attached to the measuring machine.

This machine was made by Sir W. G. Armstrong, Whitworth & Co. at their Openshaw Works, to designs prepared by the firm in collaboration with Mr. L. F. RICHARDSON, formerly of the Metrology Division of the Laboratory. It consists of a straight bed carrying two fixed headstocks with cone-centres, between which the coil, on its mandrel, can be mounted. A saddle sliding on the bed carries two measuring headstocks on a slide which is adjustable so as to bring the line of centres of these headstocks exactly perpendicular to that of the fixed headstocks. Each of the measuring headstocks contains a barrel which can be moved in and out along the line of centres by means of a carefully calibrated micrometer screw; and sliding freely in the centre of each barrel is a plunger, the front end of which constitutes the measuring face, while the rear end carries a small knife-edge pressing against a vertical lever pivoted in the barrel. At its upper end this lever carries a sensitive spirit level. The barrel is advanced by means of the micrometer screw until the plunger, being arrested by contact with the object to be measured, tilts the level far enough to bring the bubble to a definite mark. The reading is then taken with the aid of a vernier on the measuring wheel attached to the micrometer screw, to $\frac{1}{100,000}$ inch. The same operation is carried out with each of the two measuring headstocks, one at either end of a diameter, and the sum of the readings is compared with the sum of similar readings on a gauge bar of known length. For convenience, this gauge bar is mounted on the machine during the measurements, so that by simply traversing the saddle from the coil to the gauge the zero reading can be checked at any stage of the work. To provide for this, one of the cone centres is made specially large, and pierced behind the cone by a hole through which the gauge can be passed.

In the original design the control of the vertical lever carrying the level was effected

by means of coiled springs contained in the barrel. It was found, however, that this arrangement gave rise to a greater pressure than was desirable at the point of contact between the plunger and the coil. Owing to the small radius of the wire with which the coils are wound, a comparatively slight pressure would produce deformation sufficient to lead to errors in the readings. A calculation showed that in order to keep errors due to this cause within allowable limits (less than $\frac{1}{1000}$ mm. on the diameter) the pressure must not exceed four or five ounces. At the suggestion of Mr. J. E. SEARS, the springs were taken out and the vertical lever was extended downwards and provided with a weight at its lower end, thus forming it into a pendulum with a gravity control which could be adjusted to a nicety. At the same time the form of the body of the plunger was slightly modified, and its bearing surfaces lightly polished, so as to minimize friction. In this way the contact pressure was reduced to within the required limits.

Many measurements of the coils were made with a current passing through them in order to reproduce as nearly as possible the conditions under which they would actually be used. To enable this to be done, the plungers were provided with non-conducting measuring faces of polished agate. These proved highly satisfactory in use, and the measurements made with the machine were of quite remarkable accuracy.

Two pairs of Hartmann steel gauges were employed in the measurements. The following table gives their values at 0° C. :—

TABLE II.—Giving the Values of the Gauges employed.*

Approximate length of gauge in millimetres.	Sèvres value.	N.P.L. determination, using the Sèvres value of the 1000 mm. gauge as basis.
	mm. μ	mm. μ
60	60 - 1.5	60 - 1.3
160	160 - 0.7	160 - 0.7
200	200 + 0.1	200 + 0.4
300	300 - 1.1	300 - 0.9

By addition we get

	Sèvres value.	N.P.L. value.
	mm. μ	mm. μ
Gauges 300 + 60	360 - 2.6	360 - 2.2
„ 200 + 160	360 - 0.6	360 - 0.3

The Sèvres values are stated to be accurate within 0.5μ , and the probable error of

* $\mu = 1$ micron = 0.001 mm.

the N.P.L. values is of the same order. By taking the means of Sèvres and N.P.L. determinations as the best obtainable, we have

$$300 + 60 = 360 \text{ mm.} - 2.4\mu \text{ at } 0^\circ \text{ C.},$$

$$200 + 160 = 360 \text{ mm.} - 0.4\mu \text{ at } 0^\circ \text{ C.}$$

Using the dilatation equation determined at Sèvres on a bar of the same material

$$L_t = L_0 \{1 + (10.469 + 0.0035_2 t) t \times 10^{-6}\}$$

we get the following lengths at 17° C. :—

$$\text{Gauges } (300 + 60) = 360.062_0 \text{ mm.},$$

$$,, \quad (200 + 160) = 360.064_0 \text{ mm.},$$

with an increase of 3.8μ on the 360 mm. length per 1° C. increase in temperature at or about 17° C.

To obtain the mean temperature of the marble, holes were drilled in the cylinders and in these holes thermometers were inserted. The holes are parallel to the axis of the coils, and the depth of each is about half the axial length of a cylinder, so that the temperature recorded by a thermometer is very approximately the mean temperature of the cylinder.

The temperature coefficient of expansion of the marble was determined by making diametral measurements at temperatures varying from 11° C. to 20° C. Assuming that the coefficient of expansion of the steel gauges is correct, the mean coefficient of linear expansion of the marble was found to be 5.1×10^{-6} for the range 11° C. to 20° C. The observations were remarkably consistent, as will be seen from fig. 9, in which the difference of expansion of the steel and marble is plotted against temperature. During these observations we were much struck with the comparative rapidity with which the temperature of the marble followed that of its surroundings. The temperature coefficient of expansion of the marble was also deduced from measurements of the axial length of a cylinder. In this case the difference of expansion of invar and marble was directly recorded, and taking the invar as known the value obtained for marble is 4.9×10^{-6} , which is in good agreement with the value given above. When required in our work we took the mean of these values, *i.e.*, 5.0×10^{-6} , as correct.

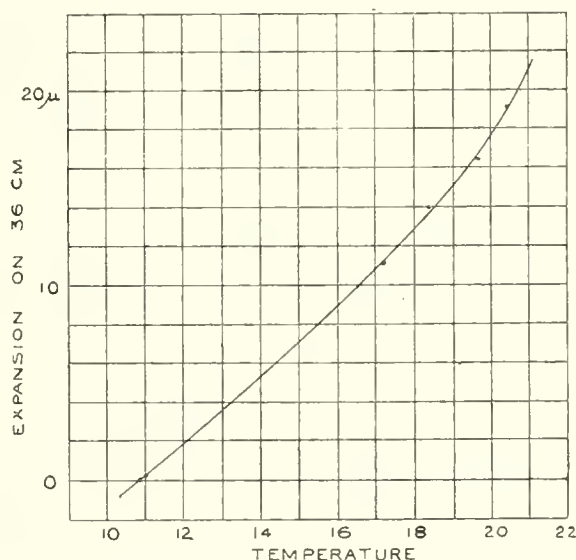


Fig. 9. Differential expansion, steel-marble.

As already stated, the contact pieces of the measuring machine are of agate, in

order that measurements of diameters may be made while a current is passing through the coils. The current used in our absolute measurements of a resistance did not exceed 2 amperes, and we experimented with such a current through the coils during our measurements of the diameters. Preliminary observations showed that the expansion of a cylinder 5 minutes after switching on the current was sufficiently great to allow of a definite measurement being made, and a thermometer recorded an increase of temperature of a whole degree in this time. A survey of the surface of the coil showed the expansion to be greatest in the mean diametral plane, and least at the ends; measurements were, therefore, made on four turns of wire, these being the 1st, 16th, 48th, and 96th strands measured from one end.

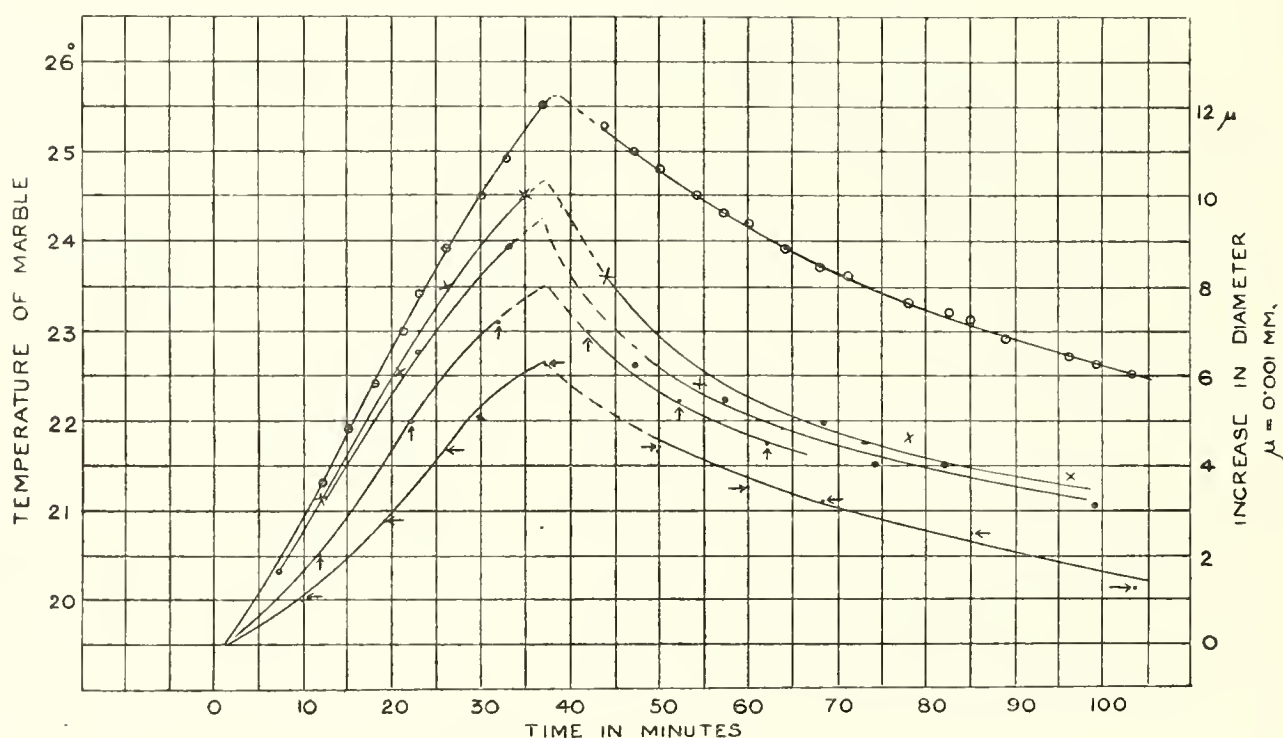


Fig. 10. Showing increase in temperature of marble cylinder and increase in diameter of certain parts of the coils when a current of 2 amperes is passed through them for 37 minutes.

○	indicates temperature observations	} measured from one end of coil.
×	„ observations on strand No. 96	
●	„ „ „ „ 48	
↑	„ „ „ „ 16	
←	„ „ „ „ 1	

The results of the measurements are given in figs. 10 and 11. In fig. 11 the expansion of the coil is given 5, 10, 15, 20, 25, and 35 minutes after a current of 2 amperes was switched on. The maximum increase in diameter is 10.4μ . Fig. 10 shows the rise in temperature recorded by the thermometer when the current was left on for 37 minutes, and shows also the fall in temperature after the circuit was broken. A relation between the mean rise in temperature due to the current and

the change in mutual inductance of the coils and the discs was worked out and found to be extremely useful when making the resistance measurements. The relation is as follows:—"If, in the resistance measurements, t_1 is the mean of the initial temperatures of the marble cylinders, and t_2 the mean of the final temperatures, the mean mutual inductance (during the resistance observations) of all the coils and the two discs is the same as when the temperature of the cylinders is uniform throughout and equal to $t_1 + 0.42(t_2 - t_1)$ other things being kept constant."

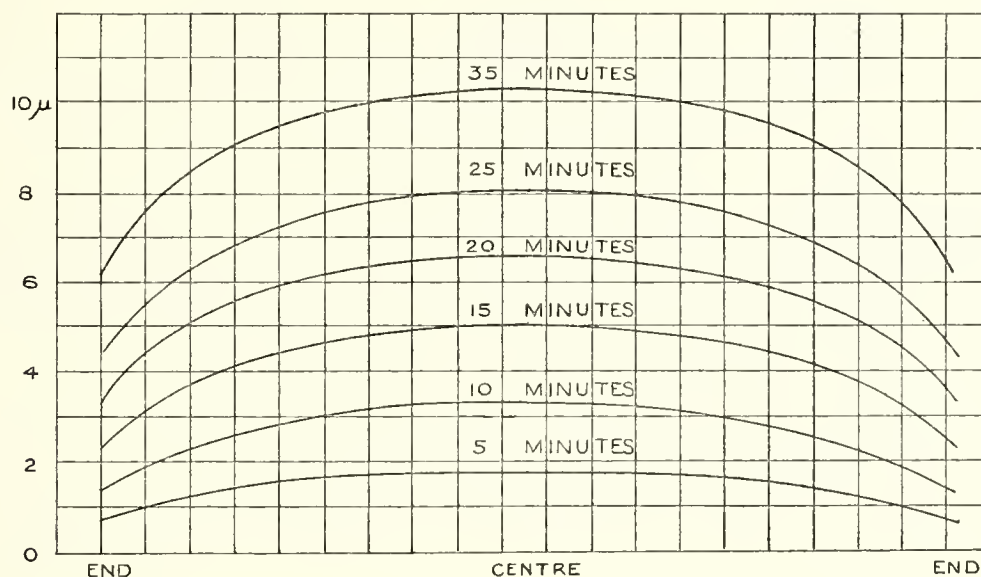


Fig. 11. Showing expansion of coils on marble cylinders at intervals from 5 to 35 minutes, when a current of 2 amperes is passed through them.

Complete measurements of the coils have been made on three occasions. The first set of measurements was made in February and March, 1912, and two sets were made in April, 1913. In the first set 192 observations were made on each coil, 16 observations being made in each of 12 axial planes 15 degrees apart. The turns of wire measured in any one plane were 1 cm. apart, and neighbouring strands were measured in succeeding planes, so that one measurement was made on every turn of wire.

In the measurements made in 1913, six measurements were made in each of 16 diametral planes 1 cm. apart, alternate sets of six measurements being made in axial planes 30 degrees apart, and the remaining measurements were made in planes midway between these. In addition, 16 observations were made in each of three axial planes 60 degrees apart, thus making 144 observations in all. Both pairs of gauge bars were used, and the difference found between them is identical within 0.5μ with that deduced from the values already given.

In the following table we give the mean diameters of the coils on the four cylinders as deduced from the measurements made in February and March, 1912, and April, 1913:—

TABLE III.—Giving the Mean Diameters of the Coils at 20°·0 C.

Coil on cylinder No.	February–March, 1912.	April, 1913.		Mean.
		(a)	(b)	
	cm.	cm.	cm.	cm.
1	35·8808 ₃	35·8807 ₂	35·8806 ₈	35·8807 ₄
2	35·8817 ₆	35·8815 ₂	35·8815 ₀	35·8815 ₉
3	35·8855 ₄	35·8853 ₀	35·8853 ₄	35·8853 ₉
4	35·8867 ₄	35·8866 ₂	35·8864 ₄	35·8866 ₀

The values recorded in columns 2 and 3 are exactly the means of 192 and 102 measurements respectively. The values in column 4 are not quite the means of the 48 observations. The cylinders are not quite circular in cross-section, and the mean of 48 measurements in three planes should, in general, be different from the mean of

TABLE IV.—Giving the Results of Measurements of the Diameters of the Coils on Cylinder No. 1. Temperature 20°·0 C.

Axial plane No. . .	1	2	3	4	5	6	7	8	9	10	11	12	
Measurements on turn No.	Diameter = 35·8750 cm. +												Mean.
1	μ . 29	μ .	μ . 14	μ .	μ . 22	μ .	μ . 27	μ .	μ . 30	μ .	μ . 30	μ .	μ . 25· ₃
12		27		26		31		36		36		23	29· ₈
24	34		30		32		36		35		35		33· ₇
36		35		37		42		41		47		45	41· ₂
48	41		39		39		46		48		49		43· ₇
60		40		42		48		47		49		48	45· ₇
72	61		57		59		61		64		63		60· ₈
84		65		66		69		70		75		65	68· ₃
96	67		65		76		74		72		70		70· ₇
108		70		69		72		69		78		66	70· ₇
120	58		61		63		65		72		66		64· ₂
132		66		65		73		74		73		68	69· ₈
144	67		63		68		69		71		72		68· ₃
156		66		70		74		76		70		73	71· ₅
168	75		67		73		71		72		71		71· ₅
180		65		67		70		72		65		66	67· ₅
192	64		65		70		74		78		70		70· ₂
Mean . . . =	55· ₁	53· ₉	51· ₂	55· ₂	55· ₈	59· ₉	58· ₁	60· ₆	60· ₈	61· ₆	58· ₄	56· ₈	57· ₂

∴ Mean diameter (centre to centre of wires) = $35·8750 + 0·0057_2 = 35·8807_2$ cm. at 20°·0 C.

measurements made in a large number of planes. This difference can be obtained from curves showing the cross-section of the cylinders, and in all cases we have estimated this difference and applied a correction.

It will be observed that the means of the 1913 measurements are about 2μ less than those of 1912. This difference cannot be due to the gauge bars employed, and we believe it is not due to errors of observation; it is probably due to a real

TABLE V.—Giving the Results of the Individual Measurements on the same Coils, when Measurements were made in Three Planes only.

Axial plane No. . .	Between 3 and 4	Between 7 and 8	Between 11 and 12	
Measurements on strand No.	Diameter = $35.8750 \text{ cm.} +$			Mean.
	$\mu.$	$\mu.$	$\mu.$	$\mu.$
2	24	28	31	27.7
12	24	29	32	28.8
24	26	32	35	31.0
36	32	35	38	35.0
48	40	42	47	43.0
60	42	45	47	44.7
72	54	66	65	61.7
84	61	66	67	64.7
96	58	61	69	62.7
108	67	71	74	70.7
120	57	69	63	63.0
132	61	67	67	65.0
144	56	75	70	67.0
156	71	79	76	75.3
168	67	80	74	73.7
180	68	75	71	71.3
192	70	79	75	74.7
Mean . . . =	51.6	58.5	58.9	56.3

$$\therefore \text{Mean diameter} = 35.8750 + 56.3\mu = 35.8806_3 \text{ cm. at } 20^{\circ}.0 \text{ C.}$$

From fig. 12 and the observations in Table IV. we conclude that the mean diameter in these three planes is less than that of the complete coils by 0.5μ . Hence mean diameter of the coils = 35.8806_3 cm. at $20^{\circ}.0 \text{ C.}$

diminution in the diameters of the coils due to a displacement of small crystals of marble under the wires. This is quite possible, as the coils are wound under tension and have been subjected to considerable changes in temperature (due to the current passing through them) since 1912.

Tables IV. and V. give the results of the individual measurements made in April, 1913, of the coils on cylinder No. 1.

The curves given in fig. 12 show to what extent the coils vary in diameter. The conicality of the coils on cylinders 3 and 4 is much less marked than in the case of those on cylinders 1 and 2, but the variation from a circular cross-section is very small for all of the coils. The variation of diameter in any cross-section is so small

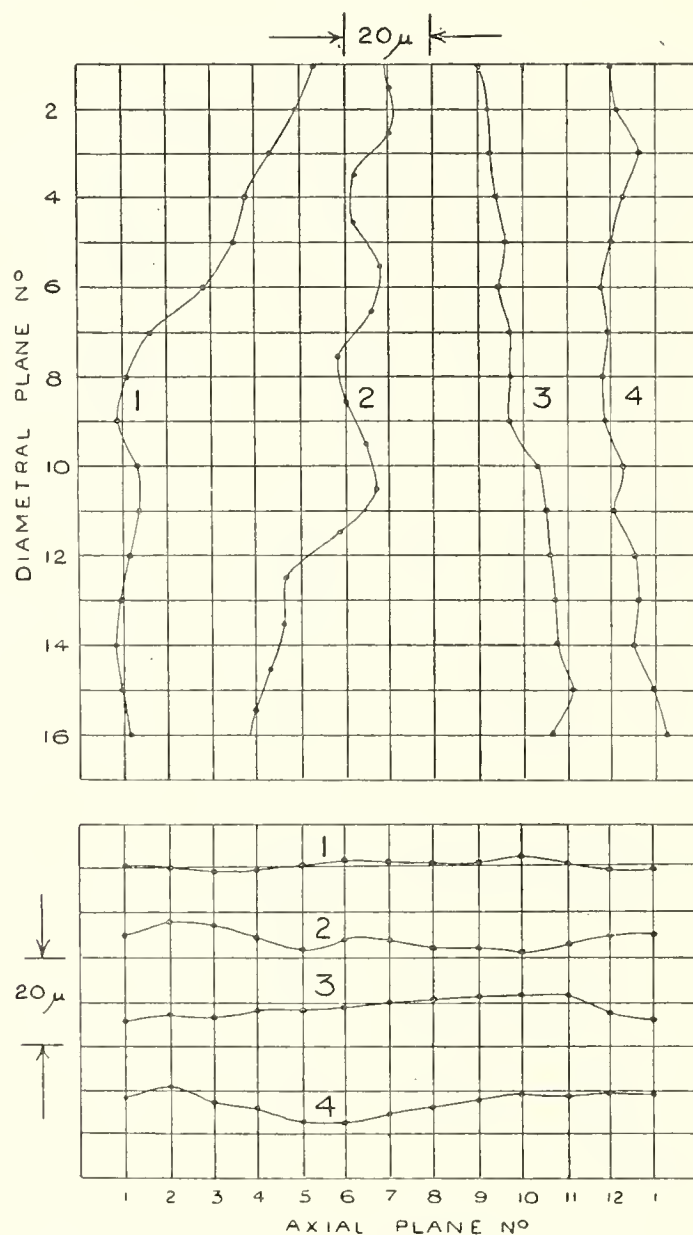


Fig. 12. Showing extent to which coils are conical, and variations in cross-section from circle.

that the mean diameter of the cross-section may be employed for the purposes of the calculation of the mutual inductance without appreciable error, but the variation in diameter from end to end of the coils necessitates the application of a conical correction.

The probable error of the mean diameter of the coils is estimated to be not greater than 1μ . The probable errors of the gauge bars used are stated on p. 54; the probable errors of the observations may be estimated from the data given in Table III.

Measurement of the Axial Lengths of the Coils and the Variation of Pitch.

The pitch measuring machine, also constructed by Sir W. G. Armstrong, Whitworth and Co., consists of a straight horizontal bed provided with dead centres on which the mandrel carrying the coil can be mounted. Parallel to the line of centres is a slide, along which a small slide rest can be traversed by means of a calibrated measuring screw, of $\frac{1}{10}$ -inch pitch. The measuring wheel which actuates this screw can be read to $\frac{1}{100,000}$ inch by means of a vernier, but the accuracy of repetition of readings is somewhat less than this. The slide carries a microscope whose optical axis intersects the line of centres of the machine at right angles. The microscope is focussed on the coil and is moved by means of the measuring screw so as to bring the consecutive turns of the coil, one after the other, into a definite position with respect to the cross wires in the eye-piece, the reading of the measuring wheel being taken at each setting. The calibration of the screw is effected by focussing a graduated line standard in the microscope and taking similar readings on the graduations of the bar. The measuring screw and slide are themselves movable on a second slide in a direction parallel to the axis of the microscope, so that by withdrawing them slightly the standard bar can be inserted at any time between the coil and the microscope, and check readings taken on it without disturbing the coil.

The standard employed was an invar metre, the history of which is so well known that the probable error over a length of 12 cm. (the axial length of a coil) is less than 1μ .

For the measurements on each cylinder, readings on successive half centimetre divisions of the invar metre were taken over a length of 14 cm., and readings over a length of 12 cm. (corresponding to the axial length of the coils), were frequently made. Any change in temperature of the screw of the measuring machine could thus be allowed for.

As already explained, the two wires on each cylinder are wound in double screw cut grooves. To measure the mean variation in pitch of the two coils it is better, therefore, to make observations on the spaces between neighbouring wires than on the wires themselves, for in the latter case twice as many observations are required. In practice, observations on the spaces proved the easier way; the white marble showed up well between the wires, and it was not difficult to bisect the white spaces by means of the cross wires of the microscope. In addition, check observations on the wires were always made. For a complete set of measurements 96 readings are required, and three such sets in different axial planes were made on each cylinder.

After correcting the readings, the mean pitch of the coils was calculated within a few parts in 10,000 by dividing the difference of the extreme readings by the number of turns. The approximate pitch thus obtained was 0.065635 inch ($= 0.166712_5$ cm.). We multiplied this number by 1, 2, 3, 4, ..., &c., up to 95, and from the resulting products the corresponding pitch readings of a coil were subtracted. If the coil had

been perfectly uniform in pitch, the differences would, when plotted, have lain on a perfectly straight line, but, instead, wavy lines result (fig. 13). As an example of the difference readings those for the coils on cylinder No. 1 are given in Table VI.

TABLE VI.—Giving the Difference Readings in Microns for the Coils on Cylinder No. 1.

μ .	μ .	μ .	μ .
16	21	36	32
11	20	31	38
7	24	36	37
12	39	47	38
28	43	51	31
31	25	42	35
23	25	36	33
17	19	25	37
5	15	27	37
8	17	35	44
3	30	45	39
23	45	53	43
27	38	40	42
37	31	35	32
27	25	29	38
22	23	23	33
10	26	28	36
12	31	33	41
15	43	45	47
28	50	44	39
46	52	41	36
42	42	34	44
35	40	30	43
28	31	27	37

From the products resulting when the numbers 1, 4, 7, 10, &c., up to 94 were multiplied by 0.065635, the half centimetre readings for the invar metre were subtracted and the differences when plotted were found to lie on a straight line as they should do if all the corrections have been properly applied. The graphs for the invar metre are shown in fig. 13 and are distinguished as Invar₁, Invar₂, &c., the numbers corresponding to the numbering of the four cylinders. A phosphor-bronze ring is screwed into one end of each cylinder, and these ends are called the "ring ends."

It is clear from fig. 13 that there is a periodic variation in the pitch of all the coils. The distance corresponding to a complete cycle is half an inch, which is equal to the pitch of the leading screw of the lathe on which the cylinders were turned. The cause of the periodicity may be in the leading screw itself, or it may be due to want of parallelism of the surfaces at which the thrust was taken, or it may be due to both of these. Such periodic variations in pitch are present in nearly all screws. In addition to these periodic irregularities, the graphs show others which are taken into consideration when the mutual inductance is calculated of a coil and disc circumference.

The graphs for the coils enable one to place the cylinders in pairs. The lengths at

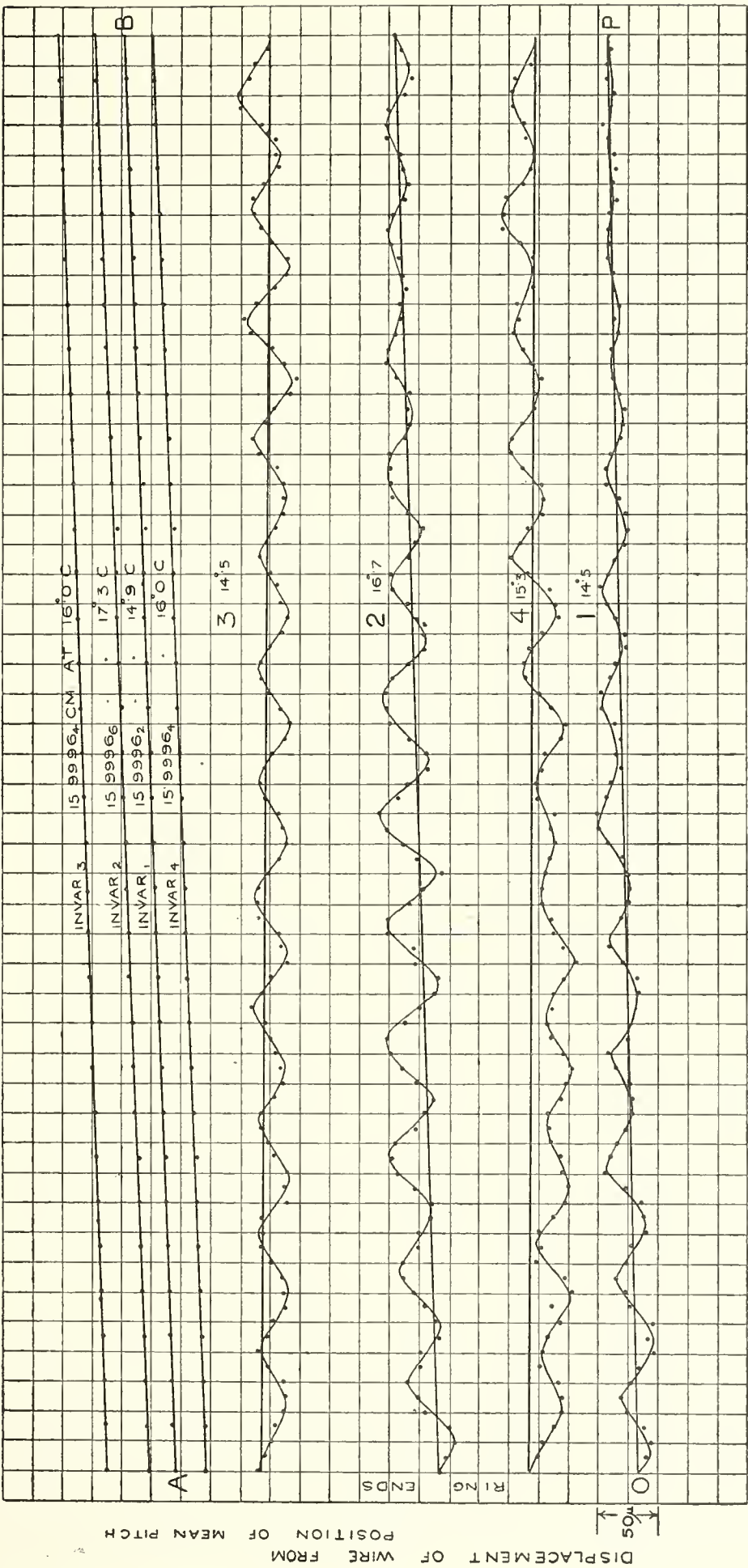


Fig. 13. Showing variations in pitch of coils.

20°·0 C. of the coils on cylinders Nos. 1 and 2 are 16·0018 cm. and 16·0013₅ cm. respectively; those on cylinders Nos. 3 and 4 are 16·0054 cm. and 16·0054 cm. respectively. The amplitude of the periodic variation dies away towards one end for the coils on cylinders Nos. 1 and 2, but for the coils on cylinders Nos. 3 and 4 it does not. The graphs for the variations in diameter of the coils lend support to this division of the cylinders into pairs, and as the same lathe and the same portion of the leading screw were used for the turning of all, the differences appear to be due to a slight difference in the skill and touch of the late Mr. TAYLERSON, who turned cylinders Nos. 1 and 2, and Mr. TRIBE, who was responsible for cylinders Nos. 3 and 4. In all cases the turning is remarkably good.

The mean axial length of the coils on one cylinder is obtained from the graph showing the periodic variation of the pitch and the corresponding graph for the invar metre. Limiting our attention to the coils on cylinder No. 1, it is clear that the extreme mean length of the coils is equal to the length of the invar metre section AB plus the length corresponding to the difference of the ordinates OA and PB. The length of the invar section is 15·9996₂ cm. and the difference of the ordinates corresponds to +17 μ . Hence the mean length of the coils on cylinder No. 1 is $15·9996_2 + 17\mu = 16·0013_2$ cm. at 14°·5 C. = 16·0018 cm. at 20°·0 C.

Table VII. gives the mean axial length of the coils at 20°·0 C.

TABLE VII.

Coils on cylinder.	Mean axial length at 20°·0 C.
	cm. .
1	16·0018
2	16·0014
3	16·0054
4	16·0054

These observations for the mean axial length were made in April, 1913. The results are identical within the limits of the errors of observation (about 3 μ) with those obtained in March, 1912. A change in the length of a coil of 20 μ produces a change in the mutual inductance of that coil and the two discs of 1 part in 100,000.

SECTION 14.—ERECTING AND ADJUSTING THE INSTRUMENT.

Before assembling the parts of the machine, a concrete block, built up of Keene's cement and Thames ballast, was prepared as a foundation. The block is non-magnetic, is 8 m. long, 80 cm. wide, and 120 cm. deep. Slide rails of gunmetal are bolted in position on this block and to secure greater stability the rails are sunk 5 cm. into the concrete. The rails are in three pairs; one pair support the motor; a second pair the fly-wheel; and the third pair, which are nearly 4 m. long, support the portions with the rotating discs. A period of twelve months was allowed for the concrete block to

assume approximately constant dimensions, and at the end of that time the upper surfaces of the rails were scraped to obtain flat surfaces and to ensure these surfaces lying in the same horizontal plane. Two special fitters from Sir W. G. Armstrong, Whitworth & Co. carried out this part of the work, and also superintended the alignment of the parts of the machine. The under surfaces of the castings supporting the fly-wheel, discs, &c., were scraped plane at the Elswick Works and the alignment of the parts was, in consequence, a comparatively easy task. Originally, the couplings were intended to be flexible ones, but after a few runs of the machine rigid couplings were found to be better and such were used. Much of the work connected with the couplings and other fittings to the machine was carried out in the Engineering Department of the Laboratory, and we are greatly indebted to Dr. STANTON, the Superintendent, for much advice on these matters.

At first there was trouble with the bearings. The clearance allowed was very small, and after the machine had run for one or two hours the expansion of the shaft was sufficiently great to cause a collar to come into contact with one of the bearings. Increased clearance was allowed and the difficulty disappeared. From that time (April, 1911) no portion of the machine has given the slightest trouble. The parts are so exactly balanced that the tremor of the concrete base is almost too small to be detected. This is fortunate, as during the observations for an absolute measurement of a resistance, two microscopes are mounted on the concrete bed and used to gauge the distance apart of two coils within a thousandth of a millimetre. These measurements were frequently made when the speed of the machine was 1040 revolutions per minute, but an accuracy within a thousandth of a millimetre was obtained with little trouble. During the progress of our work the machine has been admired by many hundreds of visitors to the Laboratory, and the kindness of Sir ANDREW NOBLE in having the heavy metal work carried out at Elswick is greatly appreciated.

To place the cylinders in position on the cradles, the portions of the shaft supporting the discs were removed from their bearings, the cylinders threaded over the shafts into approximately correct positions and the shafts restored to their proper places. To prevent damage to the cylinders during this operation, a framework of wood was built to support the shafts and cylinders and to enable them to be lifted as one piece. The cradles on which the cylinders are supported are provided with stops and clamping screws to prevent any marked relative movement of a cylinder and the cradle on which it rests.

SECTION 15.—INSULATION TESTS.

When making an absolute measurement of resistance, the greatest difference of potential between any portion of the circuit and the earth was about 130 volts, and the greatest difference of potential between neighbouring turns of wire on any one cylinder was about 16 volts.

The insulation resistance between the circuit and the earth was tested each day on which measurements of resistance were made. For the insulation test, the earth wire

(p. 85, fig. 19) was removed, and the voltage applied to the circuit was the same as that used for the resistance measurements. The insulation resistance was always greater than 1000 megohms.

To measure the insulation resistance between the coils, the latter were usually arranged in two groups, each group consisting of one coil from each cylinder. At first, with an applied voltage of 20 volts, the insulation resistance was about 10,000 ohms. The insulation resistance of coils on cylinders Nos. 2, 3, and 4 proved to be well above 1000 megohms, but that on the coils on cylinder 1 was low. We found these coils to be in close proximity to, or in contact with, a few crystals of pyrites, and after dislodging these, wholly or in part, the insulation resistance increased to above 1000 megohms. No trouble has since occurred. As the coils are of bare copper wire and are covered only with a thin wrapper of silk, we think it necessary to make an insulation test of the coils on each day that resistance measurements are made. The test occupies but a few minutes, and during our work it was regularly made.

The insulation resistance between the earth and the rotating wires attached to the discs was usually tested at 20 volts. The results in all cases were satisfactory. For certain measurements, it is essential that the rotating wires be insulated from each other and as the wires may not occupy exactly the same positions with respect to the disc and shaft when rotating, as when stationary, the tests are preferably made with the machine running. The test when the wires are rotating is of some interest. One terminal of the galvanometer is connected to one brush, and one pole of the battery to the remaining four brushes, the five brushes being in contact with the segments on one of the discs. The other terminal of the galvanometer is connected to the remaining pole of the battery. From fig. 5, given on p. 42, it is clear that when the discs are rotating one terminal of the galvanometer is always connected to one or two of the rotating wires, and the other terminal, through the battery, is connected sometimes with four and sometimes with eight of the wires. The wires are continually changing in position and the deflection of the galvanometer enables the insulation resistance to be calculated. The first set of conducting wires which were used became faulty in insulation resistance because of the action of some insulating tape which was used to bind the wires together. They were replaced by others which were supplied with double layers of silk and which we subsequently shellacked and encased in silk tubes. No trouble has since been experienced, the insulation resistance being well over 200 megohms. The insulation of the remaining part of the circuit, *e.g.*, leads to brushes, standard resistance, &c., was frequently tested, but no fault was found.

SECTION 16.—MEASUREMENT OF THE DIAMETRAL DISTANCE BETWEEN OPPOSITE SEGMENTS.

The distance between opposite segments was measured both when the discs were stationary and when running at speeds varying from 170 to 1110 revolutions per minute.

For the stationary measurements, two micrometer heads were fixed to, but insulated from, two short upright rods of brass secured to a stout bar of the same metal, the distance between the contact faces of the micrometer screws being approximately equal to the distance between opposite segments. This gauge was supported on two uprights secured to the slide rails of the Lorenz apparatus and adjusted in position for the measurement of a diameter. Between the segments and the micrometer screws, vertical wires of phosphor bronze, similar to those used for the brushes, were interposed, and the measured distance was taken as equal to the distance apart of the segments plus twice the diameter of a wire. Contact between a segment and a micrometer head was indicated by the buzzing of a telephone due to the passage of a small alternating current through a circuit including the telephone, the micrometer head, and the segments. Four measurements were made on each pair of segments lying on opposite sides of a diameter and the mean of the 20 measurements was taken as correct. Such a series of measurements was frequently taken and show the wear of the segments, due to their friction with the brushes, to be comparatively slight. As examples, we give the results of some measurements made in December, 1912, and in June, 1913.

TABLE VIII.—Giving the Diametral Distance between Opposite Segments.

Segments.	Disc No. 1.		Disc No. 2.	
	December, 1912.	June, 1913.	December, 1912.	June, 1913.
	cm.	cm.	cm.	cm.
1 and 6	53·581	53·562	53·556	53·546
2 „ 7	·579	·566	·556	·547
3 „ 8	·579	·569	·550	·546
4 „ 9	·579	·573	·556	·549
5 „ 10	·584	·570	·554	·560
Mean	53·580	53·568	53·554	53·550
Mean	53·574 cm.		53·552 cm.	
Mean radial distance } of segments . . }	= 26·787 cm.		= 26·776 cm.	

The standard of length used was an invar rod with approximately flat ends, and measurements on this rod were made in the same way as those on the segments.

When the discs were rotating, contact with the segments was made by advancing a brush by means of a micrometer screw until a telephone indicated the completion of a circuit as in the previous measurements. The micrometer head made contact with the brush holder through the medium of a small steel hemisphere attached to the

holder, and good contact was ensured by a spring attached to the supporting ring and brush, which forced the latter towards the micrometer screw. The brush wires were put under considerable tension and petrol was used as a lubricant. Measurements were made at speeds varying from 170 revolutions per minute to 1110 revolutions per minute, the speed being registered by the directly driven chronograph. The results of three sets of measurements are plotted in fig. 14. The relation between the

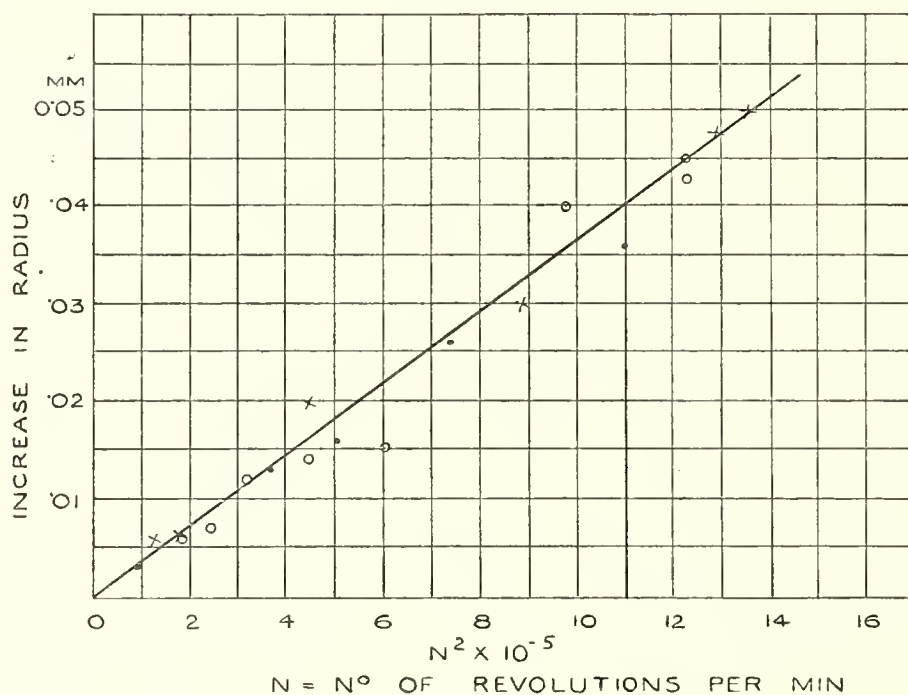


Fig. 14.

increase in radius and the square of the number of revolutions per minute is $dr = 3.7n^2 \times 10^{-8}$, where dr is the increase in radius in millimetres and n is the number of revolutions per minute. The normal speed of the apparatus when resistance measurements are in progress is about 1050 per minute, and the mean radius is then 0.04 mm. greater than the mean value given in Table VIII. The effective radial distance at $20^\circ 0$ C. of the segments is therefore $26.787 + 0.004 = 26.791$ cm. for disc No. 1 and $26.776 + 0.004 = 26.780$ cm. for disc No. 2.

SECTION 17.—DETERMINATION OF THE DISTANCE BETWEEN THE CENTRES OF THE COILS.

The distance between two coils on opposite sides of a disc has to be known with considerable precision, as a change of three-thousandths of a millimetre in this distance changes the mutual inductance of the coils and discs by 1 part in 100,000. As it is not possible to make direct measurements of this distance with both rapidity and precision, we followed the plan adopted by GLAZEBROOK* (and by Lord RAYLEIGH in 1882) of reversing the coils, without interchange, and then repeating the resistance

* 'Phil. Trans.,' vol. 174, p. 254, 1883.

measurements. Reference marks are made on the flanges of the cylinders, and when the coils are parallel the mean distance between two such marks, one on each cylinder, in the two positions of the coils, is exactly equal to the mean distance between the mean planes of the coils. If there are two marks diametrically opposite on each cylinder and two distances are measured for each position of the coils, the mean distance between the marks is the mean distance between the centres of the coils if the latter are *approximately* parallel. In our case a want of parallelism of 0.25 degree (far in excess of that met with in practice) introduces an error of less than 1μ in the determination of the distance between the centres of the coils. In practice we have four marks on each cylinder, these being at approximately equal distances from the mean plane of a coil, and the distance of each mark from such plane is known within 10μ or 20μ . The vertical distances of the marks from the axis of the coil are the same within 0.1 mm., and for the measurements under consideration the distances may be regarded as identical.

Fig. 15 represents diagrammatically two cylinders at a mean distance $00'$ apart. When the coils are parallel, the distances ac , eg , bd , and fh are approximately equal to $00'$. When the coils are reversed but not interchanged, the distances are in general different from what they were before; let the distances be $a'c'$, $e'g'$, &c. With parallel

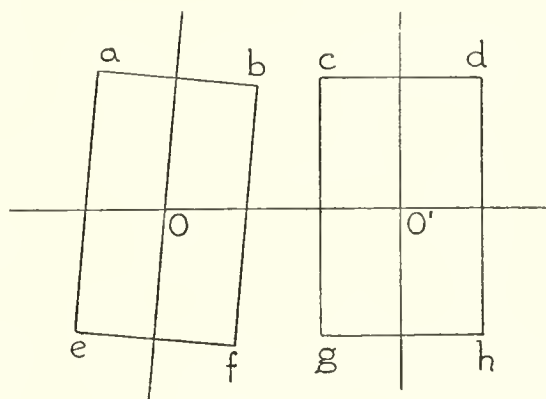


Fig. 15.

coils each of the mean distances $(ac + a'c')/2$, $(eg + e'g')/2$, &c., is equal to the average distance apart of the mean planes of the coils. If the two coils are not parallel but very nearly so, the average of the distances ac , $a'c'$, eg and $e'g'$ is equal to the average of the remaining four distances and also equal to the average distance apart of the centres of the coils.

In the flanges of each cylinder four brass plugs are screwed and cemented in position. The plugs in each flange are at opposite ends of a diameter, and the four plugs in one cylinder are contained in a common axial plane. Fig. 4 shows two of the plugs ab in cylinder No. 3, and two cd in cylinder No. 4. On the faces of the plugs thin rectangular pieces of platinum are soldered; the surfaces of these are polished, and a scale of half millimetres, cut at right angles by two horizontal lines, half a millimetre apart, is engraved on each piece. The scales were engraved by the late Mr. DONALDSON of the Metrology Department.

For the measurement of the distances, two microscopes are mounted on a special platform supported on a tripod, the base of which can be clamped to the bed of the Lorenz machine by means of the wing nuts w , w' , w'' , &c., shown in fig. 4, and by means of screws the platform can be given the necessary movements to enable both microscopes to be rapidly focussed without any change in their relative position. When the platform is clamped by the wing nuts ww' , the lines on the plugs a and c

are under observation, and when the platform is clamped by means of w' and w'' , the lines on plugs b and d can be viewed, and so on for the remaining positions.

The reference standard is a scale of half millimetres, engraved on invar. The scale was standardised by Mr. ATTWELL before making any absolute measurements of resistance, and again after their completion. The results on the divisions employed in our work agree within less than 1μ .

The general procedure in making the measurements was to observe the invar scale, then the eight plug distances, and finally the invar scale again. The first and last of the observations agreed in general within 1μ or 2μ . An absolute measurement of resistance immediately followed and then another set of observations was made on the invar scale and the plugs. The coils were not reversed in position until a large number of resistance measurements had been made.

The distance between coils Nos. 1 and 4 and between Nos. 2 and 3 is required with a much less degree of accuracy, an error of 1 mm. in both of these distances producing a change in the mutual inductance of less than 1.5 parts in 100,000. As the distances between coils Nos. 1 and 2 and Nos. 3 and 4 are known from the previous measurements within a few thousandths of a millimetre, it was only necessary to measure the distance 1 to 4, or 2 to 3. In practice we measured the distance between a plug on cylinder 2 and a plug on cylinder 3, and employed for this purpose a brass bar with half-millimetre divisions engraved at the ends; this enabled the distance to be determined within about 0.1 mm. The distances of the plugs from the mean diametral planes of the coils were known and no further measurements were therefore necessary. Instead of measuring the distance between two coils such as Nos. 2 and 3, we may, as an alternative, measure the distance between the two sets of five brushes, and a number of such measurements were made as checks.

SECTION 18.—CALCULATION OF MUTUAL INDUCTANCE OF THE COILS AND THE CONTACT CIRCLES OF SEGMENTS AND BRUSHES.

The arrangement of the coils and discs in the instrument may be represented diagrammatically in section by fig. 16. The coils are numbered 1, 2, 3, and 4, and the discs are denoted by D_1 and D_2 .

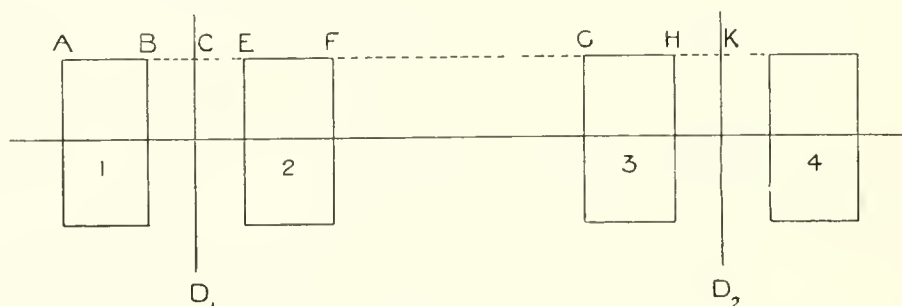


Fig. 16.

The mutual inductance M of any one coil and the contact circles is calculated in two parts. The first part gives the mutual inductance M_1 of the coil and the circumference

of the disc nearer to it, and the second part gives the mutual inductance M_2 of the coil and the circumference of the other disc. The difference of these is required, so that $M = M_1 - M_2$.

To find M_1 , two mutual inductances were calculated, viz., that between D_1 and a coil of length BC, and that between D_1 and a coil of length AC. The difference of these mutual inductances gives M_1 . If the coils on the four cylinders are exactly similar the values of M_1 are identical.

To find M_2 for coils Nos. 1 and 4, the mutual inductances were calculated between D_2 and a coil of length BK and that between D_2 and a coil of length AK. The difference is equal to M_2 . To find M_2 for coils Nos. 2 and 3, the mutual inductances required are that between D_2 and a coil of length EK and that between D_2 and a coil of length FK.

In the Lorenz apparatus, the distances of the coils from the discs can be varied. This changes the mutual inductance and the rate of change of M with variation of axial distance must therefore be known.

To find M_1 we used the following formula due to J. VIRIAMU JONES*

$$M = \Theta (A + a) ck \left\{ \frac{F - E}{k^2} + \frac{c'^2}{c^2} (F - \Pi) \right\}.$$

In this expression, Θ is the angular length of the helix, A the radius of the helix, a the radius of disc or contact circle, and x the axial length of helix.

$$\begin{aligned} c^2 &= 4Aa/(A+a)^2, & c'^2 &= 1 - c^2, \\ k^2 &= 4Aa/(A+a)^2 + x^2, & k'^2 &= 1 - k^2, \end{aligned}$$

F , E , and Π , are complete elliptic integrals of the first, second, and third kinds respectively; F and E are to modulus k , and

$$\Pi = \int_0^{\pi/2} \frac{d\psi}{(1 - c^2 \sin^2 \psi)(1 - k^2 \sin^2 \psi)^{1/2}}.$$

Putting $c'/k' = \sin \beta$, the quantity $(F - \Pi)$ can be expressed in terms of complete and incomplete integrals of the first and second kinds†; thus

$$c^{-1}k'^2 \sin \beta \cos \beta (F - \Pi) = -\frac{1}{2}\pi - F(k) F(k', \beta) + E(k) F(k', \beta) + F(k) E(k', \beta).$$

The various elliptic integrals required were calculated by interpolation from LEGENDRE'S tables, but as a check on possible misprints in the tables a number of the integrals were calculated directly by successive quadric transformation.‡

The dimensions chosen for calculating the values of M_1 are given in Table IX. The

* J. V. JONES, 'Roy. Soc. Proc.,' vol. 63, p. 198, 1898.

† CAYLEY, 'Elliptic Functions,' § 183.

‡ CAYLEY, Chapter XIII.

dimensions of the coils and discs differ from these values by small amounts dA for the radius of the coils, da for the radius of the discs, and dx for the length of the coils. Small corrections to the calculated values in Table IX. have therefore to be applied. These corrections are obtained by application of the increment formula

$$\frac{dM}{M} = q \frac{dA}{A} + r \frac{da}{a} + s \frac{dx}{x},$$

which gives the change in M due to small changes in dimensions, A being the radius of the coil, a that of the disc, and x the axial length of the coil; q , r , and s are coefficients which are given by the expressions

$$q = \frac{\Theta ck}{M} A \left\{ F + \frac{A-a}{2a} (F - \Pi) \right\},$$

$$r = \frac{\Theta ck}{M} a \left\{ F - \frac{A-a}{2A} (F - \Pi) \right\},$$

$$s = -1 - \frac{\Theta ck}{M} (A + a) \left\{ \left(1 - \frac{2}{k^2} \right) F - \frac{2}{k^2} E \right\}.$$

The sum $q + r + s$ is always equal to unity.

In Table IX. we give in columns 1, 2, and 3 the constants employed for the calculation of nine mutual inductances; in column 4 the values of the mutual inductances; and in columns 5, 6, and 7 the values of q , r , and s , which are required in our work. Table X. gives the difference values M_1 .

TABLE IX.—Calculation of Mutual Inductance.

A = radius of coil, a = radius of circle (disc), x = length of coil.

The number of turns per centimetre length of the coil is 12.

A.	a.	x.	M.	q.	r.	s.
cm.	cm.	cm.	cm.			
17·9419	26·7870	23·2650	52702·37	2·1003	-0·5737	-0·5265
17·9419	26·7870	7·2635	23682·39	2·4007	-1·2626	-0·1381
17·9419	26·7870	23·3150	52755·93	2·0998	-0·5724	-0·5274
17·9419	26·7870	7·3135	23822·74	2·3991	-1·2596	-0·1396
17·9419	26·7870	23·2900	52729·18	2·1000	-0·5731	-0·5270
17·9419	26·7870	7·2885	23752·65	2·3999	-1·2611	-0·1388
17·9419	26·7870	19·2851 ₁	47883·64	2·1404		
17·9419	26·7870	11·1678 ₈	33534·20	2·2880		
17·9419	26·7870	15·2572 ₅	41656·11	2·2001		

In Table X. the first difference values are the mutual inductances of a coil 16·0015 cm. long and a circle of radius 26·7870 cm., when the mean diametral plane

of the coil is distant from the circle 15.2642_5 cm., 15.2892_5 cm., and 15.3142_5 cm. respectively. In these three positions an increase in the radius of the coil of 10μ increases the mutual inductance by 3.001, 2.995, and 2.988 respectively, and an increase in the radius of the circle of 100μ increases the mutual inductance by -0.125 , -0.099 , and -0.072 respectively. If we have two such coils, one on each side of the circle, the intensity of the magnetic field at the circumference of the circle will be zero when the mean planes of the coils are about 30.656 cm. apart and the circle is

TABLE X.—Difference Values of Mutual Inductance.

A = radius of coil	17.9419	17.9419	17.9419	17.9419	17.9419	17.9419
a = radius of circle	26.7870	26.7870	26.7870	26.7870	26.7870	26.7870
x = Axial length of coil . .	23.2650	7.2635	23.2900	7.2885	23.3150	7.3135
M = mutual inductance . .	52702.37	23682.39	52729.18	23752.65	52755.93	23822.74
Difference value = M_1 . .	29019.98		28976.53		28933.19	
dM for $dA = 10\mu$	+6.169	+3.168	+6.172	+3.177	+6.174	+3.186
Difference value	+3.001		+2.995		+2.988	
dM for $dA = 100\mu$	-11.288	-11.163	-11.281	-11.182	-11.274	-11.202
Difference value	-0.125		-0.099		-0.072	

midway between them. The same result is of course brought about by a slight reduction in the diameter of the brush contact circle, an inevitable result of wear and re-turning of the surface of the segments. However, exact realisation of this condition is unimportant since for the positions of the coils dealt with in the calculations, the maximum change in the mutual inductance is only 4 parts in 1,000,000 for a change in the diameter of the circle of one-fifth of a millimetre. Exact centering of the circle between the coils is also unimportant as will now be shown.

When the distance of the circle from each of the mean diametral planes of the coils is exactly 15.2892_5 cm. the total mutual inductance of the two coils and the circle is 57953.0_7 . If the circle is moved so as to be 0.25 mm. nearer one of the coils the total mutual inductance is increased to 57953.1_7 , *i.e.*, about 1 part in 600,000 greater than before. Hence with a brush half a millimetre wide the variation of potential from wire to wire will not exceed 1 part in 600,000. In practice the brushes commonly used consisted of three parallel wires, the extreme width being about 0.5 mm.

Table XI. gives the constants used and the results obtained in one of the calculations dealt with in Table X. and gives also the dimensions of the Lorenz coils and brush contact circles at 20°·0 C. Table XII. gives (1) the differences of dimensions of the coils and standard, and (2) the mutual inductance M_1 of each coil and the contact circle nearer to it.

TABLE XI.

	Standard.	Coils on cylinder No.—			
		1.	2.	3.	4.
A = radius of coil in centimetres .	17·9419	17·9403 ₇	17·9408 ₀	17·9427 ₀	17·9433 ₀
x = length of coil in centimetres .	16·0015	16·0018	16·0014	16·0054	16·0054
a = radius of circle in centimetres	26·7870	26·791	26·791	26·780	26·780
d = distance of mean diametral plane of coil from the circle in centimetres	15·2892 ₅				
n = number of turns	16·0015 $\times$ 12	192	192	192	192
M_1 = mutual inductance of coil and circle	28976·54				

We summarise below our knowledge of the changes of M_1 with changes in the dimensions of the coils and disc when the distance of the mean diametral plane of the coil from the brush contact circle is equal to 15·2892₅ cm.

	Change in M_1 .
1. Increase in radius of a coil by 10 μ	+2·995
2. Increase in radius of contact circle by 10 μ	−0·0099
3. Increase in length of coil by 10 μ , the position of the mean diametral plane remaining constant, and <i>the number of turns increased so that there may still be 12 turns per centimetre</i>	+1·940
4. Correcting factor to reduce the mutual inductance to a coil of exactly 192 turns is	16/length of coil in centimetres.

The values of M_1 given in Table XII. hold good only when the mean diametral plane of the coil is 15·2892₅ cm. from the circle. For other slightly different distances we may readily calculate the corrections from the data given in Tables IX. and X. The mutual inductance of the coils on cylinders Nos. 1 and 2 and the brush contact circle No. 1 is 57939·7 when the distance between the mean planes of the coils is 30·5785 cm., and the corresponding values for the coils on cylinders Nos. 3 and 4 is

57955.4, the temperature being 20° C. For an increase in the distance apart of the mean planes of Nos. 1 and 2 by 250 μ the mutual inductance diminishes by 43.43, and for a decrease in the distance by the same amount the mutual inductance increases by 43.32. The corresponding values for cylinders Nos. 3 and 4 are practically identical, being 43.48 and 43.37 respectively.

TABLE XII.—Giving the Mutual Inductance M_1 of the Coils and Contact Circles on the Assumption that the Radius is at all Points Equal to the Mean Radius and that the Pitch is Uniform.

Coils on cylinder No.—							
1.		2.		3.		4.	
Differences.	Correc- tion to M_1 .	Differences.	Correc- tion to M_1 .	Differences.	Correc- tion to M_1 .	Differences.	Correc- tion to M_1 .
$dA = -15.3^{\mu}$ $dx = + 3$ $da = + 40$	-4.58 $+0.58$ -0.04	$dA = -11^{\mu}$ $dx = - 1$ $da = + 40$	-3.29 -0.19 -0.04	$dA = + 8^{\mu}$ $dx = + 39$ $da = - 70$	$+2.40$ $+7.57$ $+0.07$	$dA = +14^{\mu}$ $dx = + 39$ $da = - 70$	$+4.19$ $+7.57$ $+0.07$
For number of turns n }	-3.26	—	-2.54	—	-9.78	—	-9.78
Total correction }	-7.30	—	-6.06	—	$+0.26$	—	$+2.05$
$M_1 = 28969.24$		$M_1 = 28970.48$		$M_1 = 28976.80$		$M_1 = 28978.59$	
Sum = 57939.7				Sum = 57955.4			

With change of temperature the mutual inductance varies owing to the expansion of the coils and discs. If the distance apart of the mean diametral planes of the coils is kept constant, the temperature coefficient of M_1 is readily calculated to be 1.06×10^{-5} for an increase in temperature of 1° C.

We have previously stated that it is not necessary to know the axial length of a coil with great accuracy. From the data given on page 74, it is clear that if the coils increase in length by 10 μ , the change in M_1 for the coils on any one cylinder is +1.94. But this supposes the number of turns to increase in the ratio 16.0010/16.0000. Since the number of turns is constant the real change is $1.94 + 28970 - 28970 \times (16.0010/16.0000) = 0.13$. That is, a change of 10 μ in the axial length of the coils produces a change in the mutual inductance of the coils and discs of about 5 parts in 1,000,000.

Calculation of M_2 , the Mutual Inductance of the Coils, and the farther Brush Contact Circle.

The following formula, due to ROSA and GROVER,* gives the mutual inductance of a single layer coil of length x and a co-axial circle of radius A in the plane of one end of the coil :—

$$M = \frac{2\pi^2 a^2 N}{d} \left[1 + \frac{3a^2 A^2}{8d^4} + \frac{5a^4 A^4}{64d^8} X_2 + \frac{35a^6 A^6}{512d^{12}} X_4 + \frac{63a^8 A^8}{1024d^{16}} X_6 + \dots \right],$$

where

$$X_2 = 3 - 4x^2/A^2,$$

$$X_4 = 5/2 - 10x^2/A^2 + 4x^4/A^4,$$

$$X_6 = 35/16 - 35x^2/2A^2 + 21x^4/A^4 - 4x^6/A^6,$$

N = total number of turns in length x ,

a = radius of coil,

A = radius of circle,

x = axial length of coil,

$$d = \sqrt{x^2 + A^2}.$$

When the coil is long this formula is very exact and easy to use, and it was a simple matter to calculate M_2 with the precision necessary for our work. The results of 16 calculations are given in Table XIII.

In Table XIV. the values of M_2 are given for coils, the mean diametral planes of which are distant from the brush contact circle by the amounts given in column 4. The results show the variation of M_2 with change (1) in the axial distance of the coils, (2) in the radius of the coils, and (3) in the radius of the contact circle of segments and discs.

The summary on p. 78, Table No. XV., relates to the coils and brush contact circles of the Lorenz apparatus and sufficiently indicates how small corrections were made when the distance between the coils was varied. The contact circle of segments and brushes was practically in the midplane of the coils near to it; the method of ensuring this is described in Section 22.

Correction for Conicality of Coils.

The increment coefficient q (Table IX.) enables the change of mutual inductance to be calculated when the radius of a coil is changed by a small amount, but the change must be a uniform one. When the change is not uniform, the change in mutual inductance for an increase in radius of any part of a coil must be known, and the correction for conicality must be calculated in parts. In the past it has been customary to take the radius of a coil as absolutely uniform from end to end, but such a procedure invariably introduces errors into the calculation.

* 'Bur. of Standards Bull.,' vol. 8, No. 1, p. 101.

TABLE XIII.—Calculations of Mutual Inductance.

Radius of circle.	Radius of coil of 12 turns per centimetre length.	Length of coil.	Mutual inductance of coil and circle.	Mutual inductance differences = M_2 .	Sums of differences.
cm.	cm.	cm.			
26·7870	17·9419	144·1	74981·04	234·26	372·12
26·7870	17·9419	160·1	75215·30		
26·7870	17·9419	174·6	75376·21		
26·7870	17·9419	190·6	75514·07	137·86	370·14
26·7870	17·9419	144·4	74986·12	232·94	
26·7870	17·9419	160·4	75219·06		
26·7870	17·9419	174·9	75379·13	137·20	
26·7870	17·9419	190·9	75516·33		
26·7870	17·9369	144·1	74939·24	234·14	371·93
26·7870	17·9369	160·1	75173·38		
26·7870	17·9369	174·6	75334·19		
26·7870	17·9369	190·6	75471·98	137·79	
26·7370	17·9419	144·1	74985·66	233·42	370·78
26·7370	17·9419	160·1	75219·08		
26·7370	17·9419	174·6	75379·42		
26·7370	17·9419	190·6	75516·78	137·36	

TABLE XIV.—Mutual Inductances M_2 .

Radius of circle.	Radius of coil.	Length of coil of 192 turns.	Distance of mean diametral plane of coil from circle.	M_2 .
cm.	cm.	cm.	cm.	
26·7870	17·9419	16·0	152·1	234·26
26·7870	17·9419	16·0	152·4	232·94
26·7870	17·9369	16·0	152·1	234·14
26·7370	17·9419	16·0	152·1	233·42
26·7870	17·9419	16·0	182·6	137·86
26·7870	17·9419	16·0	182·9	137·20
26·7870	17·9369	16·0	182·6	137·79
26·7370	17·9419	16·0	182·6	137·36

TABLE XV.—Mutual Inductances M_2 .

Distance apart of mean planes of coils.	Distance apart of contact circles.	Radius of contact circle 2.	Radius of coils.	M_2 .
cm.	cm.	cm.	cm.	
Giving M_2 of Coils on Cylinders Nos. 1 and 2, with the Contact Circle of Disc 2.				
30·20	167·50	26·7800	17·9406	370·57
30·50	167·35	26·7800	17·9406	371·89
30·50	167·65	26·7800	17·9406	369·91
30·50	167·50	26·7800	17·9406	370·90
30·55	167·50	26·7800	17·9406	370·95
30·60	167·50	26·7800	17·9406	371·01
30·80	167·50	26·7800	17·9406	371·23
Giving M_2 of Coils on Cylinders Nos. 3 and 4, with the Contact Circle of Disc 1.				
30·20	167·50	26·7910	17·9430	370·94
30·50	167·35	26·7910	17·9430	372·27
30·50	167·65	26·7910	17·9430	370·28
30·50	167·50	26·7910	17·9430	371·27
30·55	167·50	26·7910	17·9430	371·32
30·60	167·50	26·7910	17·9430	371·38
30·70	167·50	26·7910	17·9430	371·49
30·80	167·50	26·7910	17·9430	371·60

The change in mutual inductance for an increase in radius of any section of the coil is most readily calculated by finding dM/dA for the brush contact circle and a second circle of the same radius as the coil, the distance apart of the two circles varying over the length of the coil.

TABLE XVI.

d = distance apart of the circles.	Radius of first circle = 26·787 cm. Radius of second circle =			Change in M for change of 10μ in radius of smaller circle.
	cm. 17·9141	cm. 17·9419	cm. 17·9697	
	M =			
cm. 23·3150	88·8782	89·1121	89·3456	0·0084
19·2851	—	113·5449	113·8412	0·0107
15·2573	—	145·4984	145·9000	0·0145
11·1679	—	187·1127	187·6662	0·0199
7·3135	232·7922	233·5633	234·3348	0·0278

In Table XVI. we give the values of the mutual inductance of two such circles. The radius of one circle is 26.787 cm. and the radius of the other circle varies from 17.9141 cm. to 17.9697 cm. In column 5 of the table we give the change in M for a change in radius of the smaller circle of 10μ .

The values in columns 1 and 5 were plotted and the resulting graph was employed, in conjunction with the values of the diameters used in plotting the conicality curves (fig. 12), to calculate the correction for the conicality of the coils. The method is so obvious that we need only give the results.

TABLE XVII.—Corrections to M_1 for the Conicality of the Coils.

Coils on cylinder No.	Correction to M_1 .	
	Ring end of cylinder near disc.	Ring end of cylinder away from disc.
1	-0.589	+0.942
2	-0.571	+0.494
3	+0.295	-0.326
4	+0.200	-0.153

The mutual inductance of one coil and a disc circumference is about 30,000 cm., and when cylinder No. 1 is reversed in position a change of 1.5 in M_1 , or 5 parts in 100,000, is brought about by the conicality of the coils.

Correction for Variation in Pitch.

The graph of the difference measurements of a coil absolutely uniform in pitch is a straight line such as OP, fig. 17. Such a coil may be called a perfect coil and any short section of it a "perfect section." That there is a difference in the mutual inductance of a "perfect" section such as OAP and a circle, and an actual section such as OBP and the same circle, is easily seen. With the exception of the wires at O and P, every wire in the actual section is farther from the circle than the corresponding wires in the perfect section, and the mutual inductance of the former section will in consequence be the smaller.

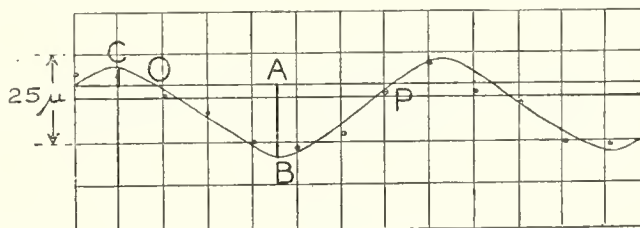


Fig. 17.

If the periodic curve is symmetrical with respect to the "perfect" curve, the reverse is true for the next section, but the difference is not so great and hence there is not perfect compensation. As we proceed along a coil with such periodic variations the

individual corrections are alternately plus and minus, and the total effect of these half-period elements depends on the difference of phase at the beginning and end as in the case of diffraction effects in light.

To obtain the corrections we have calculated the mutual inductance of small portions of the coils and the corresponding circles. The mutual inductances calculated are (1) those of the circle and of coils each of which is 0.635 cm. in length, the distance between the circle and the end of the coil nearer to it increasing from 7.3885 cm. to 23.2635 cm. in steps of 0.635 cm.; (2) those of the circle and of coils, each 25μ long, the distance between the coil and circle increasing from 8.0235 cm. to 23.2635 cm. in steps of 1.27 cm. In all cases the number of windings on a coil is at the rate of 12 turns per cm. The results of these calculations were plotted and by means of the curves we have corrected for the variation in pitch.

As an illustration, let us examine the second section of coil No. 3 when the ring end of the cylinder is nearer to the disc. For clearness, this section is reproduced in fig. 17 and is denoted by CB. The end of the section nearer to the disc is 5μ nearer than the end of the corresponding perfect section, and the other end of the section is 20μ farther from the disc than the other end of the corresponding perfect section. The ratio of the lengths of the two sections is therefore $6375/6350$ and the ratio of the current densities in the equivalent current sheets is $6350/6375$. The M of the perfect section and the disc is 1723.692; if its length is increased by 5μ at the end nearer the disc and by 20μ at its other end, M is increased by the amounts 1.382 and 5.340. (These corrections are obtained from the curve.) The total M is now 1730.414. If next we reduce the number of turns to its original value the M becomes $1730.414 (6350/6375) = 1723.628$, and since the section is now identical with the actual section, the correction for variation of pitch and displacement of section is $1723.628 - 1723.692 = -0.064$ or -3.7 parts in 100,000.

In this way we have calculated the corrections for all the sections of the coils both when the ring end is nearer to the disc and when farther from it.

The values of the corrections are as follows:—

TABLE XVIII.

Coils on cylinder No.	Correction to M .	
	Ring end of cylinder near disc.	Ring end of cylinder away from disc.
1	+0.502	-0.369
2	+0.987	-0.874
3	-0.734	+0.536
4	-1.535	+0.486

SECTION 19.—EFFECT OF ELECTRIC MOTOR ON THE MUTUAL INDUCTANCE
OF THE COILS AND DISCS.

Originally it was our intention to employ a water turbine to drive the Lorenz apparatus, but there were many difficulties in the way of a satisfactory system, and finally we decided on an electric motor. Before coming to this decision we made a number of experiments on the effect of the presence of a large mass of soft iron on the mutual inductance of two coils each of 200 turns of wire and 24 cm. in diameter. The distance between the coils could be changed and the approximate mutual inductance between them was measured by Mr. A. CAMPBELL. The iron employed was a mass built up of laminated sheets tightly pressed together, the dimensions of the mass being $35 \times 18 \times 8$ cm. When the coils were parallel and their mean planes 7 cm. apart, the following values were obtained for the mutual inductance when the iron was placed on the common axis of the coils and at a distance d from their mean plane :—

d in centimetres.	M in microhenries.
∞	4942.5
70	4942.6
60	4942.6
39	4945.2
20	4975.0
∞	4942.6

The distance apart of the coils was afterwards increased to 10 cm. and eventually to 20 cm., but in no experiment could the effect of the iron at a distance of 1 m. be detected by the mutual inductance measurements. The stray magnetic field produced by the motor is not a source of trouble and has no influence on our results; the only manner in which the motor can affect the resistance measurements is by its action, as a mass of iron, on the mutual inductance of the coils and discs.

The field magnet of the motor is not large; its length parallel to the shaft is about 30 cm., and its section at right angles to the shaft is about 1000 sq. cm. The distance from the centre of the motor to the centre of the nearest of the four coils is almost exactly 400 cm.

Dr. G. F. C. SEARLE, F.R.S., who has taken great interest in our work and to whom we tender our thanks, has very kindly calculated the effect of a sphere of soft iron on the mutual inductance of a coil and circle, both of which are some distance away. If the radius of the sphere is 20 cm. (corresponding roughly to a mass of iron equivalent to our motor) and the coil and circle are of the same dimensions and the same distance from the sphere as in the Lorenz apparatus, the effect of the sphere on the mutual inductance appears to be about 1 part in 10,000,000 which is, of course, absolutely negligible. This value is in very good agreement with two values determined experimentally.

The intensity of the magnetic force, at a point corresponding to the centre of the motor, which is produced by a current of 2 amperes through the four coils of the Lorenz apparatus, is about 0.0025 C.G.S. units. The iron of the motor becomes magnetised and the result is an increase in the total magnetic flux through the discs. We had to find the ratio of this increase to the flux produced by the current in the four coils. This ratio we have found in two ways. In both of these we magnetised the iron of the motor by winding around it a large solenoid of 16 turns of insulated copper wire. When a current was passed through this coil the magnetising field inside the solenoid was approximately in the same direction as the field due to the four coils, *i.e.*, practically parallel to the shaft. The iron was magnetised and the two discs of the Lorenz apparatus were caused to rotate in the resulting field. Since the mean field in which one disc rotated was not the same as that in which the other disc rotated, a difference of potential was produced between the edges of the two discs. In our experiments we compared the galvanometer deflection produced by this potential difference with that produced by a current in the four coils of the Lorenz apparatus when the mutual inductance of the coils and discs was changed by a known amount. The result found is that when the iron of the motor is placed in a magnetising field of 2.5 C.G.S. units, the total effective magnetic flux through the two discs is 1 part in 10,000 of that due to the current in the four Lorenz coils. With such a small magnetising field as 0.0025 C.G.S. units, it was impossible to measure any effect, but the calculated effect is $\frac{0.0001 \times 0.0025}{2.5}$, *i.e.*, 1 part in 10,000,000.

In the second experiment we wound on a framework of wood a similar coil to that surrounding the motor. Both coils were of 20 cm. radius and had 16 complete turns. When a current of 5 amperes was passed through the coil, the intensity of the axial field was measured by means of a magnetometer at distances of 1, 2, 3, and 4 metres from the mean plane of the coil. Similar measurements were made along the axis of the coil which surrounded the motor, the current through this coil being 5 amperes as before. The results are of interest and are given in Table XIX.

The nearer disc of the Lorenz apparatus is about 400 cm. from the centre of the motor, and we may take the magnetic force due to the motor as uniform over this disc. The area of one face of the disc is 2200 sq. cm. so that the total flux through it due to the iron of the motor was 2200 (0.0010–0.0003) = 1.5. Had the magnetising force on the iron of the motor been that due to 2 amperes through the four coils of the Lorenz apparatus, the total flux would be reduced to $(1.54 \times 0.0025)/2.5 = 0.0015$. The total effective flux through the two discs will be somewhat less than this, but the reduction is not important. When making a resistance measurement, the total effective flux through the two discs is about 24,000, so that the magnitude of the motor effect is $0.0015/24000 =$ about 6 parts in 100,000,000.

The intensity of the stray field of the motor when running at full load has already

TABLE XIX.—Giving the Intensities of the Axial Magnetic Fields produced by a Current of 2 Amperes in Two Solenoids One of which has the Motor as a Core.

Axial distance from centre of coil.	Intensity of magnetic field.	
	No iron present in coil.	Motor as core of the coil.
cm.	C.G.S. units.	C.G.S. units.
100	0·023	0·09
200	0·0034	0·013
300	0·0009	0·0036
400	0·0003	0·0010

The motor was not running during these measurements.

been stated to cause no trouble. The intensity of the component of this stray field parallel to the shaft of the apparatus and 400 cm. from the centre of the motor, was measured to be 0·0006 C.G.S. units.

The general conclusion is that an electric motor may safely be used for such a purpose as ours.

SECTION 20.—ARRANGEMENT OF CIRCUITS.

(a) *Connection to Brushes.*—The brushes are connected to insulated terminals fixed to the phosphor-bronze rings, and double silk-covered copper connecting wires pass from these terminals to a selector switch on the observation table. A diagrammatic representation of the connections is given in fig. 19. The selector switch consists of two conducting arms which enable the observer to complete the circuit through any one of the five pairs of brushes by moving the contacts from stud to stud. When it is desired to place the ten brushes in series and thereby get an induced voltage five times as great as with one pair of brushes, conducting straps are placed in the positions SS SS indicated by the dotted lines and the turning head makes contact with the studs 1' and 5. Any one of the five pairs of brushes may still be selected and observations may be made to test if the brush contacts are satisfactory. When the ten brushes are placed in series the thermo-electric effects are of course added and particular care has therefore to be taken with the brushes. When trouble is experienced it greatly facilitates the work to select pairs of brushes and so detect the faulty ones.

To place the brushes in two sets of five in parallel, conducting straps connect together each set of five terminals TT, &c., on the selector switch. The position of the turning head is immaterial.

(b) *Multiple Commutator and Plug Board* (fig. 19).—This is designed after the manner of the multiple commutator used for the Ayrton-Jones current weigher, and a short description will therefore suffice. A commutator allows of the reversal of the

current in both coils on any one cylinder, and the plug board allows of the reversal of the current in any one or more of the helices. Each helix is designated by a number and a letter which are marked on an ebonite bridge at the top of the multiple commutator; the turning heads are also numbered to enable changes to be rapidly made without likelihood of error. The lower commutator reverses the current in all of the coils.

By suitable conducting straps the coils on one cylinder may be placed in parallel with those on another cylinder. In many of our observations the coils on cylinders 1 and 2 were placed in parallel with the coils on cylinders 3 and 4.

(c) *The Reversing Switch*.—Preliminary experiments indicated that the make and break of the current through the coils had a considerable inductive action on the galvanometer circuit. The deflection of the galvanometer thus produced amounted at times to 50 cm., on a scale at a distance of 2 m., and prohibited such rapid reversal of the current as we desired to make. Similar inductive action is remarked on by Prof. J. V. JONES in the 'Report of the British Association for 1890.' Had the galvanometer circuit been a stationary one, we might have introduced a compensating system, but this was not possible with the system of rotating conductors we had installed. It was evident that the galvanometer circuit should be broken before making or breaking the current circuit, and as the position of rest of the galvanometer coil on open circuit is, in general, different from that when the circuit is closed, it appeared desirable to have the galvanometer excessively damped when not included in the main circuit. The switch shown in fig. 19 enables this to be done. All the connections are shown in the figure and it is not necessary to describe the switch in detail. On moving from stud 1 to stud 2 the galvanometer circuit is shunted by a negligible resistance and the main galvanometer circuit is broken. On moving to stud 3 the current is broken—on to stud 4 the current is made again but in the reverse direction—and when the movement is continued to stud 5 the galvanometer is again placed in the brush circuit. This switch proved very convenient in practice and greatly facilitated our work.

(d) In addition to the commutators already described, a simple commutator was added to reverse the potential leads attached to the standard resistance. Such reversals were made to eliminate electrostatic effects.

(e) *Galvanometer*.—This was of the Ayrton-Mather type and was very kindly lent to us for the work by The Cambridge Scientific Instrument Company, Limited. The resistance of the galvanometer is 16·5 ohms and the period of the coil is 5·2 seconds. At a distance of 2000 scale divisions the sensitiveness is 57 divisions per microvolt. The external resistance for aperiodic working is 50 ohms.

When the current through the eight coils of the Lorenz apparatus is 2 amperes and a direct measurement of a resistance of 0·01 ohm is in progress, the difference of potential on the standard resistance is 0·02 volts. In making the measurements the current is reversed, and on reversal there is a change in the rest point of the

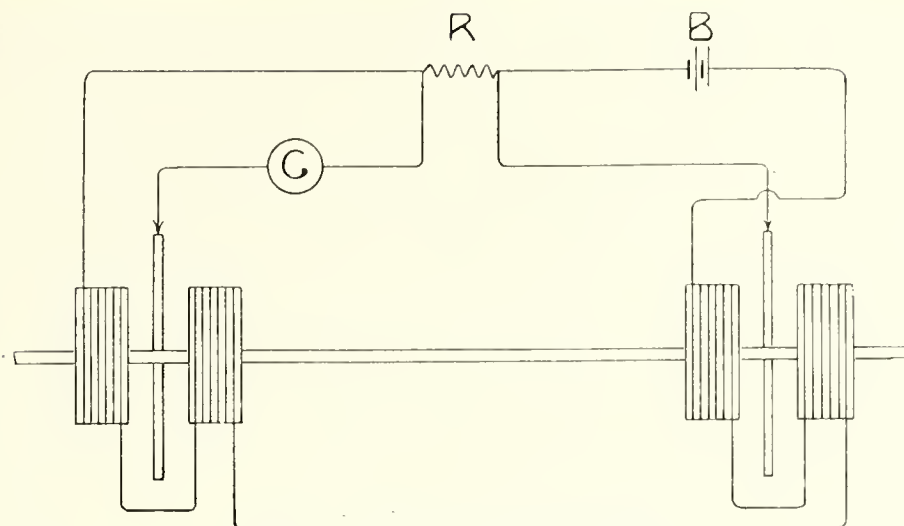


Fig. 18. Simple diagram of circuit.

R, resistance to be measured ; B, battery ; G, galvanometer.

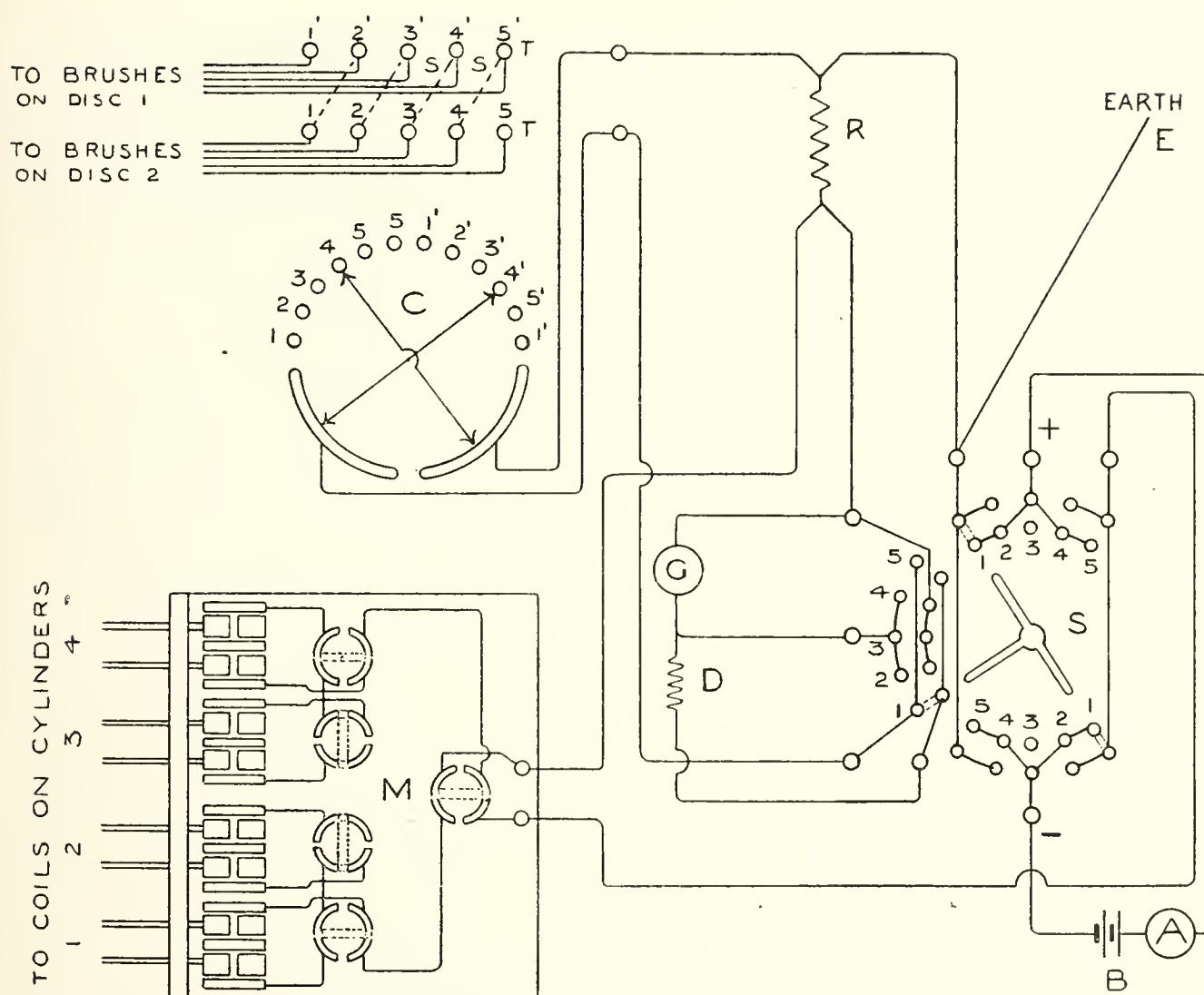


Fig. 19. Detailed diagram of circuit.

A, ammeter ; B, battery ; C, brush selector switch ; D, resistance in galvanometer circuit ; E, earthed point on circuit ; G, galvanometer ; M, multiple commutator and plug board ; R, resistance to be measured ; S, reversing switch.

galvanometer spot of 57 mm. (at 2 m.) per microvolt. To obtain a precision of 1 part in 100,000 on a single reading it is necessary, therefore, to note the change of deflection when the current is reversed with an error not greater than 11.4 mm. When the motion of the coil is made aperiodic by introducing additional external resistance, the sensitiveness is about one-quarter of the previous value, and the error of the difference reading must not be greater than about 3 mm. For measurements of a resistance of 0.001 ohm the difference of potential on the standard resistance was about 0.004 volt and the change of deflection had to be read with an error not greater than 0.6 mm. For a single measurement of a resistance about fifty different readings were taken and the mean of these is used to calculate the result (see p. 98).

Fig. 18 is a simple diagram of the circuit and requires no explanation. Fig. 19 is a more detailed diagram and shows the connection of the circuit to earth.

SECTION 21.—STANDARD RESISTANCES.

The resistances measured in absolute measure were of nominal values 0.001 ohm, 0.002 ohm, and 0.01 ohm. The first and last of these were standard manganin resistances capable of carrying currents of 30 and 10 amperes respectively without increasing in temperature by an appreciable amount. The currents used in our measurements did not exceed 4 and 2 amperes respectively, and the heating effect due to these currents could not be detected.

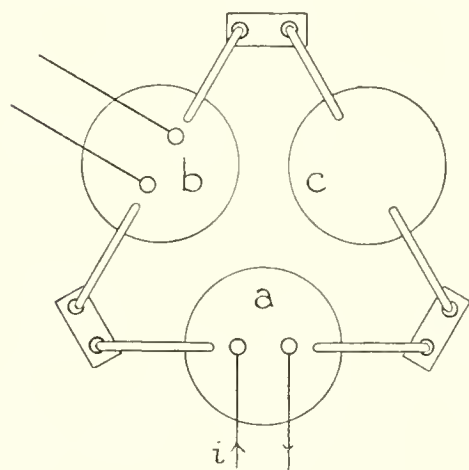


Fig. 20.

To obtain an effect corresponding to that of a resistance of 0.002 ohm, three standard resistances were arranged in a triangular fashion (fig. 20) as first suggested by Lord RAYLEIGH.* The resistances are of the well-known type introduced by the Physikalisch-Technische Reichsanstalt; they are of manganin and the mean temperature coefficient in the neighbourhood of 20° C. is about +15 parts in 1,000,000

per degree rise of temperature. The resistance (*a*) fig. 20 is a coil of 2 ohms resistance, (*b*) is a 1-ohm standard, and (*c*) consists of a coil of 1000 ohms shunted by others having values from 100,000 to 500,000 ohms. If the current in the main circuit is *i* the current through *b* is $\frac{ai}{a+b+c+L}$ where *L* is the summed value of the resistances of the current leads to the three coils. The difference of potential at the extremities of *b* is $\frac{abi}{a+b+c+L}$. The quantity $\frac{ab}{a+b+c+L}$ thus takes the place of *R* in the formula $R = Mn$ (p. 35) and is called the effective resistance.† The two ohm

* 'Phil. Trans.,' 1883.

† A complete treatment on the combinations of a four-terminal resistance with other resistances is given by G. F. C. SEARLE, 'The Electrician,' March 31 to April 21, 1911.

coil is of special construction; it is divided into two equal coils each of which can be compared with the standard coils.

The coils were supported from mercury cups in a bath of well stirred paraffin oil maintained at a constant temperature of $20^{\circ}0$ C. The stirring was produced by blowing dry air through the oil. A spiral toluene thermostat was used to control the temperature and the heating of the oil was produced by an electric current flowing through a resistance coil supported on a large frame at the bottom of the bath. These arrangements are those in common use in the Electrical Standards Department, and the results obtained are exceedingly satisfactory.

The methods adopted for the accurate comparison of the coils with other wire standards and with mercury standards of resistance are published elsewhere,* and it is not necessary to describe them here. All the resistances were frequently compared with standard manganin coils which are hermetically sealed and the secular changes in which are exceedingly small.

The new mercury standards of resistance of the National Physical Laboratory are nine in number. They have spherical end vessels 4 cm. in diameter and were made in accordance with the specification† of the London Conference on Electrical Units and Standards 1908. The resolution relating to the international ohm is as follows:—

“The international ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 gr. in mass, of a constant cross-sectional area and of a length of 106.300 cm.”

Mercury standards of resistance have also been made at the Physikalisch-Technische Reichsanstalt and the Bureau of Standards at Washington. Recent comparisons show that the units of resistance so derived agree with the unit derived at the National Physical Laboratory within about 2 parts in 100,000. The exact figures are not yet to hand.

SECTION 22.—SETTING OF THE COILS TO BE COAXIAL WITH THE SHAFT AND AT APPROXIMATELY EQUAL DISTANCES FROM THE BRUSH CONTACT CIRCLES.

(1) *Setting of the Axes of the Coils to be Parallel to the Axis of the Shaft.*

Mechanical Method.—At the time of turning the marble cylinders, a phosphor-bronze ring was let into one end of each, and the surface turned at right angles to the axis. When the cylinders were in position on the cradles of the Lorenz apparatus, a radial arm supporting a micrometer head was clamped in a suitable position on the shaft and contact between the micrometer screw and each of the rings in turn was made at three points 120 degrees apart. The making of a contact was

* ‘B.A. Elect. Stands. Committee Report,’ 1906.

† ‘B.A. Elect. Stands. Committee Report,’ 1909.

indicated by a telephone, and readings could be repeated within about $10\cdot01$ mm. Each cylinder was adjusted in position until the three readings were identical and the axes of the cylinders were then approximately parallel to the axis of the shaft. The method was sensitive, but irregularities in the thrust-bearing surface of the shaft must introduce errors which are difficult to eliminate.

(2) *Setting of the Coils to be Coaxial with the Shaft.*

(a) *Mechanical Method.*—A direct-reading spring indicator was used to measure the perpendicular distance of the inner surface of each marble cylinder from the surface of the shaft, and each cylinder was adjusted in position until the readings at all points were practically identical. The indicator was sensitive to a difference of about $0\cdot02$ mm. and the adjustment of a cylinder occupied only a few minutes. If the inner surface of a cylinder is not coaxial with the coil, then of course an error is introduced.

(b) *Electrical Method.*—The calculations we have made give only the mutual inductance of the coils and the brush contact circles, and it is not possible for us to give other than general diagrams indicating the manner in which the intensity of the magnetic field varies along the radius of a circle and other diagrams indicating the difference of potential between the axis and any point on a radius which is produced by the rotation of that radius. Such diagrams will, however, serve to explain the electrical method of setting the coils.

In fig. 21 (a), DD' represents the plane of rotation of one of the discs, and BB' the position of the brush contacts. The intensity of the magnetic field produced by a current in the coils on both sides of the disc is practically zero at BB' and changes in sign as we pass radially outwards from the disc. The form of the intensity curve is roughly shown by the curve EFGH. When the disc rotates, the difference of potential between the centre O and any point A on a radius increases with increase of the distance OA to a maximum value at B and B', but afterwards it decreases. The potential difference is given by $2\pi n \int_0^y B y dy$ where OA = y and B is the magnetic flux at a distance y from O. The potential difference can be represented as an area if the flux be multiplied by y before it is plotted. The area enclosed by the line DD' and the curve KCLM shows roughly how the potential difference varies; thus the difference of potential between O and A is represented by the area OCA. Areas to the left of DD' are counted as positive and those to the right are counted as negative. In our apparatus the brush and segment at B are insulated from those at B' and it is therefore possible to measure the difference of potential between the segments and so determine whether the field produced by the current in the coil is symmetrical with respect to the axis of the shaft. Without the insulation of the segments, *i.e.*, by the use of a disc alone as in the old forms of apparatus, eddy currents in the disc produce disturbing effects.

The effect of displacing the coils parallel to the shaft is diagrammatically shown in fig. 21 (*b*). The area $OB'CS$ is greater than the area $OLR-RNB$ and the difference of potential between O and B' is greater than that between O and B . The relation between the mean of these voltages and that obtained when the coils and shaft are coaxial will be seen presently.

A similar change in the difference of potential between B and B' is produced by a rotation of a coil about a vertical axis passing through its centre. Fig. 21 (*c*) shows

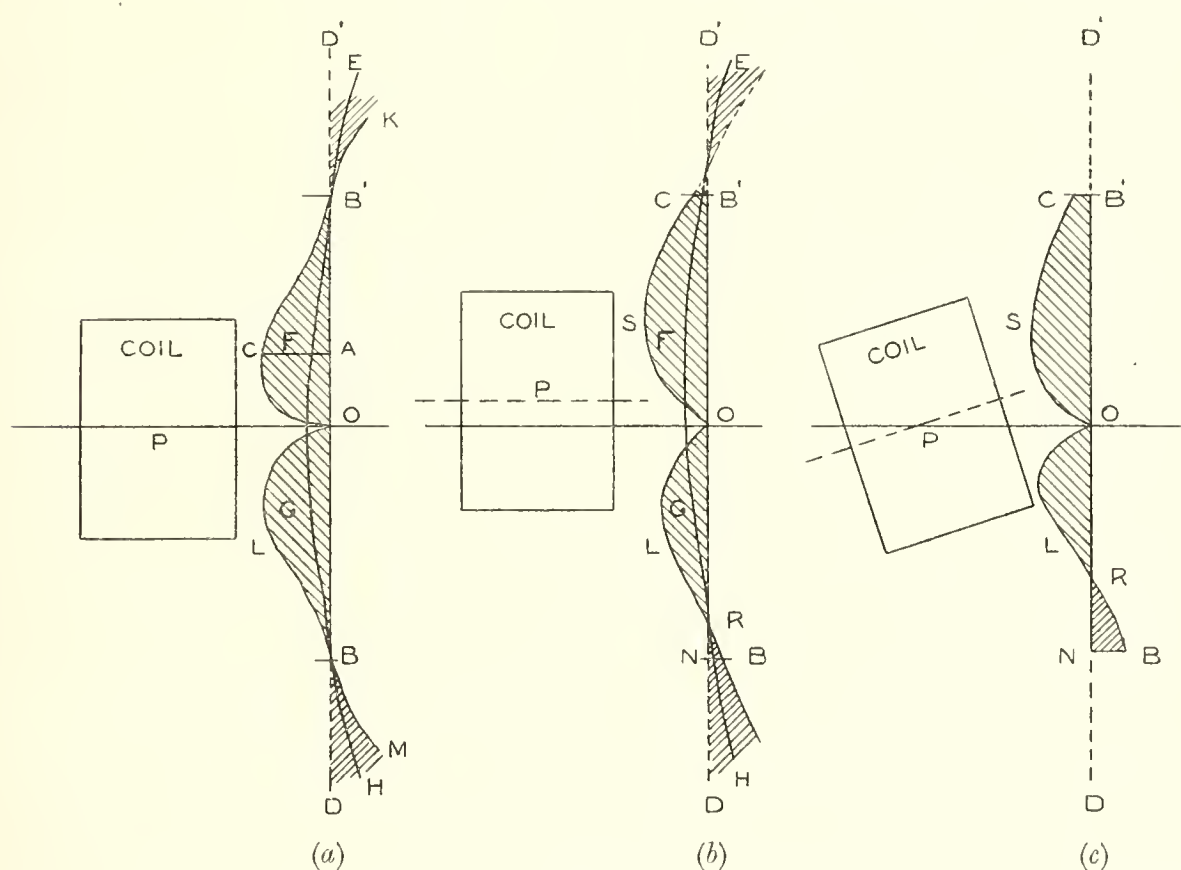


Fig. 21.

generally the effect produced and makes it clear that the mean diametral plane of the coil may be set parallel to the brush contact circle by observations of the differences of potential at such points as BB' .

The first coils to be set coaxial with the shaft by these electrical methods were those on cylinders Nos. 1 and 4. Afterwards the mutual inductance of the No. 1 coils and the brush contact circles was made equal to that of the No. 4 coils and the same circles by passing a current in the same direction through both and altering the distance between the No. 1 coils and the disc nearer to them until on reversal of the current in the coils there was no change in the difference of potential between the two contact circles. The object of this procedure was to enable us to measure directly the amount by which a potential difference such as $OB'CS$, fig. 21 (*b*), exceeded the normal potential difference such as $OB'C$, fig. 21 (*a*). The circuit formed will be

clear from the small diagram at the top of fig. 22. One brush made contact with one (or two) segments on each disc and the circuit was completed through the rotating conducting wires. The displacements were read to 0.001 mm. and the changes in

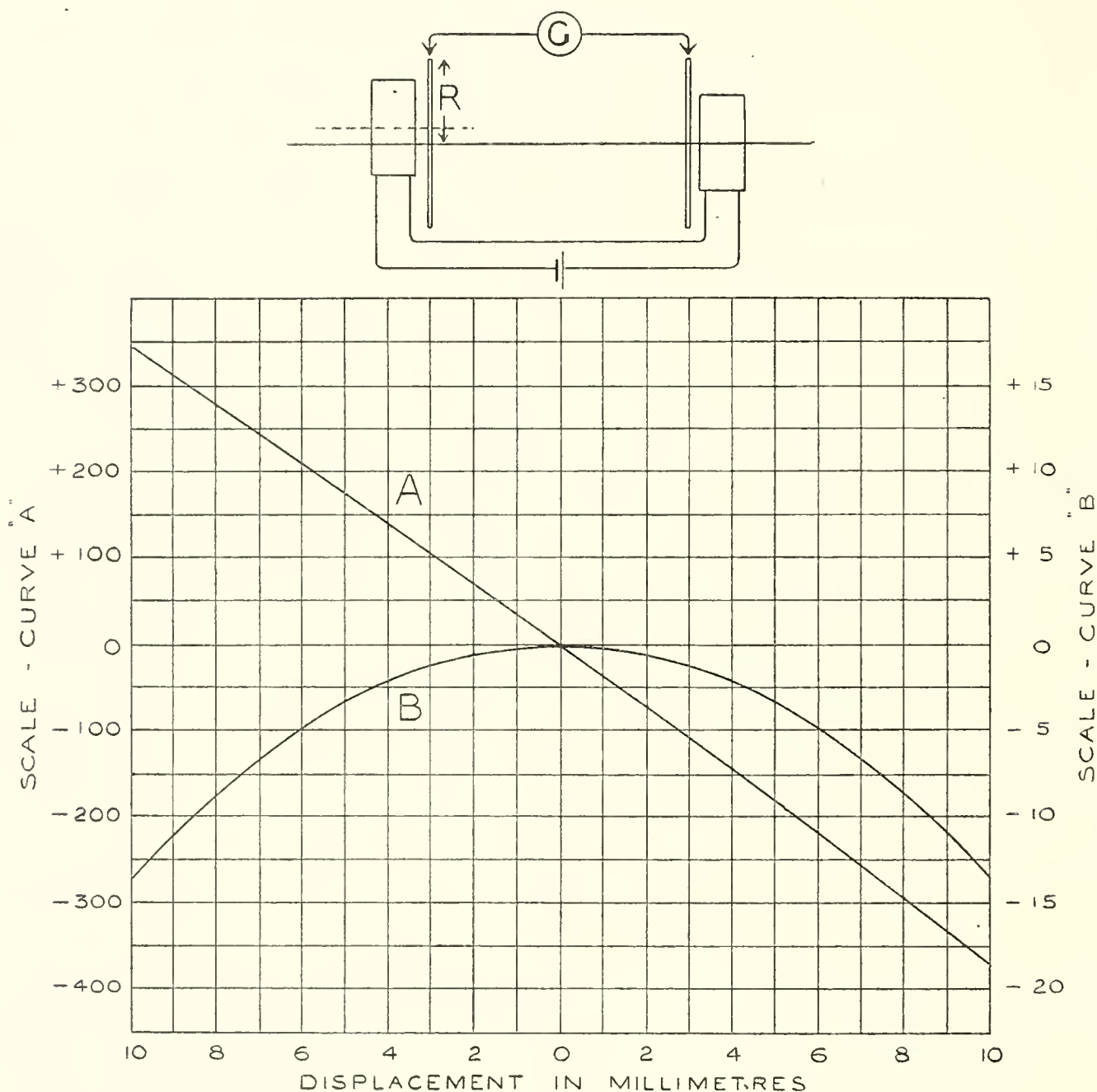


Fig. 22.

A, change (parts in 100,000) in voltage induced by rotation of radius R due to displacement of cylinder in direction perpendicular to its axis; B, change in *mean* voltage induced when five brushes are used, touching points 72 degrees apart on edge of disc.

potential were calculated from the changes in the galvanometer deflection which were produced on reversal of the current in the coils.

A displacement of 10 mm. increases the difference of potential between O and B',

fig. 21 (a) and (b), by about 343 parts in 100,000, and diminishes that between O and B, fig. 21 (a) and (b) by about 371 parts in 100,000. If one brush only were employed on the segments during measurements of a resistance it would therefore be necessary to ensure coincidence of axes of coils and shaft within about $\frac{3}{1000}$ of a millimetre, in order that the error associated with the setting should not exceed 1 part in 100,000.

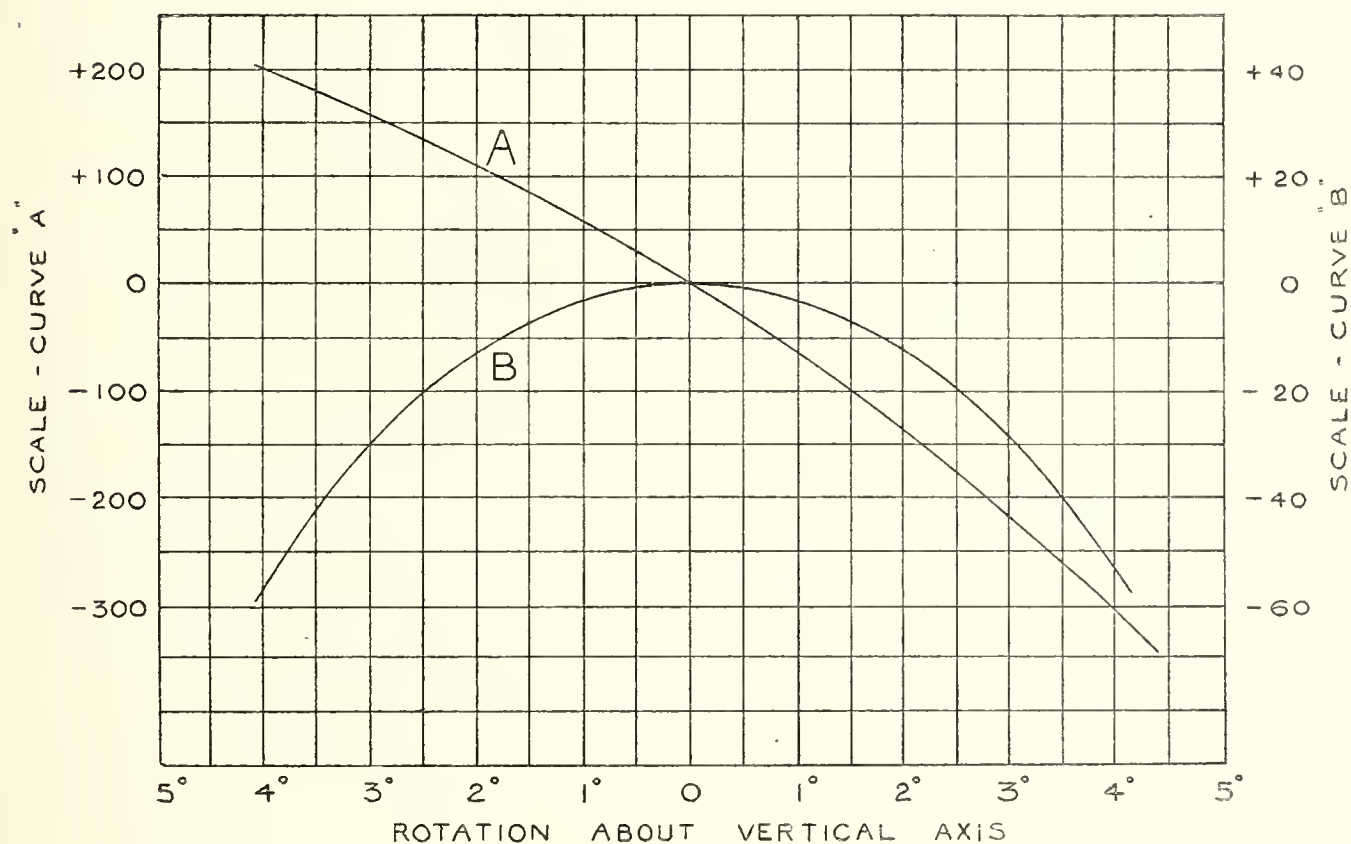
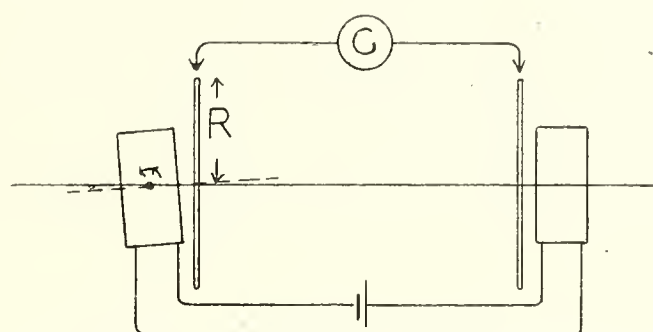


Fig. 23.

A, change (parts in 100,000) in voltage induced by rotation of radius R due to angular displacement of cylinder about vertical axis passing through mean diametral plane; B, change in mean voltage induced when five brushes are used, touching points on edge of disc 72 degrees apart.

In practice, however, five brushes are used on each disc, and when these are used in series with the five brushes on the other disc, a displacement of 10 mm. diminishes the voltage by $(371 - 343)/2 = 14$ parts only in 100,000. When the brushes on each disc are placed in parallel the mean potential difference measured by the galvanometer

is nearly the mean of the separate potential differences and the error of setting is about the same as when the brushes are placed in series. With the brushes in parallel it is better, however, to include a resistance in series with each brush before connecting the brushes together, for then any difference of resistance between the brush contacts is rendered negligibly small. The observed differences of potential due to displacements when the brushes were placed in parallel are plotted in fig. 22, and the equation of the resulting curve is $y = 0.132x^2$ where y is the change in voltage in parts in 100,000 produced by a displacement of x mm. from the coaxial position. An identical curve results when the brushes are placed in series. It is clear that the observations enable the coil and shaft to be set coaxial within less than 0.1 mm. and an error of this amount introduces an error in the resistance measurements of about 1 part in 100,000,000.

Similar observations were made when one of the cylinders was rotated about a vertical axis P. The results are plotted in fig. 23. The equation when the brushes are placed in series or in parallel is $y = 3.5x^2$ where y is the change in voltage in parts in 100,000 produced by a rotation of x° . Thus, if $x = 0.1^\circ$ (a large amount) the error introduced is 4 parts in 10,000,000.

(3) *Setting of the Coils on opposite Sides of a Disc to be at the same Mean Distance from the Contact Circle of Segments and Brushes.*

The current through the coils on cylinder No. 1 was caused to circulate in the same direction as the current through the coils on cylinder No. 4, and the resulting flux through disc No. 1 was therefore in the same direction as that through disc No. 2. The galvanometer circuit was completed without the inclusion of the standard resistance R, and reversals of the current through the coils were made in the usual manner to eliminate all effects but that due to a difference of flux through the discs, caused by the current. In general there was a want of balance and in such case the distance of one of the coils was changed until balance was secured or very nearly so. With care equality could be obtained within about 2 parts in 100,000, but in general we were content to ensure equality within about 25 parts in 100,000. This is equivalent to equality, within about 0.07 mm. of the mean distance of the No. 1 coils from No. 1 disc, and that of the No. 4 coils from the No. 2 disc. A similar setting of the No. 2 and No. 3 coils was next made, and afterwards the current through coils Nos. 1 and 3 was made to circulate in the opposite direction to that in coils Nos. 2 and 4. If all the coils are at equal distances from the discs, there is now no deflection of the galvanometer and the positions of coils Nos. 1 and 4 or of Nos. 2 and 3 are changed by equal amounts until a balance is secured. The coils on all of the cylinders are thus placed at the same mean distance, within about 0.1 mm., from the brush contact circles and such equality is sufficient.

SECTION 23.—DETERMINATION OF RESISTANCE IN ABSOLUTE MEASURE.

(1) *Preliminary Tests.*

(A) *Elimination of Error due to the Thermo-electric Effects at the Brush Contacts.*—The thermo-electric effects are eliminated by observing only the change of deflection of the galvanometer with reversal of the current in the coils. If the magnitude of the changes of the thermo-electric effects is considerable, and great accuracy is desired, a large number of reversals are necessary. It becomes important, therefore, to determine the general magnitude of the error introduced by making a comparatively small number of reversals.

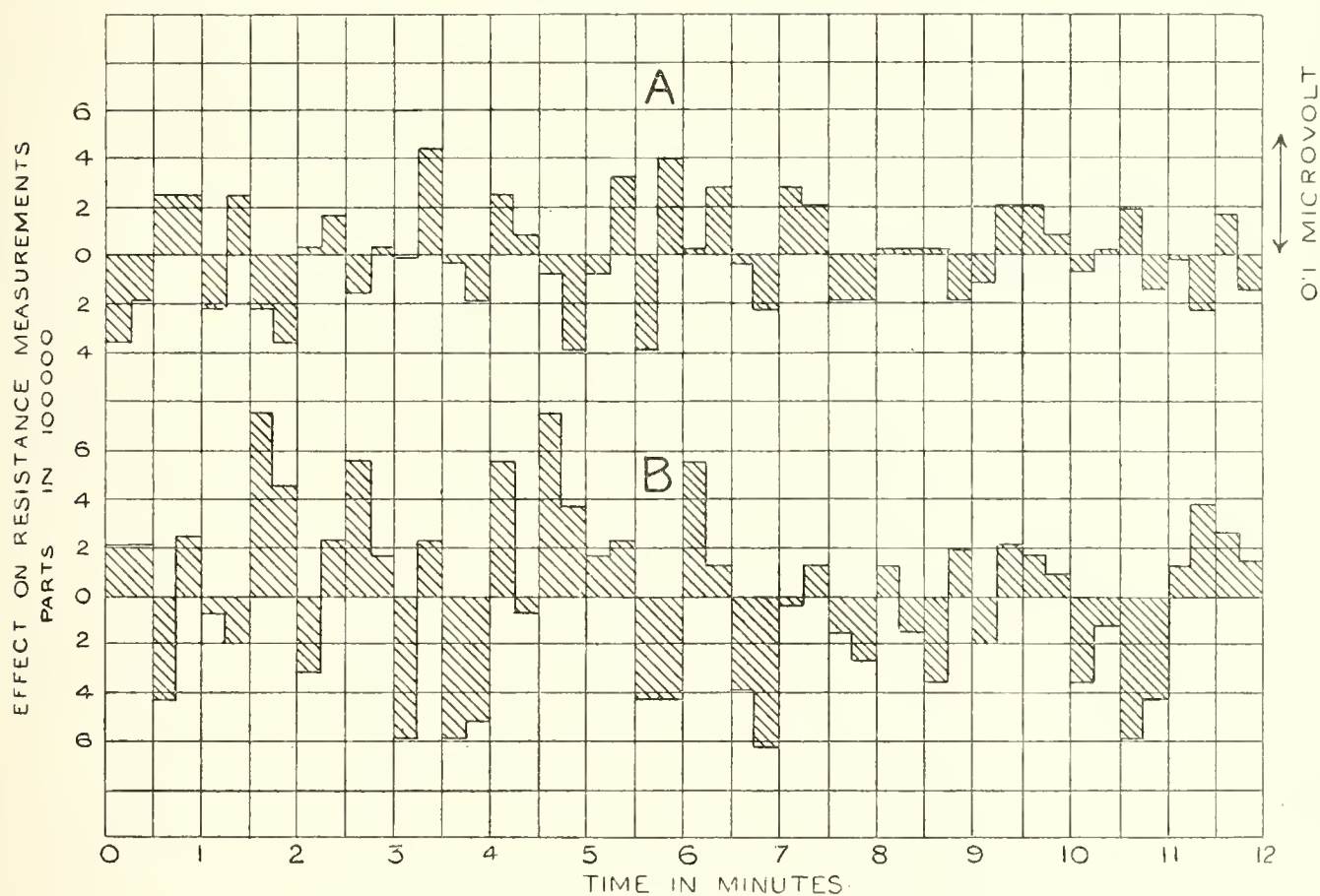


Fig. 24.

A, variations in thermo-electric effects; B, variations in thermo-electric effects *plus* variations in speed.

Our normal procedure in making resistance measurements was to reverse the current in the coils at intervals of 15 seconds, and in practically all cases 48 reversals were made. To determine to what extent this procedure eliminated the thermo-electric effects, we operated the reversing switch with no current flowing in the coils and noted the galvanometer deflection at the end of every 15 seconds.

The differences obtained in one such experiment are plotted against time in graph (A) fig. 24. The mean difference from the mean is ± 0.03 mm., which would introduce an error in a measurement of resistance of something less than 1 part in

1,000,000. The mean difference from the mean irrespective of sign is ± 0.8 mm. These differences are typical of those obtained in our experiments and from them we conclude that the thermo-electric effects introduce no error in our final result.

(B) *Elimination of Error due to Electrostatic Effects.*—We have already described the connections of the circuit to earth and the reversals made in order to eliminate any electrostatic effect on the galvanometer, but we thought it desirable to make tests from time to time of the efficiency of our arrangements. In these experiments the coils were removed from the circuit and an equivalent resistance substituted, which resistance was placed in such a position that the current through it produced no magnetic field in the neighbourhood of the discs. The standard resistance remained in the circuit, but the leads connecting it to the galvanometer were connected to one and not both of the potential terminals. Observations of the change of deflection were made as before at intervals of 15 seconds. The mean difference from the mean of such a set of measurements was always negligibly small and indicated that no error due to electrostatic effects could influence our results.

(C) *Constancy of the Speed.*—The records of speed are remarkably good and show the variations to be very small. Each record enables the mean speed, during the measurement of resistance associated with it, to be calculated with great accuracy; but a fluctuation in speed of a few parts in 100,000 lasting for a comparatively short time cannot be detected. The method adopted to control the speed is described in Section 10, and it is evident that the speed must oscillate in value, the magnitude of the oscillations depending on the cause of the variations, the sensitiveness of the bridge, the sluggish movement of the galvanometer, and the ability of the operator. Now the galvanometer in the bridge circuit is sluggish in its movements and unsuited to detect changes in speed lasting for a few seconds only, but the Cambridge galvanometer used in the Lorenz circuit has a short period (5.2 seconds), and the changes in its deflection during a measurement of resistance show that variations in speed lasting for a few seconds only are of common occurrence.

An idea of the magnitude of these sudden changes of speed is afforded by a comparison of the two graphs (A) and (B), fig. 24. Graph (A) shows the variation in the thermo-electric effects, while graph (B) shows the variation in the thermo-electric effects plus the variations in speed. The differences plotted in graph (B) are those taken during a measurement of a resistance. These two graphs are typical of the results generally obtained. In all our experiments the combined variation of thermo-electric effects and changes of speed were of about twice the magnitude of the changes in thermo-electric effects alone, and it appears reasonable to conclude that small oscillations in the speed were frequent. The magnitude of these oscillations appears to be about 5 parts in 100,000. However, such fluctuations had practically no effect on the final results.

(D) *Effect of the Leads.*—It was possible that the current in the leads to and from the coils might produce a magnetic field of sufficient intensity to affect the results.

This was tested by completing the main circuit through the leads only (omitting the coils) and taking a few complete sets of observations. No effect could be measured and we conclude that the current in the concentric leads and remaining portions of the circuit other than the coils have no influence on our results.

(2) *Normal Procedure in making an Absolute Measurement of a Resistance.*—In making a determination of a resistance in absolute measure, we determined (*a*) the insulation resistance of the various parts of the circuits; (*b*) the mean distance between the mean diametral planes of the coils; (*c*) the temperature of the marble cylinders; (*d*) the want of equality between the product of speed and mutual inductance and the resistance, the value of which was desired; (*e*) the temperature of the marble cylinder; and (*f*) repetition of (*b*).

To measure (*b*) and (*f*) we commenced with observations on the invar line standard; we then observed the eight plug distances and concluded with further readings on the invar. These observations lasted about twenty minutes.

The temperatures (*c*) and (*e*) of the cylinders were taken by means of thermometers, and the effective temperature was calculated by means of the formula given in Section 13. In general, the difference of temperature (*e*)—(*c*) was about $2^{\circ}2$ C. To determine (*d*) the commutators and plugs were correctly set, the galvanometer circuit closed, and the reversals of current, &c., made which are indicated in Section 20. At the same time the speed was maintained constant within a few parts in 100,000 and a record of the speed taken. The time occupied was usually from 17 to 20 minutes. In general, the first 12 reversals of current produced changes in the deflection due to a difference $Mn - R$ where R is the value of the standard resistance in absolute measure. The next 8 reversals were made when R was reduced in value by 1 part in 1000 by shunting it with another standard resistance. Then followed 24 more readings with R unshunted, 8 with R shunted, and the final 12 measurements were made with R again normal. The sensitiveness of the arrangement is, of course, directly proportional to the current through the coils, and as this was not constant from day to day we thought it best to determine it on every occasion.

Before proceeding with a measurement, the brushes were wiped with wash-leather and the tension on the wires adjusted. The variations in the thermo-electric effects were then observed and the petrol lubrication attended to. Bad lubrication on disc No. 1 produced a deflection of the galvanometer in one direction, and bad lubrication on disc No. 2 produced a deflection in the reverse direction. With a good supply of petrol on the edges of the discs there was practically no deflection when the brushes were in good condition. When the brushes were somewhat worn—usually after 6 runs—it was not possible to take good observations, and rather than waste time in making indifferent measurements we preferred to wait until new brushes had been inserted. Careful tests of the brushes were regularly made, and it is no doubt due to this fact that we are able to record the result of every completed measurement. At times the thermo-electric and speed variation effects were a little troublesome, but we

never considered them sufficiently serious to justify us in discarding the results. It is true that in two experiments we were not able to calculate the resistance; in the first of these, the apparatus for recording the speed was not put in gear and so no record was made; in the second, an interference with the battery connected to the motor produced a change of speed beyond our control and we had to abandon the experiment. In all other cases when a set of measurements was commenced, it was completed and the result is given in this paper.

At a speed of about 17.4 revolutions per second, the product Mn is nearly 2×10^6 , and the corresponding resistance is therefore about 0.002 ohm. We have already described the combination of coils which gives such an effective resistance, and we used the apparatus for its measurement on ten occasions.

To measure a resistance of 0.001 ohm, the coils on cylinders Nos. 1 and 2 were placed in parallel with those on cylinders Nos. 3 and 4. The latter coils were found to be equal in resistance to the former within the limits of the errors of the measurements made, and a division of the main current into two parts, equal within about 1 in 3000 could be ensured. In general, equality of the divided currents, within 1 per cent. would have been sufficient. The current through the standard resistance is now twice the mean value of the current through the coils and the resulting equation is $iMn = 2iR$. When $Mn = 2 \times 10^6$, R is 0.001 ohm. This arrangement was most convenient. A resistance of 0.001 ohm is more readily compared with 1-ohm standards than a resistance of 0.002 ohm, and as our standard of 0.001 ohm was of thick manganin strip, a current of 4 amperes could be passed through it without an appreciable heating effect.

When the brushes are placed in series, a resistance of 0.01 ohm can be measured, as the equation $5iMn = iR$ then holds good. Although this arrangement is very sensitive, a slightly greater pressure is required on each brush and this frequently produced trouble.

A sample series of readings, taken on June 7, 1913, gives a good idea of the measurements involved in a single determination:—

June 7, 1913.

- (1) Observations on invar line standard. Temperature = $15^{\circ}.5$ C. Lines 4–616.
Length at $15^{\circ}.5$ C. = 30.6016₇ cm.

Microscope readings (corrected).

	Left.	Right.	Diff. (L - R).	
Taken before readings on plugs	1154.9	946.6	+ 208.3 μ	} Mean = + 208.3 μ .
	1135.0	1028.0	+ 207.0	
	1133.4	1025.4	+ 208.0	
Taken after readings on plugs	979.4	770.9	+ 208.5	}
	982.4	774.1	+ 208.3	
	1035.1	825.2	+ 209.9	

(2) Observations on plugs.

Microscope readings (corrected).

Plug.	Left.	Plug.	Right.	Diff. (L - R).	Corr.,* <i>d</i> .	(L - R) - <i>d</i> .
(7)	927.3	(16)	179.2	748.1	- 750	- 1.9 μ
(8)	1551.1	(15)	834.1	717.0	- 716	+ 1.0
(4)	1052.0	(11)	1185.2	- 133.2	+ 107	- 26.2
(3)	328.9	(12)	2447.9	- 2119.0	+ 2094	- 25.0
(10)	1042.6	(1)	1232.3	- 189.7	+ 177	- 12.7
(9)	1167.6	(2)	1126.0	41.6	- 55.5	- 13.9
(13)	294.3	(6)	1199.8	- 905.5	+ 887	- 18.5
(14)	1409.9	(5)	757.5	652.4	- 669	- 16.6
Mean						- 14.2 μ

 $\therefore$ Mean distance between the mean diametral planes of the coils

$$= 30.6016_7 \text{ cm.} - 14.2\mu - 208.3\mu = 30.5794_2 \text{ cm.}$$

(3) Temperature of Marble Cylinders.

No.	Before resistance measurements. ° C.	After resistance measurements. ° C.	Difference. ° C.
1	16.6	18.8	+ 2.2
2	16.55	18.75	+ 2.2
3	16.5	18.8	+ 2.3
4	16.55	18.9	+ 2.35
Mean . . .	16.5 ₅	Mean . . .	+ 2.2 ₅

$$\therefore \text{Effective temperature} = 16^{\circ}.5_5 \text{ C.} + 0.42 (2^{\circ}.25 \text{ C.}) = 17^{\circ}.5 \text{ C.}$$

* The correction is equal to the difference of the distances of the plugs from the mean diametral planes of the coils, these distances being deduced from the metrology measurements. Thus we obtained from these measurements:—Distance (d_1) of plug 7 from centre = 9.3892 cm., distance (d_2) of plug 16 from centre = 9.3142 cm. Hence, when the coils are parallel, the distance of plug 16 from plug 7 is greater than the distance between the mean diametral planes of the coils by $9.3892 - 9.3142 = 750\mu$ and a corresponding correction must therefore be applied. For our purpose it is necessary to measure four distances between four pairs of plugs, but, in practice, we measured eight distances between eight pairs of plugs in order to obtain a check. The agreement was always good.

(4) Changes in the Deflection of the Galvanometer produced on Reversal of the Current in the Coils.

Set No. 1.	Sensitiveness (a).	Set No. 2.		Sensitiveness (b).	Set No. 3.
mm.	mm.	mm.	mm.	mm.	mm.
+4.2	-36.4	+0.2	+6.0	-38.4	+2.2
4.2	40.0	4.4	3.8	38.4	4.2
1.0	38.6	0.2	1.2	36.6	4.0
4.4	38.0	0.6	0.0	40.2	3.6
2.8	41.0	6.0	3.0	41.0	1.4
2.2	37.0	2.8	3.8	36.0	2.6
7.0	40.0	7.0	2.4	37.2	0.0
5.4	36.2	5.0	1.8	36.2	1.0
1.6		4.0	3.8		3.8
4.4		4.4	2.4		5.0
6.0		1.0	1.4		4.4
4.0		1.0	4.2		3.8

Mean of Sets 1, 2, 3 = $+3.2_0$ mm.

Mean of Sets (a) and (b) = -38.2 mm.

$\therefore$ Change of deflection for a diminution of the resistance of 100 parts in 100,000

$$= -38.2 - 3.2 = 41.4 \text{ mm.}$$

$\therefore$ The resistance is greater than the product Mn by $\frac{100 \times 3.20}{41.4} = +7.7$ parts in 100,000.

(4) Sets of readings similar to (1) and (2) on the invar standard and the plugs were made after the resistance measurements, but these need not be given here. The mean distance obtained is identical within 1μ with that already given. Such remarkable agreement was in general found between the two sets of measurements that we conclude the expansion of the cylinders does not affect the position of the mean diametral planes of the coils. In such a case, with uniform expansion of the four cylinders, the plug distances keep absolutely constant. Because of this we did not always take a second set of readings. Constancy over seven or eight hours was, however, rarely obtained, for the bed of the machine usually increased in temperature, and its expansion resulted in a change in the distance between the cylinders. Of course, the distance was frequently changed intentionally to alter the value of M .

(5) The temperature of the discs was about 17°C . and the distance between the brush contact circles was 167.5_0 cm. The speed was found from the record to be 17.3835_7 revolutions per second.

(6) Calculation of Mn . :—

(a) The value of M_1 is 115893.7 when the temperature of the coils and discs is 20°C . and the distance between the mean diametral planes of the coils is 30.5785 cm.

(b) The value of M_2 is 742.4 for a distance between the brush contact circles of 167.50 cm.

(c) When the distance between the diametral planes is increased to 30.5794_2 cm., M_1 is diminished by 3.19 ; and when the temperature of the coils is diminished to 17.5 C., M_1 is reduced by 3.06 .

(d) When the temperature of the disc is reduced to 17° C., $M_1 - M_2$ is increased by 0.12 .

(e) Hence the value of M is

$$\begin{aligned} M = M_1 - M_2 &= 115893.7 - 742.4 - 3.17 - 3.06 + 0.12 \\ &= 115145.2 \text{ cm.} \end{aligned}$$

(f) Mn is therefore $115145.2 \times 17.3835_7 = 200163_5$.

(7) Calculation of R :—

We have

$$iMn = 2iR \{1 + (7.7 \times 10^{-5})\}.$$

Hence

$$\begin{aligned} R &= 200163_5 / 2 + 7_7 \\ &= 100089_5 \text{ cm./sec.} \\ &= 0.00100089_5 \text{ ohm.} \end{aligned}$$

This value of R is on the assumption that the position of the mean diametral planes has been correctly estimated from the metrology measurements. To remove the assumption it is necessary to reverse the coils without interchanging, and this was done after 28 measurements of the 0.001 ohm standard had been made. The effect of reversal is very small and indicates that the assumed mean position of the diametral planes is very nearly correct. The mean result obtained in the two positions is taken as the value of R . When the coils Nos. 1 and 2 are used independently of Nos. 3 and 4, an appreciable difference—about 4 parts in 100,000—is obtained on reversal of the coils. The conclusion is that the estimated position of the mean diametral planes of the coils Nos. 1 and 2 is incorrect by about 0.006 mm. The same is true for the coils Nos. 3 and 4.

In Tables XX. and XXIII. we give the data relating to measurements of a resistance of nominal value 0.001 ohm, and Tables XXI. and XXII. give the results only of the measurements of a resistance of 0.01 ohm and an effective resistance of 0.002 ohm.

The observed values in absolute measure and the values in international ohms (new N.P.L. Mercury Standards of Resistance) are given in Table XXIV.

Probable Errors.

The mean observational error of the results given in Table XX. is about 2 parts in 100,000 for a single observation. This error includes all the errors arising from an inaccurate estimation of the distance apart of the coils, of variations in the speed, of an erroneous estimate of the mean speed, of faulty temperature observations, and the

TABLE XX.—Measurement of 0.001 ohm at 20° C. Part A. (Ring Ends of Cylinders away from Discs.)

 D_1 = Mean distance between the mean diametral planes of coils 1 and 2 and of coils 3 and 4. M = Mutual inductance between the coils and the discs. n = Number of revolutions per second. R = Value of resistance in absolute measure, as deduced from the observations. $(R > Mn)$ = Value deduced from mean change in galvanometer deflection on reversal of current.

The values given for D_1 , M , &c., are not quite correct. They are subject to small corrections owing to the exact positions of the mean diametral planes of the coils not being known. These corrections are of exactly the same magnitude in Part B of the table, but they are of opposite sign. The mean value of R (Parts A and B) is the true one.

The distance between the brush-contact circles was practically constant and equal to 167.50 cm.

Date.	D_1 . cm.	t_1 = tempe- rature of coils. ° C.	t_2 = tempe- rature of discs. ° C.	M .	n .	$Mn/2$.	$R > Mn$.	R .	Difference from mean. Parts in 100,000.
5.3.13	30.5794 ₉	16.6	17	115146.0	17.3890 ₂	100113 ₈	-22 ₄	100091 ₄	-0.6
5.3.13	30.5795 ₅	18.0	18	115147.4	17.3888 ₇	100114 ₂	-21 ₆	92 ₆	+0.6
5.3.13	30.5802 ₀	21.1	19	115148.9	17.3892 ₂	100117 ₅	-30 ₂	87 ₃	-4.7
11.3.13	30.5790 ₃	16.9	17	115147.9	17.3812 ₁	100070 ₅	+19 ₇	90 ₂	-1.8
12.3.13	30.5782 ₉	12.8	13	115145.6	17.3928 ₇	100135 ₆	-41 ₈	93 ₈	+1.8
12.3.13	30.5951 ₅	17.3	15	115092.6	17.3925 ₂	100087 ₅	+5 ₂	92 ₇	+0.7
13.3.13	30.5955 ₃	16.5	17	115090.3	17.3932 ₂	100089 ₅	+3 ₃	92 ₈	+0.8
14.3.13	30.5957 ₉	17.2	17	115090.2	17.4024 ₈	100142 ₅	-51 ₃	91 ₂	-0.8
17.3.13	30.5943 ₁	15.0	15	115092.7	17.3928 ₇	100089 ₆	+3 ₁	92 ₇	+0.7
17.3.13	30.5924 ₇	17.7	17	115102.3	17.3930 ₂	100098 ₈	-7 ₀	91 ₈	-0.2
18.3.13	30.5939 ₅	12.2	12	115090.7	17.3951 ₇	100101 ₁	-8 ₈	92 ₃	+0.3
18.3.13	30.5885 ₉	15.1	14	115112.7	17.3951 ₂	100120 ₀	-28 ₈	91 ₂	-0.8
18.3.13	30.5892 ₆	16.8	15	115112.4	17.3945 ₂	100116 ₂	-21 ₀	95 ₂	+3.2
19.3.13	30.5888 ₀	13.9	14	115110.5	17.3939 ₆	100111 ₃	-21 ₆	89 ₇	-2.3
27.3.13	30.5886 ₉	14.6	15	115111.7	17.3896 ₈	100087 ₈	+3 ₂	91 ₀	-1.0
28.3.13	30.5888 ₃	13.4	14	115109.8	17.3935 ₇	100108 ₅	-13 ₉	94 ₆	+2.6
28.3.13	30.5886 ₄	15.9	15	115113.5	17.3937 ₈	100112 ₇	-15 ₅	97 ₂	+5.2
7.4.13	30.6180 ₃	12.9	13	115008.0	17.4058 ₅	100090 ₆	-2 ₆	88 ₁	-3.9
7.4.13	30.6084 ₈	15.8	15	115044.6	17.4057 ₈	100121 ₈	-31 ₀	90 ₈	-1.2
7.4.13	30.6266 ₆	17.7	17	114983.8	17.4060 ₀	100069 ₁	+26 ₁	95 ₂	+3.2
7.4.13	30.6265 ₄	19.4	18	114986.3	17.4057 ₈	100072 ₆	+17 ₄	90 ₀	-2.0
8.4.13	30.5291 ₇	14.6	15	115318.7	17.3635 ₂	100116 ₉	-26 ₃	90 ₆	-1.4
8.4.13	30.5289 ₄	17.0	17	115322.3	17.3629 ₃	100116 ₇	-23 ₆	93 ₁	+1.1
8.4.13	30.5298 ₁	18.3	18	115320.8	17.3629 ₃	100115 ₃	-25 ₀	89 ₄	-2.6
8.4.13	30.5303 ₃	19.8	18	115320.9	17.3641 ₃	100122 ₄	-27 ₂	95 ₂	+3.2
9.4.13	30.5298 ₂	15.8	16	115317.8	17.3637 ₅	100117 ₅	-26 ₅	90 ₉	-1.1
9.4.13	30.5308 ₁	18.0	17	115317.0	17.3641 ₅	100119 ₁	-25 ₄	93 ₇	+1.7
14.4.13	30.5294 ₇	16.0	16	115319.6	17.3644 ₂	100122 ₉	-31 ₂	91 ₇	-0.3
							Mean . . .	100092 ₀	±1.8

TABLE XX. (continued).—Part B. (Ring End of Cylinders near Discs.)

Date.	D ₁ .	t_1 = temperature of coils.	t_2 = temperature of discs.	M.	n .	$Mn/2$.	$R > Ma$.	R.	Difference from mean. Parts in 100,000.
	cm.	° C.	° C.			cm./sec.	cm./sec.	cm./sec.	
4.6.13	30.5521 ₈	18.6	19	115241.1	17.3752 ₅	100117 ₁	-22 ₇	100094 ₄	+3.2
4.6.13	30.5996 ₁	19.1	19	115077.0	17.4019 ₂	100128 ₀	-37 ₂	90 ₈	-0.4
5.6.13	30.5998 ₀	18.0	18	115075.0	17.3962 ₃	100093 ₃	-4 ₃	89 ₃	-1.9
5.6.13	30.6012 ₁	19.7	19	115072.2	17.3955 ₂	100087 ₀	+5 ₇	92 ₇	+1.5
5.6.13	30.6007 ₉	20.1	19	115074.1	17.3941 ₂	100080 ₃	+10 ₉	91 ₅	+0.3
6.6.13	30.5797 ₄	16.7	17	115143.0	17.3867 ₈	100098 ₃	-7 ₀	91 ₃	+0.1
6.6.13	30.5797 ₄	18.2	17	115144.9	17.3862 ₈	100097 ₁	-6 ₁	91 ₀	-0.2
6.6.13	30.5799 ₉	18.5	18	115144.3	17.3861 ₈	100096 ₀	-7 ₀	89 ₀	-2.2
7.6.13	30.5797 ₃	15.8	16	115142.0	17.3839 ₂	100081 ₀	+8 ₀	89 ₀	-2.2
7.6.13	30.5794 ₂	17.5	17	115145.1	17.3835 ₇	100081 ₆	+7 ₇	89 ₃	-1.9
10.6.13	30.5793 ₀	15.3	16	115142.9	17.3852 ₂	100089 ₂	-2 ₁	87 ₁	-4.1
10.6.13	30.5889 ₅	17.2	17	115111.7	17.3927 ₇	100105 ₆	-13 ₈	91 ₈	+0.6
10.6.13	30.5891 ₂	17.5	17	115111.5	17.3917 ₂	100099 ₄	-7 ₄	91 ₇	+0.5
10.6.13	30.5892 ₄	18.9	18	115112.7	17.3916 ₈	100100 ₂	-8 ₁	92 ₁	+0.9
11.6.13	30.6201 ₈	16.3	17	115002.3	17.4118 ₃	100120 ₀	-26 ₈	93 ₂	+2.0
11.6.13	30.6200 ₈	18.2	18	115005.0	17.4064 ₂	100091 ₃	+3 ₀	94 ₃	+3.1
11.6.13	30.6199 ₉	18.7	18	115005.9	17.4062 ₇	100091 ₂	-3 ₀	89 ₂	-2.0
11.6.13	30.6204 ₁	18.7	18	115004.4	17.4068 ₉	100093 ₄	-3 ₈	90 ₁	-1.1
11.6.13	30.6201 ₃	20.9	19	115008.1	17.4071 ₃	100098 ₀	-7 ₅	90 ₅	-0.7
13.6.13	30.6202 ₀	17.3	17	115003.5	17.4036 ₅	100074 ₀	+17 ₃	91 ₃	+0.1
14.6.13	30.5445 ₈	16.2	16	115264.7	17.3667 ₃	100088 ₅	+5 ₀	93 ₅	+2.3
14.6.13	30.5442 ₀	18.6	18	115268.9	17.3667 ₃	100092 ₂	+0 ₈	93 ₀	+1.8
14.6.13	30.5442 ₉	20.2	19	115270.5	17.3660 ₂	100089 ₅	+3 ₂	92 ₇	+1.5
16.6.13	30.5449 ₃	18.1	18	115265.7	17.3632 ₅	100069 ₄	+21 ₉	91 ₃	+0.1
16.6.13	30.5451 ₄	20.2	19	115267.5	17.3670 ₁	100092 ₆	-1 ₆	91 ₀	-0.2
16.6.13	30.5453 ₄	21.9	20	115268.9	17.3676 ₆	100097 ₆	-5 ₂	92 ₄	+1.2
17.6.13	30.5462 ₇	24.0	22	115268.1	17.3677 ₂	100097 ₂	-3 ₉	93 ₃	+2.1
18.6.13	30.5281 ₈	19.1	19	115325.0	17.3595 ₃	100099 ₄	-10 ₇	88 ₇	-2.5
							Mean . . .	100091 ₂	±1.5

$$\therefore \text{Resistance} = \frac{100092_0 + 100091_2}{2} = 100091_6 \text{ cm./sec. at } 20^\circ \cdot 0 \text{ C.}$$

Probable error of observations = 3 parts in 1,000,000.

Resistance in international ohms (N.P.L.) = 0.00100039₃ at 20°·0 C.

Difference :—Absolute measure - international measure $\times 10^9 = 52.3$ parts in 100,000.

TABLE XXI.—Results of Measurements of 0.01 ohm at 20°·0 C.

Part A (rings away).			Part B (rings near).		
Date.	Measured value of R in cm./sec.	Difference from mean. Parts in 100,000.	Date.	Measured value of R in cm./sec.	Difference from mean. Parts in 100,000.
13.2.13	100092 ₂₀	+1.8	18.6.13	100093 ₂₀	+3.5
4.3.13	90 ₄₀	±0.0	19.6.13	87 ₈₀	-1.9
28.3.13	88 ₃₀	-2.1	19.6.13	90 ₉₀	+1.2
29.3.13	90 ₅₀	+0.1	19.6.13	91 ₅₀	+1.8
31.3.13	93 ₂₀	+2.8	20.6.13	89 ₂₀	-0.5
31.3.13	88 ₆₀	-1.8	20.6.13	88 ₈₀	-0.9
31.3.13	93 ₄₀	+3.0	21.6.13	91 ₅₀	+1.8
1.4.13	93 ₀₀	+2.6	24.6.13	90 ₂₀	+0.5
1.4.13	91 ₉₀	+1.5	25.6.13	86 ₇₀	-3.0
3.4.13	88 ₅₀	-1.9	26.6.13	87 ₅₀	-2.4
4.4.13	87 ₈₀	-2.6	26.6.13	90 ₂₀	+0.5
5.4.13	87 ₅₀	-2.9	27.6.13	89 ₂₀	-0.5
Mean . . .	100090 ₄₀	±1.9	Mean . . .	100089 ₇₀	±1.5

$$\therefore \text{Resistance} = \frac{100090_{40} + 100089_{70}}{2} = 100090_{10} \text{ cm./sec. at } 20^{\circ}\cdot 0 \text{ C.}$$

Probable error of observations = 5 parts in 1,000,000.

Resistance in international ohms (N.P.L.) = 0.0100038₃ at 20°·0 C.

Difference :—Absolute measure – international measure $\times 10^9$ = 51.8 parts in 100,000.

TABLE XXII.—Results of Measurements at 20°·0 C. of an Effective Resistance of 0.002 ohm.

Part A (rings away).					Part B (rings near).				
Date.	R in cm./sec.	R in international ohms.	Difference. Abs. – int. (10 ⁹) parts in 100,000.	Difference from mean.	Date.	R in cm./sec.	R in international ohms.	Difference. Abs. – int. (10 ⁹) parts in 100,000.	Difference from mean.
6.3.13	200479 ₇	200368 ₂	55.8	+1.8	28.1.13	200469 ₀	200365 ₀	52.0	±0.0
6.3.13	472 ₃	368 ₂	52.1	-1.9	28.1.13	199457 ₉	199354 ₀	52.0	±0.0
6.3.13	472 ₄	368 ₂	52.1	-1.9	4.2.13	200465 ₄	200365 ₀	50.2	-1.8
7.3.13	478 ₆	368 ₂	55.2	+1.2	5.2.13	468 ₆	365 ₀	51.8	-0.2
7.3.13	478 ₁	368 ₂	55.0	+1.0	6.2.13	473 ₇	365 ₀	54.3	+2.3
		Mean . .	54.0	±1.6			Mean . .	52.0	±0.9

Probable error of observations = 6 parts in a million.

Difference :—Absolute measure – International measure $\times 10^9$ = (54.0 + 52.0)/2 = 53.0 parts in 100,000.

TABLE XXIII.—Results of Measurements at 20°0 C. of a Resistance of 0·001 ohm
(a) when the Coils on Cylinders 1 and 2 only were employed; (b) when the
Coils on Cylinders 3 and 4 only were employed.

(a) Coils on cylinders 1 and 2 used.				(b) Coils on cylinders 3 and 4 used.			
Date.	Ring position.	R in cm./sec.	Difference from mean. Parts in 100,000.	Date.	Ring position.	R in cm./sec.	Difference from mean. Parts in 100,000.
15.4.13	Away	100089 ₇	1·1	15.4.13	Away	100093 ₇	1·1
15.4.13	„	87 ₄	1·2	15.4.13	„	95 ₉	1·1
	Mean . . .	100088 ₆			Mean . . .	100094 ₈	
17.6.13	Near	100092 ₁	0·1	17.6.13	Near	100089 ₁	0·4
18.6.13	„	92 ₄	0·2	18.6.13	„	88 ₄	0·3
	Mean . . .	100092 ₂			Mean . . .	100088 ₇	
Grand mean = 100090 ₄ cm./sec.				Grand mean = 100091 ₈ cm./sec.			
Resistance in international ohms = 0·0100039 ₃ . Difference :—Absolute value – international value $\times 10^9$ = 51·1 parts in 100,000.				Resistance in international ohms = 0·0100039 ₃ . Difference :—Absolute value – international value $\times 10^9$ = 52·5 parts in 100,000.			

TABLE XXIV.

No. of observations in each position of coils.		Resistance.			Difference. Parts in 100,000 (abs. $\times 10^{-9}$) – (int.).
		Standard.	Absolute measure.	In international ohms.	
28		ohm.	cm./sec.		
12		0·001	100091 ₆	0·00100039 ₃	52· ₃
5		0·01	100090 ₁₀	0·0100038 ₃	51· ₈
2	Coils 1 and 2 used	0·002	See Table XXII.		53· ₀
2	Coils 3 and 4 used	0·001	100090 ₄	0·00100039 ₃	51· ₁
2		0·001	100091 ₈	0·00100039 ₃	52· ₅

The agreement is most satisfactory. Weighted mean . . . 52·₀

error arising from the observation of the galvanometer deflections. The probable observational error of the mean value in absolute measure of the 0.001 ohm resistance standard is about 3 parts in 1,000,000.

The probable error of the diametral dimensions of the coils has been estimated in Section 13 to be not greater than 1μ plus the errors of the gauges employed. Inspection of the results given in Table III. shows that the mean value taken as correct differs from the results obtained in February and March, 1912, by about 1.4μ , and from those obtained in April, 1913, by about 0.7μ . The gauges employed are believed to be accurate within 0.5μ , so that the maximum probable error of the diametral dimensions is about $\pm 1.5\mu$ which corresponds to a probable error in the mutual inductance of about 8 parts in 1,000,000. This assumes that the error is of the same sign for all the coils.

The probable error of the mean axial lengths of the coils has been estimated in Section 13 to be about 3μ , and a change of this amount in all the coils and in the same direction produces a change in the mutual inductance of 2 parts in 1,000,000.

Any error due to lack of knowledge of the position of the mean diametral planes of the coils is practically eliminated by the reversals. An analysis of the plug measurements shows that the error cannot be so great as 2 parts in 1,000,000, and we may therefore dismiss it from consideration.

The growth in the dimensions of the coils due to the passage of the current is not really large. The mutual inductance of the coils and brush contact circles is nearly 3 parts in 100,000 greater after the current has been left on for 20 minutes, but the mean mutual inductance is only about 1.5 parts in 100,000 greater than that at the start. The error of estimation of the increase must have an exceedingly small effect on our final results and need not be considered.

The mean diametral distance between opposite segments on the discs diminished by 0.08 mm. during our experiments. Such a change would be a serious one in the old form of Lorenz apparatus, but in our instrument the change in the mutual inductance due to such a reduction in the mean diameter is less than 4 parts in 1,000,000.

Evidence in favour of a small probable error is afforded by the satisfactory agreement between the results obtained when coils Nos. 1 and 2, and 3 and 4, were used independently. The resistance of the 0.001 ohm standard was found to be 100090₄ cm./sec. when coils Nos. 1 and 2 only were used, and 100091₈ cm./sec. when coils Nos. 3 and 4 were used. The difference is 14 parts in 1,000,000 and part of this is possibly due to the errors of observations, for only four observations were made in each case.

The electrical method of setting the coils in position has been shown to be subject to errors less than 1 part in 1,000,000; the magnetic susceptibility of the parts of the apparatus, excluding the motor, is too small to be measured with the apparatus at our disposal; and the effect of the motor on the mutual inductance of the coils and discs has been proved to be negligibly small. The errors of speed cannot be all of one

sign and must be quite negligible apart from a constant error in the clock rate. But this latter is clearly impossible in the case of a standard clock the error of which is taken daily. The daily rate was small and time comparisons were made with another standard clock the daily rate and error of which were also known. At any hour of the day the difference between the clocks agreed, within the possible error of the observations, with that calculated from the errors and rates of the clocks. This agreement is evidence that both clocks were going uniformly or that there was a similar want of uniformity in the going of both. Such similarity is very improbable. As an additional precaution the resistance observations were made at times ranging from 9 a.m. to 6 p.m. but no systematic differences were observed.

The possibility of error due to our coils being of wire of finite section has not been overlooked. The formula developed by J. VIRIAMU JONES gives the mutual inductance when the coils can be treated as infinitely fine helical filaments, and small corrections may be necessary. The case of the coils of the Ayrton-Jones current balance was examined by Dr. G. F. C. SEARLE, F.R.S.,* who showed the correction to be negligible in that instrument. In the present instance, no special treatment is necessary as the mutual inductance calculations made during the work are sufficient to show that no correction need be made. To illustrate this, consider the mutual inductance M of a helical filament coinciding with the axis of the wire and the nearer brush contact circle. Next consider two helical filaments of the same diametral dimensions as the previous one, but let one helix be nearer to, and the other farther from, the circle by 0.25 mm. Table X. shows that the mean mutual inductance of these two helical filaments and the circle is greater than that of the central helical filament by 2 parts in 1,000,000. The wire with which the coils are wound is about 0.557 mm. in diameter, so that strictly we ought to consider filaments 0.28 mm. away from the central one, but in view of the result obtained we think it unnecessary to calculate the small and certainly negligible difference. Next consider two circles coaxial with the disc and at equal distances from it, and let the diameter of one circle be greater than that of the other by 0.556 mm., the mean diameter being 35.88 cm. At distances of 7.3135 cm. and 23.3150 cm. the mean mutual inductance of these two circles and the circumference of the disc does not differ by more than 2 parts in 1,000,000 from the mutual inductance of the disc and circle of radius equal to the mean of the two previous ones (see Table XVI.). The coil of wire may therefore be treated as a helical filament.

The determination of a resistance in absolute measure is therefore subject to a number of small errors, the greatest of which is associated with the determination of the mean radius of the coils. This error is probably not greater than 1 part in 100,000, and if the remaining errors were all of the same sign it is unlikely that their sum would exceed another part in 100,000.

We believe, therefore, that the absolute measurements of resistance which we have made are correct within 2 parts in 100,000.

* 'Phil. Trans.,' A, 207, pp. 541-544.

The resistances which we have employed have been compared with nine new mercury standards of resistance constructed in accordance with the Specifications of the London Conference on Electrical Units and Standards (1908). These mercury standards of resistance practically realise the international ohm then defined, and the agreement between the nine standards is very good.* There are, however, certain sources of error in the construction of such standards which must always be of the same sign, and the probable error associated with the practical realisation of the international ohm has been estimated to be not less than 2 or 3 parts in 100,000.

The mean of the results given in Tables XX., XXI., XXII., and XXIII. leads to the conclusion :—

A resistance of 1 international ohm is equal to $1\cdot00052 \pm 0\cdot00004$ ohm (10^9 cm./sec.), the probable error $\pm 0\cdot00004$ being approximately the sum of those involved in the realisation of the ohm and the international ohm.

The international ohm, as defined by the London Conference on Electrical Units and Standards (1908), is the resistance at 0° C. of a column of mercury, $14\cdot4521$ gr. in mass of a constant cross-sectional area and of a length of $106\cdot300$ cm. As stated in the Introduction, the cross-section of such a column is equal to 1 sq. mm. or nearly so.

Since the international ohm is equal to $1\cdot00052 \pm 0\cdot00004$ ohms, the mass of the column of mercury of the same cross-sectional area as the international ohm and having

a resistance of 1 ohm will be $\frac{14\cdot4521}{1\cdot00052 \pm 0\cdot00004} = 14\cdot4446 \pm 0\cdot0006$ gr.,† while the length

of the column will be $\frac{106\cdot300}{1\cdot00052 \pm 0\cdot00004} = 106\cdot245 \pm 0\cdot00004$ cm.†

We may sum up our results by stating that :—

The ohm 10^9 cm./sec. is represented by the resistance at 0° C. of a column of mercury $14\cdot4446 \pm 0\cdot0006$ gr. in mass, of a constant cross-sectional area (the same as for the international ohm) and having a length of $106\cdot245 \pm 0\cdot004$ cm.

The Historical Introduction shows a number of determinations, notably those of RAYLEIGH (corrected values marked (S) Table I.), GLAZEBROOK (corrected value (S) Table I.), WIEDEMANN, DORN, and HIMSTEDT, in close agreement with that now obtained. These results are as follows :—

1882, RAYLEIGH	106·26	cm.
1882, GLAZEBROOK	106·25	„
1883, RAYLEIGH	106·24	„
1885, WIEDEMANN	106·265	„
1889, DORN	106·243	„
1892, HIMSTEDT	106·259	„

* ‘Report of the National Physical Laboratory for 1912.’

† The probable errors in these two values are so related that an error in either value is necessarily associated with an equivalent proportionate error in the other.

On the whole, the remaining results in Table I. give the length of the column of mercury representing the ohm as greater than 106.25 cm., and while a discussion of the methods and apparatus might do much to explain some of the differences, much greater detail of the apparatus than is readily accessible is necessary for this to be done.

The ohm and the international ohm differ by about 5 parts in 10,000, and since the ampere (10^{-1} C.G.S. unit) has been realised with considerable accuracy, we may express the electromotive force of the Weston normal cell in absolute measure. The value found in 1908 by AYRTON, MATHER and SMITH at the National Physical Laboratory for the E.M.F. of the Weston normal cell in terms of the ampere (10^{-1} C.G.S. unit) and the international ohm is 1.01830 volts at 17° C., with a probable error of 2 parts in 100,000. The deduced value at 20° C. is 1.01818 volts. Since that time measurements have been made by JANET, LAPORTE, and JOUAST* at the Laboratoire Central d'Électricité; by Prof. GUILLET† and Prof. PELLAT,‡ by HAGA,§ and by ROSA and DORSEY§ at the Bureau of Standards. The results are:—

AYRTON, MATHER, and SMITH	1.01818
JANET, LAPORTE, and JOUAST	1.01836
GUILLET	1.01812
PELLAT	1.01831
HAGA	1.01825
ROSA and DORSEY	1.01822
Mean	<u>1.01824 at 20° C.</u>

These results include the errors of the resistance standards employed as well as the errors of the determination of the current in absolute measure. In addition small differences existed between the Weston cells. Considering all these circumstances the agreement is very remarkable and testifies to the great care taken in the measurements. There appears to be little doubt that the value 1.01824 at 20° C. is correct, within 1 part in 10,000. We conclude that the value of the cell in volts (10^8 C.G.S. units) is $1.0188 \pm .0001$ at 20° C. This value may serve for the present for those engaged in absolute measurements.

SECTION 24.—CONCLUSIONS.

From the measurements recorded in the previous sections, we conclude that the instrument we have described may be used for absolute measurements of resistance

* 'Bull. de la Soc. Internat. des Electriciens' (2), vol. 8, p. 459, 1908, and 'Comptes Rendus,' 153, p. 718, 1911.

† 'Bull. de la Soc. Internat. des Electriciens,' 1908.

‡ 'Konink. Akad. Wetensch. Amsterdam Proc.,' p. 587, 1910.

§ 'Bull. Bureau of Standards,' vol. 8, p. 269, 1912.

with a precision satisfying all present demands whether purely scientific or technical. We believe the instrument fully realises the desires of those who were responsible for such measurements being made, and the results justify Lord RAYLEIGH's belief that the ohm, as defined in absolute measure, can be realised with a precision comparable with that of the international ohm. The instrument can be used at any time; the dimensions of all its parts can be checked when desired, the probable error associated with a single measurement is small, and the observations do not unduly tax the experimenter. We have formed an estimate, based on the measurements already made, of the time necessary to devote to a complete re-determination supposing the cylinders to be stripped of the coils. The re-winding of the coils would occupy three days, the metrology measurements would extend over six days, and sufficient resistance observations could be made in eight more days. Absolute measurements of resistance may therefore be placed in the front rank of precision measurements.

In conclusion, we desire to express our sincere thanks to the Drapers Company of London for providing a large sum of money towards the cost of the instrument, and to Sir ANDREW NOBLE, F.R.S., for his generous help with the heavy metal work.

Our most hearty thanks are hereby tendered to our Director, Dr. R. T. GLAZEBROOK, C.B., F.R.S., who has not only given his very valuable help and advice throughout the work, but has fully appreciated the many difficulties which have arisen and which required much time and patience to remove.

Our best thanks are also due to Lord RAYLEIGH for his keen interest in the investigation, to Dr. STANTON for superintending the turning of the marble cylinders, and to many of the staff of the National Physical Laboratory, particularly Mr. A. CAMPBELL and Mr. DYE, for suggestive aid throughout the investigation.

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THE INFLUENCE OF MOLECULAR CONSTITUTION AND
TEMPERATURE ON MAGNETIC SUSCEPTIBILITY.

BY

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III. *The Influence of Molecular Constitution and Temperature on Magnetic Susceptibility.*

By A. E. OXLEY, B.A., M.Sc., *Coutts Trotter Student, Trinity College, Cambridge.*

Communicated by Prof. Sir J. J. THOMSON, O.M., F.R.S.

Received December 17, 1913,—Read January 29, 1914.

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INTRODUCTION.

THE present communication is a continuation, and in some respects a completion, of several preliminary researches which have been published from time to time during the past three years.

On the hypothesis that a molecule may be distorted by the near approach of its neighbours, it has been shown* how the presence of molecular groups, which vary in

* 'Proc. Camb. Phil. Soc.,' vol. XVI., p. 486, 1912.

complexity with change of temperature, may modify the temperature coefficient of the magnetic property of a substance. The large change of molecular constitution during the transition from the liquid to the crystalline state should therefore be accompanied by a sudden change of the magnetic property.

The relation between molecular complexity and specific diamagnetic susceptibility χ_D may be written,* on the above hypothesis, in the form

$$\chi_D = \frac{1}{H} \sum n_p \cdot \Delta M_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where ΔM_p is the total diamagnetic moment produced in a complex of type p by the application of an external magnetic field H . n_p is the number of complexes of the type p per gramme. During a transition from one degree of complexity to another, several types of complexes may coexist. The summation is extended over all such types.

The corresponding expression for the specific paramagnetic susceptibility χ_P is

$$\chi_P = \sum \frac{n_p \cdot c_p}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where c_p is the Curie constant per particle of type p . Since, in the above equations, n_p is a function of the temperature, characteristic for each substance, it is probable that the irregularities of the temperature susceptibility curves obtained by Prof. HONDA,† Prof. DU BOIS,‡ and Mr. OWEN§ are due to the presence of this variable factor.

The application of (1) and (2) to the case of aqueous solutions of salts of the ferromagnetic elements, for which n_p is approximately a linear function of the temperature, gives indirect evidence of the existence of molecular complexes in solution|| (see also footnote, p. 134 *infra*).

It is intended in the present research to test more fully the hypothesis stated above, which is involved in (1) and (2), by investigating the magnetic susceptibility of a number of substances over a wide range of temperature. To simplify the problem, the substances chosen are all diamagnetic and consist chiefly of organic compounds. The small electrical conductivity of the latter allows us to neglect any effect which may arise from the presence of moving intermolecular charges.¶ The diamagnetic property will therefore be a concern of the internal structure of the

* *Loc. cit.*, p. 489.

† ‘Ann. der Phys.,’ IV., 32, p. 1027, 1910.

‡ ‘Versl. Kon. Ak. v. Wetensch.,’ Amsterdam, XII., p. 596, 1910.

§ ‘Versl. Kon. Ak. v. Wetensch.,’ Amsterdam, XIV., p. 637, 1912.

|| ‘Proc. Camb. Phil. Soc.,’ vol. XVII., p. 65.

¶ Sir J. J. THOMSON, ‘Rapports du Congrès Int. Paris,’ 1900, vol. III., p. 148; also SCHRÖDINGER, ‘Sitz. d. k. Akad. Wien,’ CXXI., p. 1305, 1912. [See, however, the article by H. A. LORENTZ, ‘Mathematische Vorlesungen an der Universität Göttingen,’ VI., p. 189, 1914.—*Note added March, 1914.*]

molecule and of such modification of that structure as is produced by the mutual actions between the molecules.

Thus stated, the diamagnetic property of a substance depends upon the molecule as a whole. The above hypothesis postulates no definite structure of the atom or molecule beyond that it is capable of assuming a slightly distorted configuration due to the influence of neighbouring atoms or molecules.

The interpretation of the experimental facts (described in Part I. of this communication) by means of the electron theory of magnetism will be withheld until the results have been discussed from the molecular standpoint.

PART I.

(1) THE MEASUREMENT OF MAGNETIC SUSCEPTIBILITY AT AIR TEMPERATURE.

The magnetic potential energy of a body placed in a magnetic field may be written

$$W = -\frac{1}{2}\chi MH^2$$

where χ is the specific susceptibility of the substance, M the mass of the body, and H the intensity of the magnetic field. For a small displacement of the body,

$$\delta W = -\chi MH \delta H.$$

If the body be so suspended that its displacement causes work to be done on the suspended system against a mechanical force, F , acting through a distance, δr , we have

$$\delta W + F \cdot \delta r = 0.$$

Therefore

$$F = \chi MH \frac{\delta H}{\delta r}. \quad \dots \dots \dots (3)$$

In the present experiments the body, which was spherical, was placed with its centre at P (fig. 1) with regard to the pole pieces A and B . The mechanical forces acting on the body are therefore by (3)

$$X = \chi M \left(H_x \frac{\partial H_x}{\partial x} + H_y \frac{\partial H_y}{\partial x} + H_z \frac{\partial H_z}{\partial x} \right)^* \quad (4)$$

along Ox , with similar expressions for the forces Y and Z , acting along Oy and Oz . If the body were truly spherical, with its centre at P , the forces Y and Z would be zero. In practice this is never the case, and Y and Z , although small, are not zero. Since, however, in these experiments

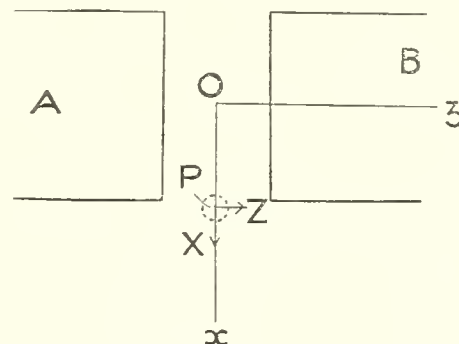
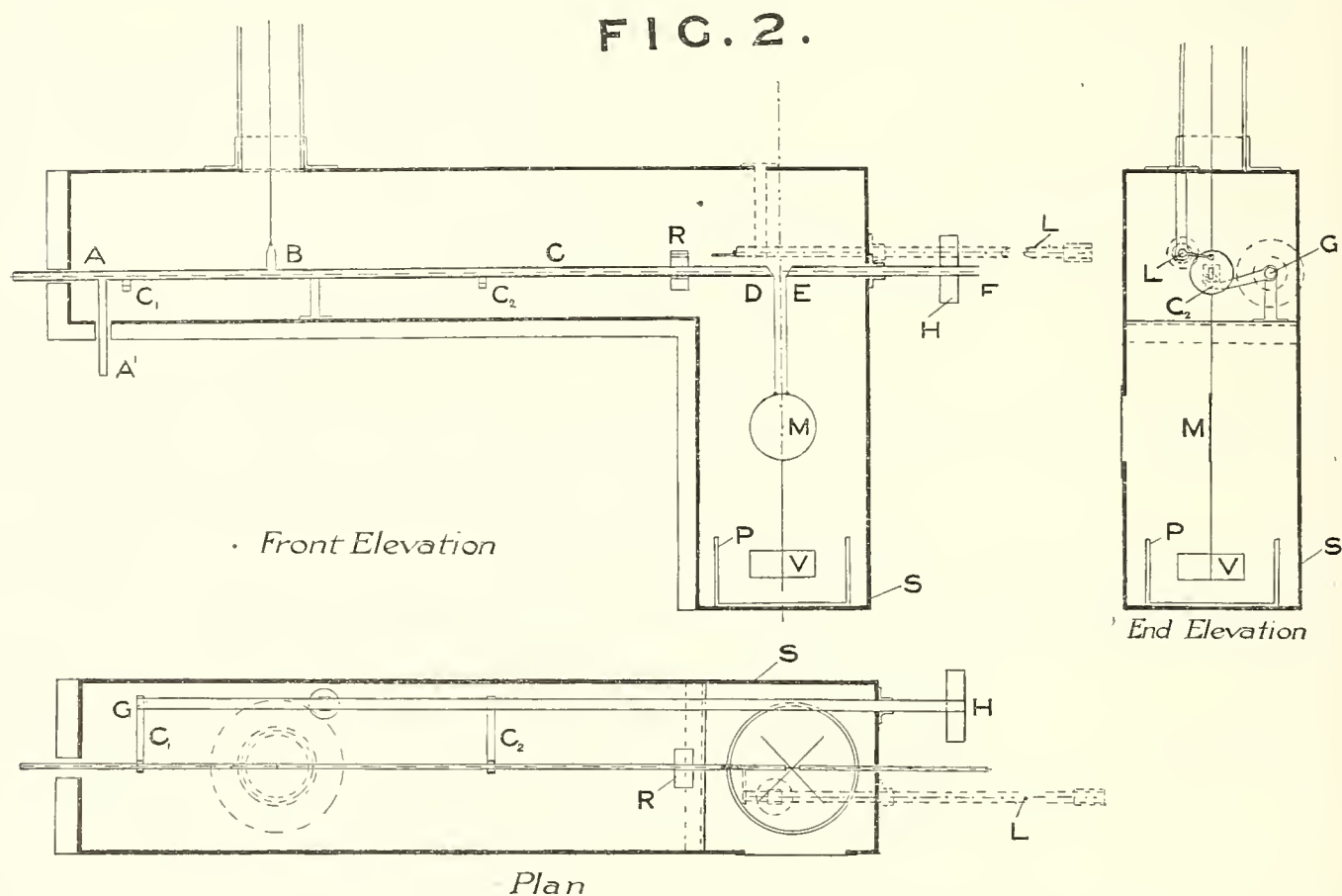


FIG. 1

* It should be noted that the term involving H_x is large in comparison with the terms involving H_y and H_z .

the resistance to motion in the directions Oy and Oz is great, the disturbance of the equilibrium of the suspended system, due to these forces, may be neglected.

The method of measuring the force X , or rather the method of comparing X for different substances, is shown in fig. 2. The torsion arm, $ABCD$, is made of aluminium. The angle through which the torsion arm was twisted when the magnetic field was excited was measured by the double suspension mirror DME . The dash-pot P contained a mixture of paraffin-oil and heavy mineral oil in which



four vanes, V , hung. The arm EF is the extremity of a three-dimensional adjustment stage. The brass case S , which surrounded the torsion arm and suspension mirror, was rigidly attached to a brass carrier which was provided with a screw motion and slid upon rails. These rails were screwed to a heavy slate bed which was supported by two large blocks of stone. To prevent the transmission of vibrations to the suspended system from outside, two layers of indiarubber were inserted between the stone blocks and the slate bed. The rod GH provided with the cranks, C_1 and C_2 , was used to rest the torsion arm while the substance under investigation was being attached to the end A (see plan and end elevation). To preserve balance of the torsion arm when different masses of substance were used, a rider R was provided which could be moved by the rod L into different positions along the torsion arm. A very fine phosphor-bronze wire, attached to the projection B , suspended the movable parts of the system.

The displacement of the substance, due to the action of the force X , was usually about $\frac{1}{200}$ mm. This will not appreciably alter the sensitiveness of the bifilar suspension, and, therefore, the use of torsion to bring the body back to its initial position is not necessary.

The substance was placed in a small glass phial (fig. 3). Form (a) was used for liquids, form (b) for both solids and liquids; (a) was provided with inlet and outlet tubes of fine bore, which admitted only a minimum contact of the substance under investigation with the air, and thereby reduced as much as possible any effect due to evaporation. The stopper consisted of a short piece of indiarubber tubing closed at one end by a glass plug. By this device the bulb could be thoroughly cleaned and easily filled. To fill the phial, the stopper was removed and the liquid drawn into the bulb. By tilting the bulb slightly, the liquid ran out of the shorter limb and the stopper was placed in position. The liquid in the longer limb was removed by means of a fine capillary tube. In this way it was possible to obtain a sphere of the liquid under investigation. This type of phial has been used with some substances which are crystalline at ordinary temperatures by filling the phial in a high temperature enclosure. The type (b) was devised primarily for use with crystals which could not be introduced into (a). It has been used for liquids as a check against any effect which might arise from the motion of the liquid in the lower limb of type (a).

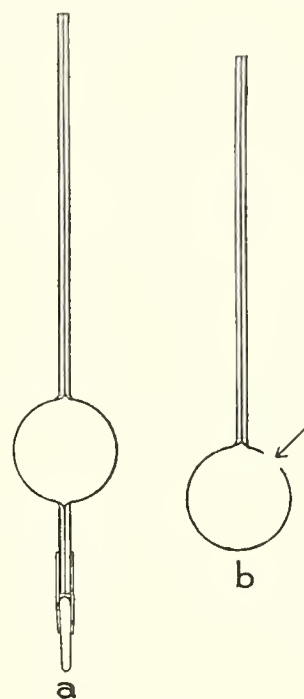


FIG. 3
Crystals introduced at arrow

Let d_1 be the deflection due to phial and contained air.

d_2	„	„	„	„	water.
d	„	„	„	„	substance.
χ_a	„	specific susceptibility of air.			
χ_w	„	„	„	„	water.
χ	„	„	„	„	substance.

Now $X = pD$, where D is the deflection on the scale, and p is some constant, provided the deflection of the torsion arm is small. Hence from (4)

$$p(d_2 - d_1) = (\chi_w m_w - \chi_a m_a) \left(H_x \frac{\partial H_x}{\partial x} + H_y \frac{\partial H_y}{\partial x} + H_z \frac{\partial H_z}{\partial x} \right), \quad . \quad . \quad . \quad (\alpha)$$

$$p(d - d_1) = (\chi M - \chi_a M_a) \left(H_x \frac{\partial H_x}{\partial x} + H_y \frac{\partial H_y}{\partial x} + H_z \frac{\partial H_z}{\partial x} \right) \quad . \quad . \quad . \quad (\beta)$$

if the phial be placed in the same position relative to the pole pieces. Here M is the mass of the substance taken, M_a that of the air which fills the same volume, m_a is the

mass of air which fills the volume m_w occupied by the water. In all the experiments described below the current was kept constant at 8.6* amperes, to within 1 part in 500. Hence the factor involving the magnetic field on the R.H.S. of (α) and (β) is constant, and we get by division

$$\chi M = \chi_a M_a + (\chi_w m_w - \chi_a m_a) \frac{d - d_1}{d_2 - d_1} \quad \dots \quad (\gamma)$$

The specific susceptibilities of water χ_w and air χ_a were taken as 7.25×10^{-7} and 210×10^{-7} (at 20° C.) respectively.

(2) EXPERIMENTS DOWN TO THE TEMPERATURE OF LIQUID AIR.

The apparatus designed for the measurement of susceptibility at low temperatures is shown in fig. 4.

The phial P, supported from the torsion arm at A', was surrounded by a thin-walled glass tube G, the latter being sealed at the lower end and supported by the

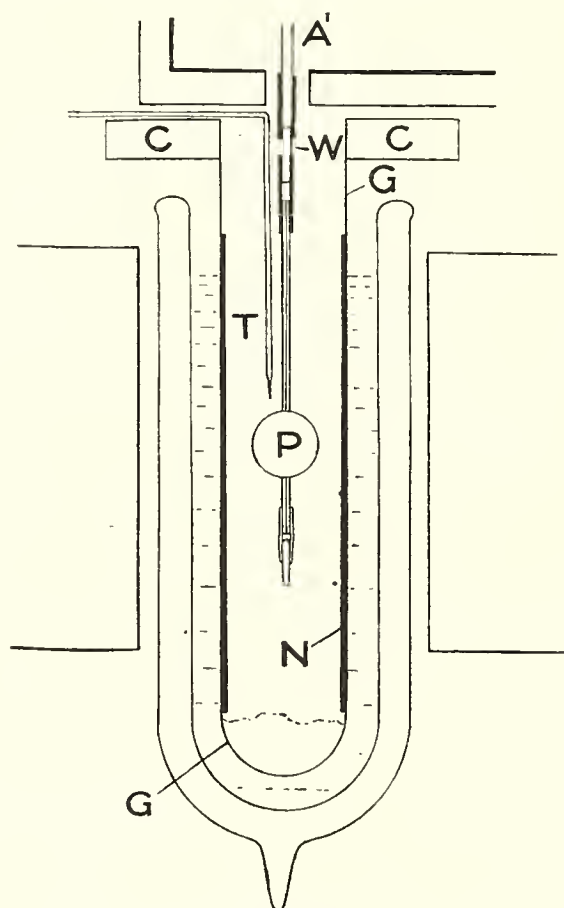


FIG. 4

cork flange CC. Conduction of heat from or to the torsion arm was prevented by inserting a strip of wood W between the phial and the projection of the torsion arm. A copper cylinder N, which just fitted into the glass tube, served to keep uniformity

* In the last two experiments the current was increased to 9.0 amperes.

of temperature throughout the volume occupied by the phial. A layer of phosphorus pentoxide was placed at the bottom of the glass tube to absorb any moisture and thereby prevent the deposition of water on the phial during cooling. The liquid air tube was brought up from below into the position shown. The flange of cork CC was very broad and prevented any convection currents from disturbing the equilibrium of the suspended system. A copper German-silver thermocouple T was used to measure the temperature, one junction being situated just above the phial. The thermocouple circuit included a delicate D'Arsonval galvanometer. A slight kick, due to induction effects, was observed when the magnetic field was excited. This quickly died away and readings of the deflection of the phial and temperature were taken simultaneously.

(3) EXPERIMENTS UP TO 250° C.

The method of suspension used is essentially that devised by CURIE,* and fig. 5 is a diagram of the apparatus, looking along the direction of the axis of the electro-

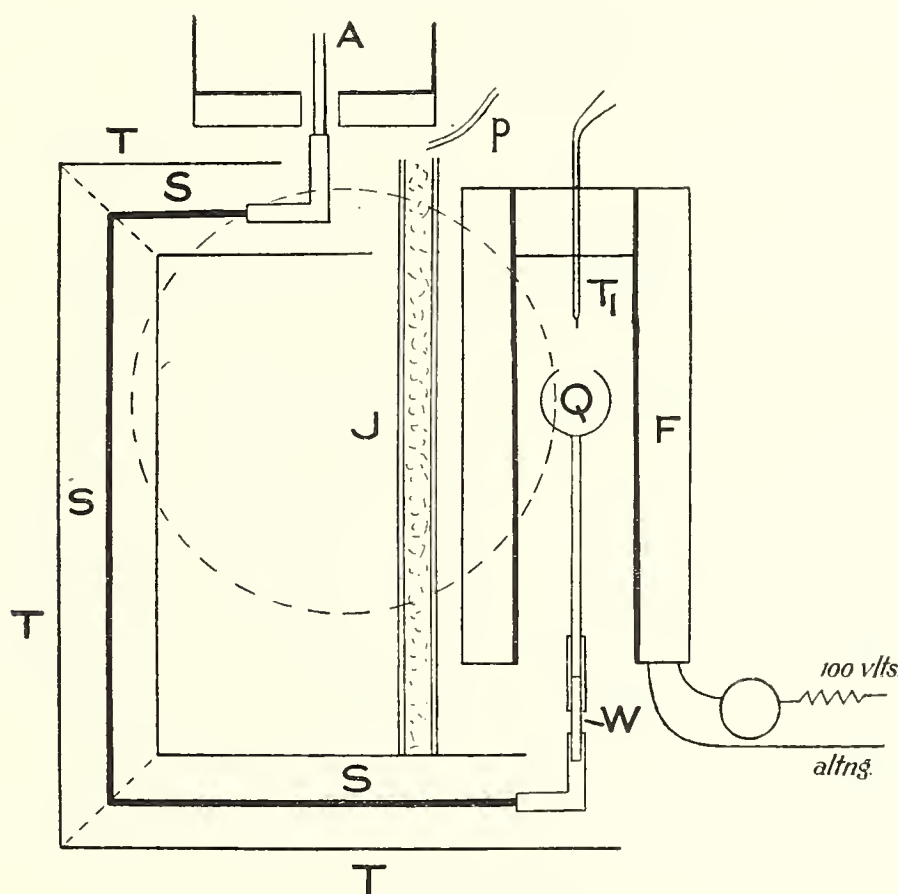


FIG. 5

magnet. The phial which was used in these experiments was open above and closed below, as shown at Q. The suspension rod, ASSS, was made of brass and was

* 'Ann. de Chim. et de Phys.,' VII., vol. 5, p. 298, 1895.

surrounded by glass tubes, TTT, to prevent disturbances due to air currents. A screen J, formed of two pads of asbestos board, with an intervening layer of cotton-wool kept wet by water from the pipe p , served to prevent disturbances due to air currents from the walls of the electric furnace F. The temperature attained was measured by the copper German-silver couple, T_1 fitted into the top of the furnace, as shown. A support of wood W was inserted between the phial and the main brass suspension to prevent conduction of heat and the accompanying thermal disturbances. In both series of experiments the whole apparatus had to be surrounded with pads of cotton-wool to keep a steady zero.

(4) DISCUSSION OF POSSIBLE ERRORS.

In the measurement of magnetic properties over wide intervals of temperature it is necessary to consider the effect of a number of possible sources of error and these will now be considered.

(a) *The Effect of Alteration of Volume during Fusion or Crystallization on the Volume of Air Displaced by the Substance.*

We shall denote the specific susceptibility of the liquid by χ_l , that of the crystals by χ_c . From equation (γ), by division,

$$\begin{aligned} \frac{d_c - d_l}{d_l - d_1} &= \frac{\chi_c M - \chi_a (M_a + \partial M_a)}{\chi_l M - \chi_a M_a} = 1 + \frac{M \partial \chi - \chi_a \cdot \partial M_a}{\chi_l M - \chi_a M_a} \\ &= 1 + \frac{M \cdot \partial \chi}{\chi_l M - \chi_a M_a} + \frac{\chi'_a \cdot \partial V_a}{7.0 \times 10^{-7} *} \text{ where } \partial \chi = \chi_c - \chi_l. \end{aligned}$$

∂M_a is the additional mass of air displaced owing to the change of volume of the substance in passing from one state to another, ∂V_a the corresponding change of volume, and χ'_a is the susceptibility of air per unit volume. (0.25×10^{-7}).

For most substances, $\partial V_a \doteq 0.1$,† if $V_a = 1$, i.e., for the volume of substance experimented on in this research. Hence the variation of deflection due to this cause is $0.25 \times 10^{-7} \times 0.1 / 7.0 \times 10^{-7}$, or 4 parts in 1000, and is beyond the limit of experimental accuracy. In the above it is assumed that there is no appreciable change of temperature of the system while fusion takes place, so that no effect due to variation of χ_a with temperature enters.

* -7.0×10^{-7} is taken as a mean value of $\chi_l \cdot M - \chi_a M_a$.

† *Vide* G. TAMMANN, 'Kristallisieren u. Schmelzen,' Leipzig, 1903, pp. 204 *et seq.* The values of ∂V in cubic centimetres per cubic centimetre for some of the substances investigated in this research are:—

Benzene, 0.10;	Naphthalene, 0.14;
Benzophenone, 0.09;	Mercury, 0.036.
Nitrobenzene, 0.08.	

(b) *The Effect of Variation of Temperature on χ_a and the Influence of Thermal Expansion over the Wide Range of Temperature used in these Experiments.*

Since

$$\frac{\partial}{\partial \mathfrak{S}} (\chi_a M_a) = \frac{\partial}{\partial \mathfrak{S}} (\chi'_a V_a),$$

and

$$\frac{\partial}{\partial \mathfrak{S}} (\chi_{c,l} \cdot M) = \frac{\partial}{\partial \mathfrak{S}} (\chi'_{c,l} \cdot V_{c,l}),$$

where the subscript c, l refers to the crystalline or liquid state, we have, dashing the deflections $d_{c,l}$ and d_1 for the higher or lower temperature,

$$\frac{d'_{c,l} - d'_1}{d_{c,l} - d_1} = \frac{\chi_{c,l} \cdot M - \chi_a M_a + (\mathfrak{S} - \mathfrak{S}_0) \left[\frac{\partial}{\partial \mathfrak{S}} (\chi'_{c,l} \cdot V_{c,l}) - \chi'_a \frac{\partial V_a}{\partial \mathfrak{S}} - V_a \frac{\partial \chi'_a}{\partial \mathfrak{S}} \right]}{\chi_{c,l} \cdot M - \chi_a M_a},$$

which is the ratio of the deflections due to thermal expansions of the substance and air, and the variation of the susceptibility of the substance and air with temperature over an interval $\mathfrak{S} - \mathfrak{S}_0$. The necessary data for calculating the magnitude of this correction are not known for most of the substances which have been used in these experiments, but the largest factor involved is that due to the variation of density and susceptibility of the contained air. (The variation of the susceptibility of the air external to the phial is eliminated since the difference between $d'_{c,l}$ and d'_1 is taken.) For a temperature interval of 150° C. , this factor amounts to 2 per cent.* (approximately) of the deflection, $d_{c,l} - d_1$, due to the substance. This error is, however, distributed over a range of 150° C. When the data of thermal expansion of the liquid and crystalline states are known, there will be two corrections, one for each state, which will be distributed throughout the range of temperature for which that state exists. It is to be observed that in this and the foregoing correction the thermal expansion of the glass and the variation of its susceptibility with temperature disappear, since they are equally involved in $d'_{c,l}$ and d'_1 , the difference of which is taken.

(c) *The effect of displacement of the substance (liquid), as a whole, during cooling was tested in two ways:—*

- (i) By actually displacing the liquid and repeating the observations;
- (ii) By using, in some of the later experiments, a phial in which the displacement was prevented.

* This factor is $\frac{-0.25 \times 10^{-7} \times 0.00365 \times 150 + 1.0 \times 0.25 \times 10^{-7}/150}{7.25 \times 10^{-7}}$.

(d) *Correction due to Dissymmetry of the Phial.*

The phials were nearly spherical and were placed with their centres as nearly as possible at the point P (fig. 1) where the force on them is a maximum and error due to displacement a minimum. The present measurements are differential ones, and provided the phial is replaced in the same position, no appreciable error can creep in from this source.

(e) *Latent Heat and Thermal Conductivity.*

The possible errors due to latent heat of change of state and thermal conductivity will be considered in more detail when the preliminary experimental curves are discussed.

(f) *Effect of Air Dissolved in the Substance.*

The error due to the presence of dissolved air in the substance is reduced to a minimum by heating the substance strongly. In those experiments performed at air temperature, an inappreciable amount of air was evolved on fusion. The error arising from this cause is negligibly small; for if a quantity of air equal in volume to that of the phial were expelled on fusion, it would account for only one-third of the actual effect found. But a small trace of air dissolved in the substance will, in all probability, remain in the substance if a considerably higher temperature is not sufficient to remove it; and although such a trace of air would have a small influence on an absolute measurement of the susceptibility, it will not appreciably affect the difference of susceptibility between the liquid and crystalline states.

(5) METHOD OF EXPERIMENT.

Fig. 6 shows how the value of the force acting on the suspended system varies with x , (a) when the phial is filled with air, (b) when it is filled with water. In each figure the dotted circle shows the size of the phial, and OM is a distance equal to the radius of the pole piece on the same scale. It will be seen that if the phial moves a small fraction of 1 mm. either towards or away from O, the resultant deflecting force remains very nearly constant, because one-half of the substance and phial has a slightly larger force acting on it, while the other half has a slightly smaller force.

The pole pieces were cylindrical, 8 cm. diameter and 4.65 cm. apart, so that the liquid air tube could be inserted for low temperature work and a small electric furnace for high temperature work. The advantage of large pole pieces is that the size of the phial is of less consequence. The hysteresis effect was not appreciable and the deflections were very consistent; a small increase of the current was readily followed by a small increase of the deflection. A Grassot fluxmeter showed that the field intensity was 3000 gauss for a current of 8.6 amperes.

Measurements were first made with the phial filled with air, water, and the

substance, at room temperature. If the substance were a liquid it was cooled by liquid air, and after about 1 hour observations were begun and taken at a large number of temperatures as the substance gradually warmed to room temperature. A series of such observations occupied a period of 10 hours. Frequently the heating was too slow to allow the complete interval to be bridged during the day, and in such cases the curve was completed the following day. It will be observed that the lag in temperature of the substance behind the thermocouple, with such a small rate of cooling, will be small, and this lag will be uniform so long as there is no sudden change of the state of the substance. At temperatures considerably higher or lower

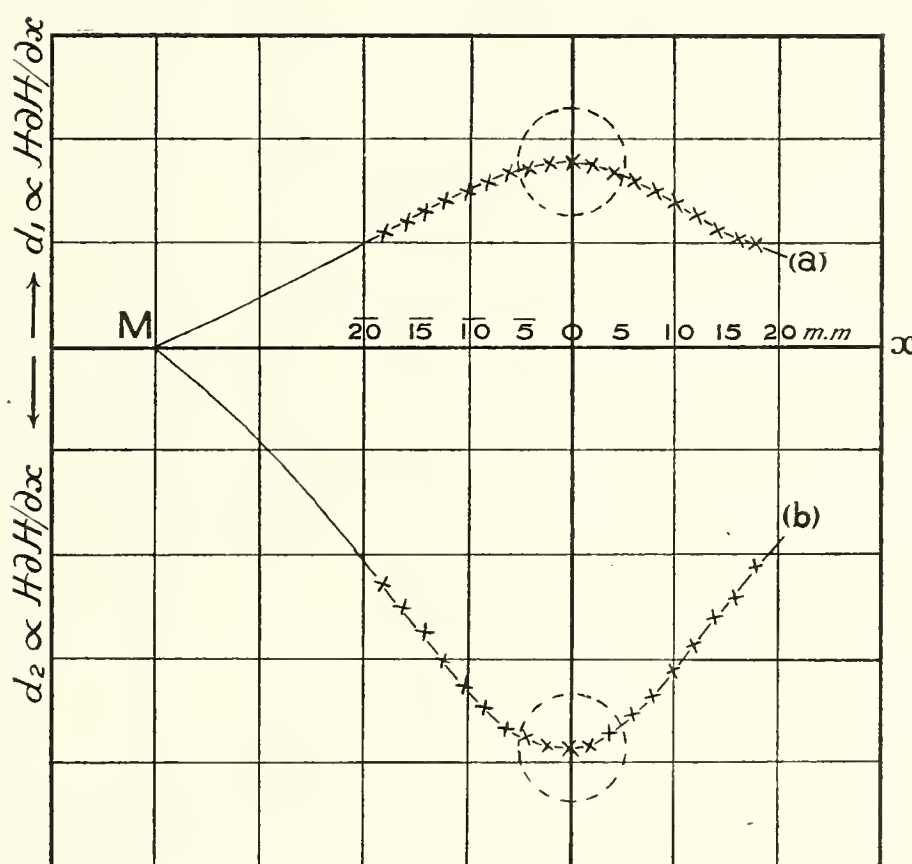


FIG. 6.

than the temperature of fusion, the lag will generally be greater than it is at the fusion-point itself. It can easily be seen from the curves given below that the effect of this will be to lessen the difference of susceptibility between the liquid and the crystalline states. The actual difference of susceptibility cannot, therefore, be attributed to this source even if the lag were considerable. A similar series of readings was taken for the phial alone, which, corrected for the variation of the susceptibility of the contained air with change of temperature and subtracted from the readings obtained above, will give the variation of the specific susceptibility of the substance with temperature.

Similar experiments have been made, using hot water and other liquids in the Dewar tube, but it was difficult to take readings at temperatures higher than 100°C .

without the aid of the electric furnace and the inverted system of suspension, due to CURIE, described above. With the latter, experiments have been made up to $250^{\circ}\text{C}.$, and observations could be taken both with rising and falling temperatures.

(6) THE EXPERIMENTS.

A sufficiently large number of compounds has been investigated to establish a general theory of the dependence of magnetic property on the transition from the

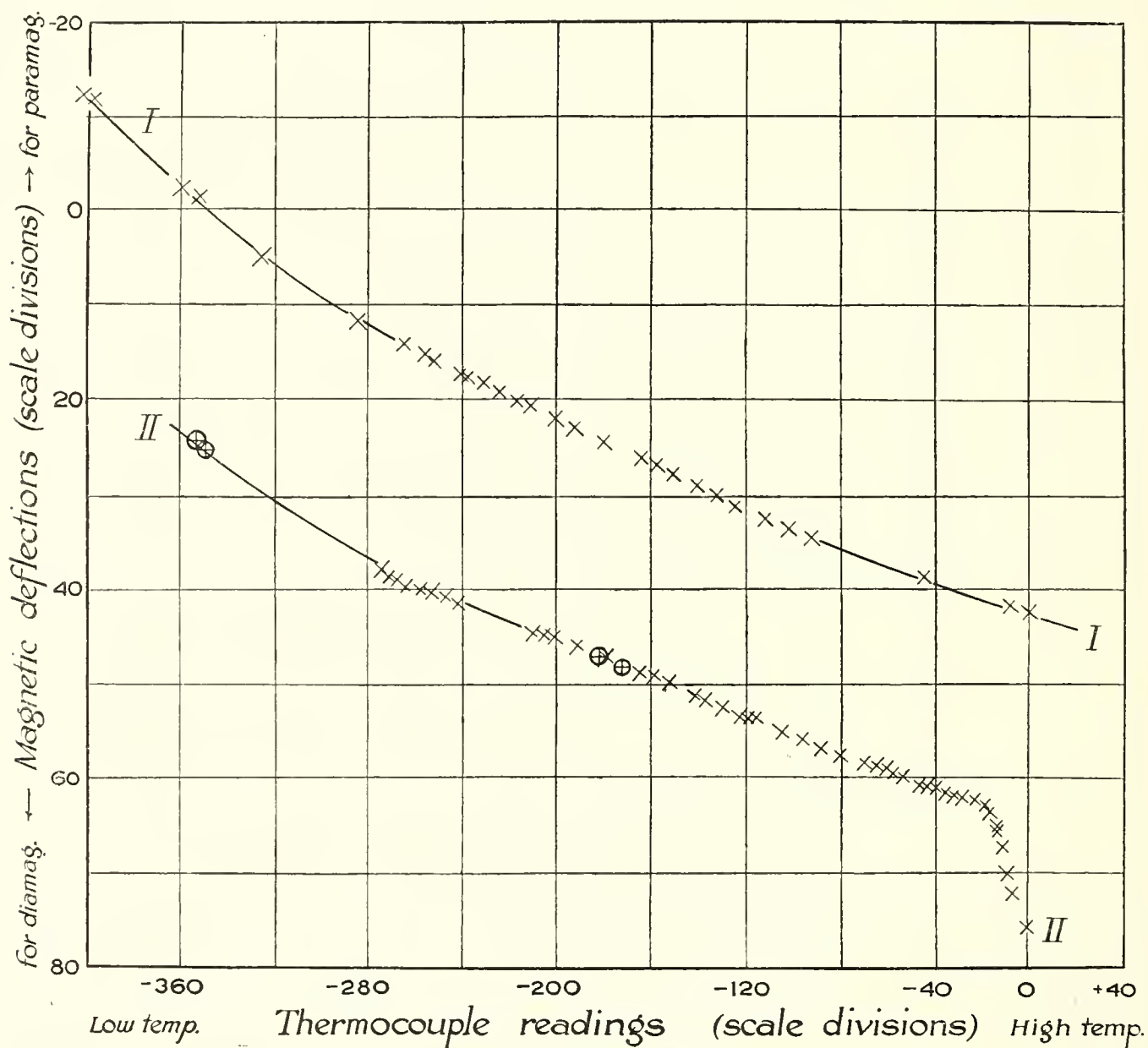


FIG 7

The ordinates of curve I have been reduced 70 scale divisions, so that the two curves may be more readily compared.

The four points marked $\oplus$ on curve II were obtained by recooling after the crystals had been melted.

liquid to the crystalline state. Preliminary experiments were made on the phial filled (1) with air, (2) with nitrobenzene. The glass from which the phials were

constructed was slightly paramagnetic, and the variation of its susceptibility with temperature is represented in fig. 7, Curve I. In this figure the ordinates are deflections in millimetres, and are proportional to the forces acting on the phial at any temperature. The abscissæ are temperatures in divisions of the thermocouple scale (T). Some of the observations are given in Table I.

TABLE I.

- T.	Deflection (d_1).	- T.	Deflection (d_1).	- T.	Deflection (d_1).
483	- 103·0	246	- 53·5	159	- 43·0
469	- 102·5	239	- 52·5	152	- 42·5
447	- 95·0	231	- 52·0	142	- 40·9
404	- 82·0	225	- 51·0	133	- 39·7
355	- 71·5	212	- 48·8	125	- 38·5
322	- 65·5	201	- 48·0	113	- 36·2
288	- 58·5	193	- 47·0	93	- 35·5
264	- 56·0	180	- 45·5	8	- 28·0
254	- 54·3	165	- 43·6	0	- 27·5

The current was adjusted to 8·6 amperes for each observation. Weight of phial = 1·223 gr.

The corresponding observations when the phial was filled with nitrobenzene are plotted in fig. 7, Curve II. Some of the observations are given in Table II.

TABLE II.

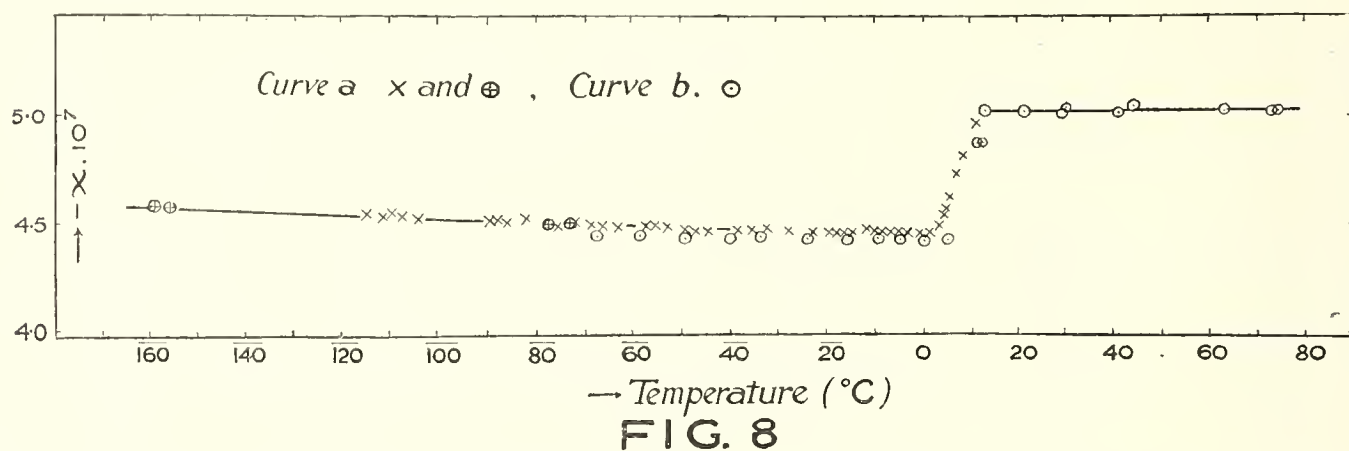
- T.	Deflection (d).	- T.	Deflection (d).	- T.	Deflection (d).
275	38·0	115	54·0	19	63·0
257	39·7	97	56·0	17	64·0
248	40·7	89	57·0	13	65·5
205	44·7	80	57·8	11	67·5
192	46·0	70	58·7	9	70·5
180	46·7	65	59·0	7·5	72·5
171	48·0	59	59·7	1	76·0
160	49·0	51	60·0	- 2	76·0
153	49·8	40	60·9	182	47·0
142	51·5	35	61·8	173	48·0
130	52·7	30	62·0	353	25·2
122	53·4	23·5	62·5	349	24·5

The current was adjusted to 8·6 amperes for each observation. Weight of phial and nitrobenzene = 2·273 gr.

T = 0 corresponded to atmospheric temperature, 16° C.

It will be seen that the curves are approximately parallel until a point corresponding to the temperature 5° C. is reached, which is the melting-point of the crystals. It appears, therefore, that the liquid is more diamagnetic than the crystals. By

subtracting the ordinates, we get the values of $d-d_1$ at different temperatures. These, in conjunction with the value of d_2-d_1 for water at air temperature, give the specific susceptibility* of nitrobenzene at different temperatures. The two curves in fig. 8 show how the specific susceptibility of nitrobenzene varies with temperature; curve (a) for phial of type (a), curve (b) for phial of type (b).



As the crystals were very gradually melted by heat conducted through the walls of the phial, no effect due to latent heat would be expected to enter. Moreover, if the crystals were taking heat from the phial faster than it was supplied by the surrounding enclosure, the temperature of the phial would be colder than that indicated by the thermometer. The phial could not get hotter than the enclosure unless the latent heat were negative. The only way in which latent heat might affect the observations, therefore, is by cooling the phial below the temperature indicated by the thermometer. But this effect would increase the deflection due to the glass and accordingly reduce that due to the diamagnetic substance contained in it. This result would make any correction due to latent heat of opposite sign to the difference actually found. A good test of the absence of any error due to this cause is obtained by showing that the change of susceptibility is permanent. If the effect were due to absorption or emission

* The calculation is as follows:—

$$d_2 \text{ for } 0.75 \text{ gr. water} = 80 \text{ scale divisions.}$$

$$d_2 - d_1 = 80 + 28 = 108 \text{ scale divisions; } d - d_1 = 76 + 28 = 104 \text{ scale divisions.}$$

$$M = 2.273 - 1.223 = 1.050 \text{ gr.; } m_w = 0.75 \text{ gr.; density of nitrobenzene} = 1.22.$$

Using equation (γ), we find

$$\chi = \frac{1}{1.050} \left[\frac{0.25 \times 1.050}{1.22} - \left\{ 7.25 \times 0.75 + 0.25 \times 0.75 \right\} \frac{104}{108} \right] 10^{-7} = -4.98 \times 10^{-7}.$$

Also

$$\partial \chi = \frac{1}{1.050} \left\{ 0.25 \partial V_a - \frac{7.5 \times 0.75}{108} \cdot \delta(d - d_1) \right\} \cdot 10^{-7}.$$

At the fusion-point

$$\partial V_a \doteq 0.08; \delta(d - d_1) = 13 \text{ scale divisions.}$$

Hence

$$\partial \chi / \chi \doteq 12 \text{ per cent.}$$

of heat on change of state, this could not be the case. The change of susceptibility cannot be ascribed to an alteration of thermal conductivity for, as will be seen later, the same difference has been obtained at air temperature between crystals and super-cooled liquid, each having stood long enough to take up the temperature of the room.

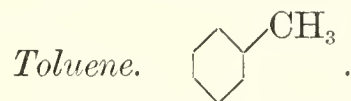
The experiments on about twenty substances* will now be briefly summarised, for the most part in the order in which they have been investigated. In one or two cases, substances, which are chemically closely related, have been classed together.

The results are shown graphically in figs. 8–14. A short account of any peculiarity will be given and the curves will be more fully discussed later. For brevity, in what follows, the specific diamagnetic susceptibility will be referred to generally by χ , the absolute temperature by ϑ , and a curve showing the relation between χ and ϑ will be called a χ – ϑ curve.

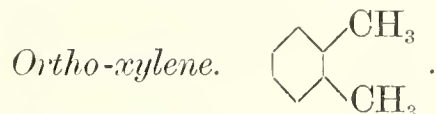
About seventy experiments have been made and some of the substances have been investigated several times, using both types of phial described above (fig. 3).

(a) *A Study of Benzene Derivatives.*

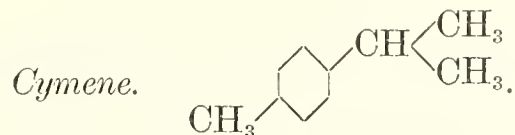
For the most part, the simpler benzene derivatives have been selected, because in many cases their crystalline forms have been investigated and it was hoped that an interpretation of the results would thereby be rendered more feasible.



Liquid toluene is more diamagnetic than crystalline toluene, $\partial\chi/\chi$ amounting to 5 per cent. Two curves for this substance have been obtained and the break in the curve occurred in each case at -96°C . The melting point is at -92°C . (fig. 9a).

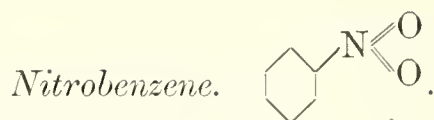


The liquid is more diamagnetic than the crystals, the change of χ taking place at -26°C . and $\partial\chi/\chi = 4.8$ per cent. The melting point of the crystals is at -28°C . (fig. 9b).



The curve for the phial filled with cymene was parallel to that for the empty phial over the interval 20°C . to -150°C . (see note under *pyridine* below).

* Over thirty substances have been investigated. An account of these additional experiments will be given in a future paper.



Two additional experiments have been made on this substance (1) using phial of type (a); (2) using phial of type (b). In each case the liquid was found to be more diamagnetic than the crystals, the value of $\partial\chi/\chi$ being 13 per cent. The results obtained in the preliminary experiments with this substance are therefore confirmed. The melting point deduced from the magnetic property is 4°C . (fig. 9c).

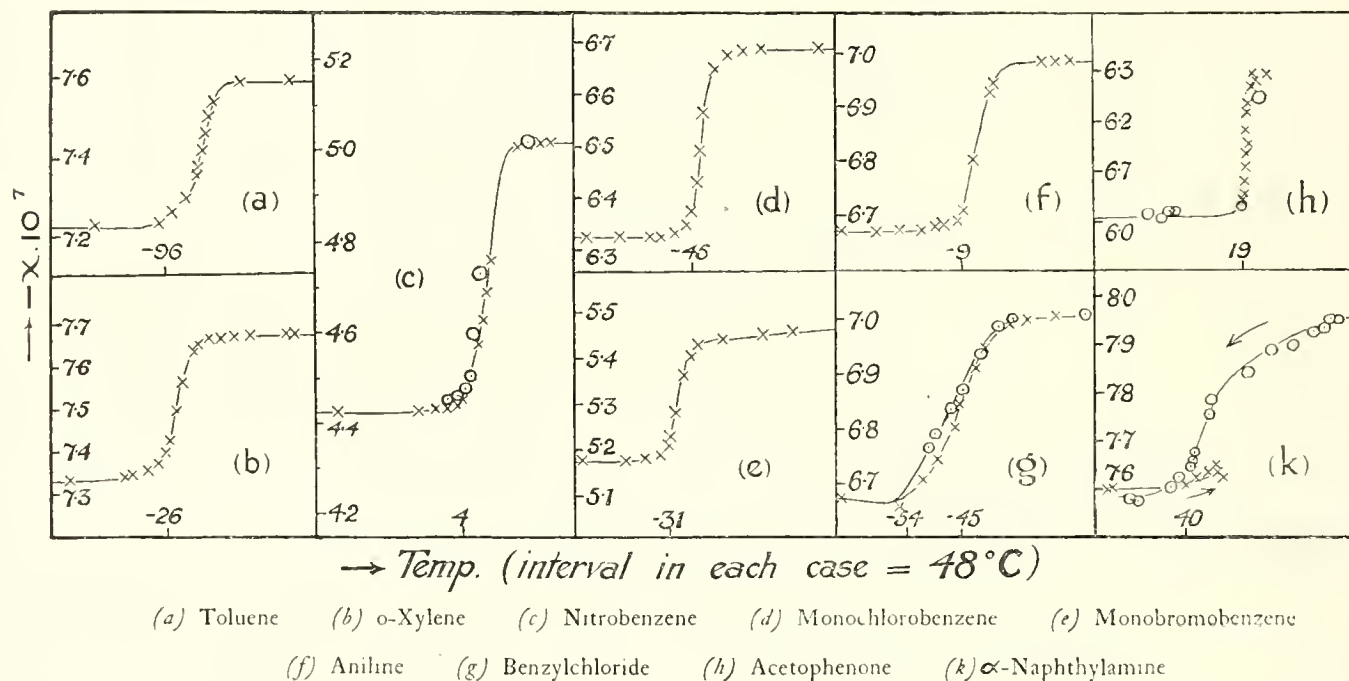
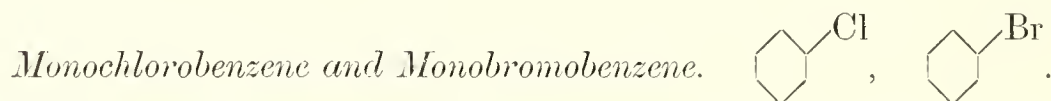
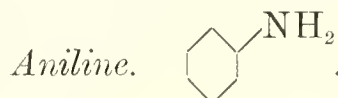


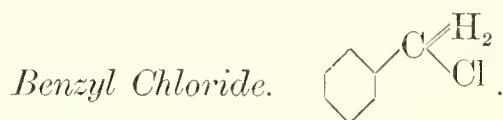
FIG 9



Each liquid is more diamagnetic than the corresponding crystals. $\partial\chi/\chi$ in each case is 5 per cent. Type (a) of phial was used with the chlorine derivative, type (b) with the bromine derivative. The melting point of the chlorine compound, as deduced from the dip of the χ - ϑ curve, is -45°C .; that of the bromine compound is -31°C . (figs. 9d and 9e).

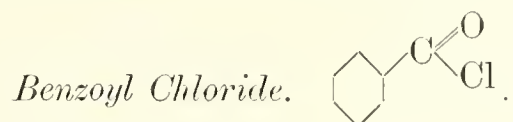


Liquid aniline is 5 per cent. more diamagnetic than crystalline aniline; the melting point derived from the χ - ϑ curve is at -9°C . (fig. 9f).



Liquid benzyl chloride is more diamagnetic than the crystalline form. The transition,

as fig. 9*g* shows, appeared to be less abrupt than usual.* The melting point, if it corresponds to the beginning of the dip, is at -54°C . The middle of the dip is at a temperature -45°C ., which is very nearly equal to the value of the melting point (-43°C .) obtained by other methods. The value of $\partial\chi/\chi$ is 5 per cent.



This substance showed a departure from the general rule that for benzene compounds the liquid state is more diamagnetic than the crystalline state. Five experiments have been made with this substance and two of the curves are shown in fig. 10, Curve I. The liquid, which is oily, is seen to be less diamagnetic than the crystals

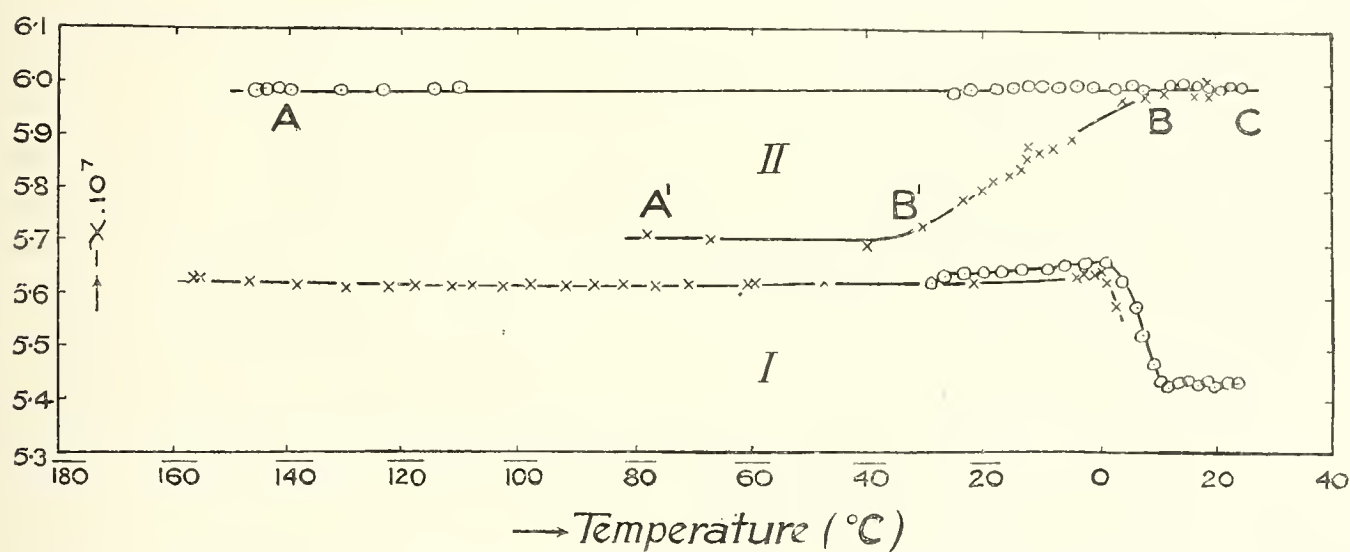
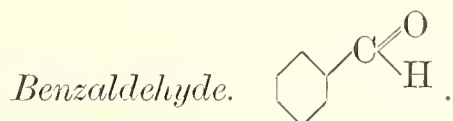


FIG. 10

by an amount 5 per cent. of the value of χ . Taking the point where the decrease of χ begins, as the temperature of fusion, we deduce -3°C . for the melting point. Other methods give -1°C .

This substance will be considered later in conjunction with several other compounds showing similar properties.

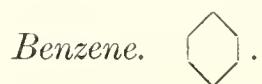


The first experiment made on this substance showed that it possessed the hitherto unique property of constancy of the magnetic susceptibility through the normal fusion point down to the temperature -150°C ., as shown by the curve ABC (fig. 10, Curve II). But on taking away the metal cylinder (used to preserve a uniform temperature over

* This may be an effect of too rapid heating of the enclosure combined with low conductivity of the substance.

the space occupied by the substance) during a repetition of this experiment, it was discovered that the benzaldehyde had not crystallized but had passed into a transparent jelly-like mass. By plunging the phial containing the benzaldehyde into liquid air, the benzaldehyde again assumed this jelly form, but on warming, followed by a further slight cooling, a white crystalline mass was obtained. This was now cooled to the temperature of liquid air, withdrawn, suspended from the torsion balance and quickly surrounded by liquid air. An experiment carried out on the benzaldehyde after this treatment is shown in fig. 10, Curve A'B'BC. It will be seen that approximately the normal dip, 5 per cent., was obtained, the liquid being more diamagnetic than the crystals, but the transition stage was considerably drawn out and appeared to have an intermediate step or stable state.

Shortly after this difference in magnetic property between the gelatinous and the crystalline states of benzaldehyde had been established, a research appeared by M. CHAUDIER,* who had discovered that the specific magnetic rotation of organic substances behaves in a similar way. He found that if the substance passed into a gelatinous state, on being cooled below the normal fusion point, there was no discontinuity of the magnetic rotation in passing from the liquid to the gelatinous state. The rotation, on the other hand, disappeared totally if the substance passed into a crystalline form on being cooled. The two phenomena—magnetic rotation and magnetic susceptibility—are doubtless closely allied and each is affected by a transition from the liquid to the crystalline state. An attempt is made in Part II. to connect theoretically these effects. It does not appear that CHAUDIER obtained the same substance in both gelatinous and crystalline forms, and it would be interesting to carry out an experiment on the magnetic rotation of a substance capable of existing in either form at a given temperature. CHAUDIER shows that the discontinuity exists for benzene, aniline, and nitrobenzene on crystallization, and that the rotation remains constant in the cases of amyl-alcohol, glycerine, and essence of terebene, which pass into the gelatinous form on cooling below the normal freezing point. The phenomenon with the last substance is made complex on account of optical activity.



This substance shows an anomaly different from that of any other benzene compound investigated. Five experiments have been made and one of the curves is drawn in fig. 11. The pit in the χ - ϑ curve occurs in every case at the normal fusion point, 5° C. On either side of this point, the curve rises, the distant parts of the curve corresponding to the liquid and crystalline states being continuous, as shown by the dotted line. This dotted line appears to correspond to no real physical state as it was found impossible to make the substance pass along this path. The depth of the pit

* 'Comptes Rendus,' vol. 156, p. 1529, May, 1913.

was not always the same—this may be explained by the slightly different rates of cooling in the different experiments—for the outer layers of the sphere of benzene crystals will melt first, and these may have reached the stage where an increase of temperature causes a rise of χ , while the inner layers of the sphere are still falling to pass through the pit.

This fall and rise of χ may be connected with the perfect symmetry of the benzene molecule.

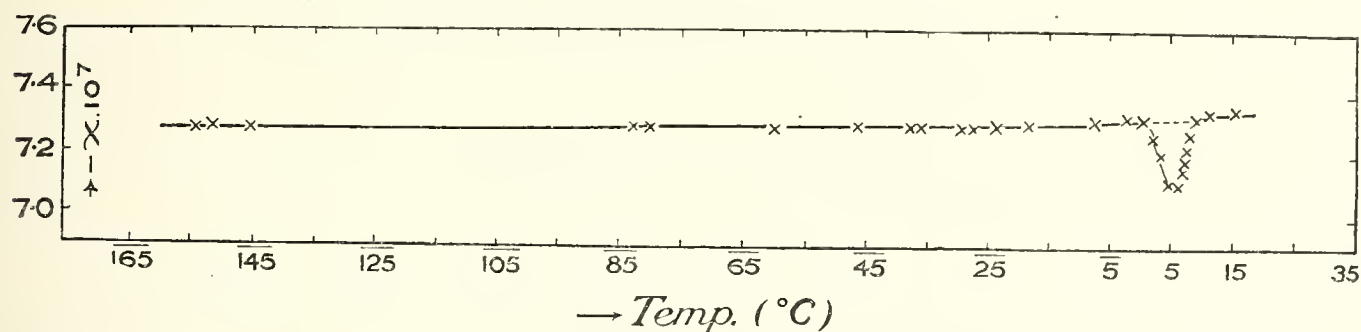
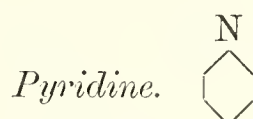


FIG. 11



The curve for the phial filled with pyridine was parallel to the curve for the empty phial throughout a range of temperature, 20° C. to -150° C. In liquid air, the pyridine passed into a stiff jelly state, as in the case of benzaldehyde. On slightly warming and again cooling, a white crystalline mass appeared.

Similar freezing experiments were made on cymene. The gelatinous state appeared turbid and on recooling, crystallization took place; the substance being transformed into a white mass which quickly melted on withdrawal from the liquid air.

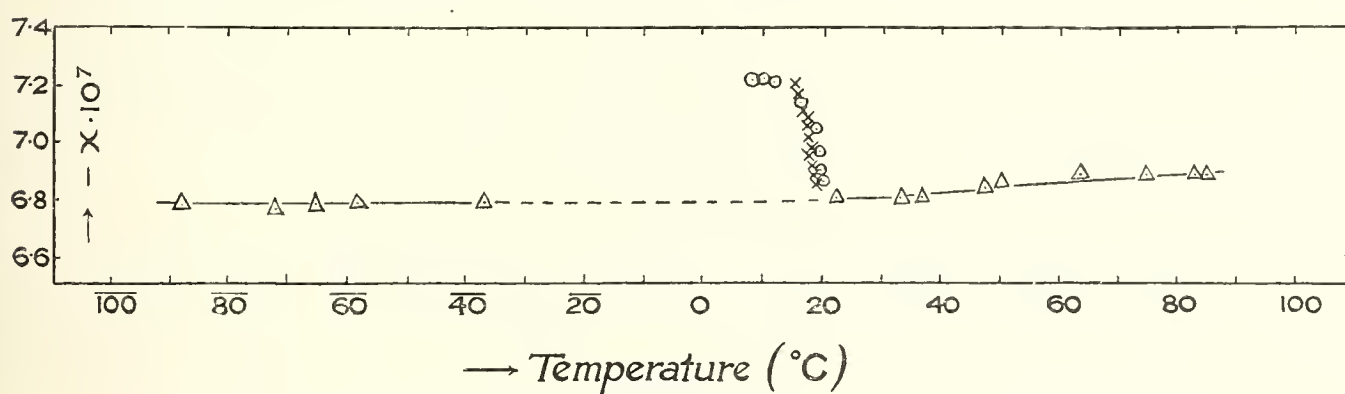
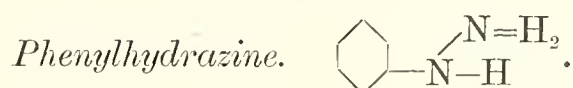
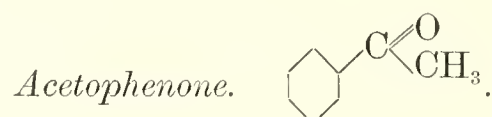


FIG. 12

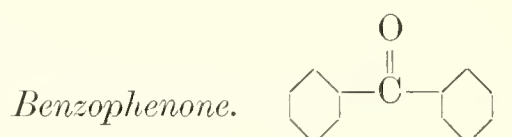


Two curves for the transition from the crystalline to the liquid state and one curve for the gelatinous state down to the temperature -90° C., are shown in fig. 12. As

phenylhydrazine melts at 20° C., there was no need to use the copper cylinder, which ordinarily surrounded the phial, and so it was possible to watch the substance throughout the transition. The crystals were found to be more diamagnetic than the liquid and the mean value of $\partial\chi/\chi$ was 4.7 per cent. The gelatinous form was stable at low temperatures and had a susceptibility equal to that of the liquid above the normal fusion point. The crystals obtained at low temperatures gave rise, on warming, to a large number of small bubbles, which prevented observations being made on the crystals at temperatures much below the freezing point.



The liquid was frozen to a white crystalline mass which melted at 19° C., accompanied by an increase of diamagnetism amounting to 4.5 per cent. On allowing the liquid to stand for two days crystals again appeared, but these were transparent and very much larger. The susceptibility of these crystals was smaller than that of the liquid by 4 per cent. The crystals were again tested after three days but no further decrease of diamagnetism was observed (fig. 9h).



Two curves for this substance are shown in fig. 13. It will be seen that in each case the benzophenone passed into the gelatinous state on cooling. This state appears

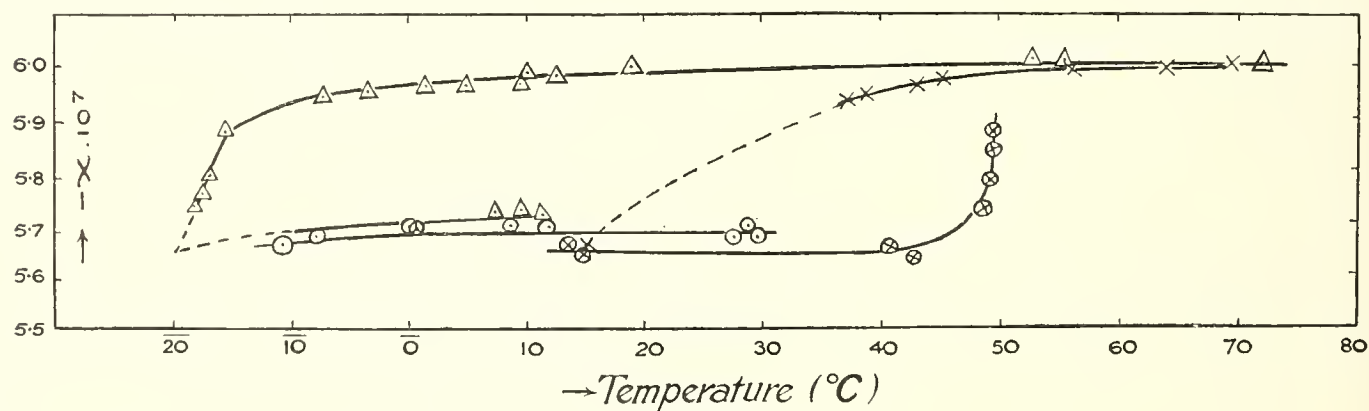


FIG. 13

to be unstable and, on cooling sufficiently, crystallization sets in, which is accompanied by a decrease of χ amounting to 6 per cent. (approximately). In the first experiment the transition took place when the liquid had been super-cooled about 30° C.; in the

second experiment, crystallization took place at a temperature 70° C. below the normal melting point. In each case, when the crystals were heated, fusion took place at the normal temperature, 48° C. The substance was melted by hot water contained in a Dewar tube, and the temperatures below that of the room were obtained with a tube which had been cooled by liquid air.

It is apparent that with this substance a definite hysteresis effect, due to temperature, is obtained, similar to that which was discovered by the late Prof. HOPKINSON*, in the case of nickel-steels. If we regard the paramagnetic state of the nickel-steel as isotropic†, the temperature hysteresis effects in the two cases are completely analogous. The paramagnetic solid corresponds to the diamagnetic liquid. On cooling, the nickel-steel passes the critical temperature‡ without crystallizing just as the benzophenone passes the normal fusion point and assumes a gelatinous form. Continued cooling in each case causes the more stable crystalline form to appear. This is accompanied by a large rise of the paramagnetism of the nickel-steel (ferro-magnetism) and by a fall of the diamagnetism of the benzophenone. If now the nickel-steel be heated, it loses its ferro-magnetism at the critical temperature—a temperature which is characteristic for a given alloy and is lower the greater the proportion of nickel in the steel—while the benzophenone regains its larger diamagnetic property at the normal fusion point—a temperature which is lower the greater the impurity present. Further, the heat produced (recalescence) when the ferro-magnetic state appears on crystallization of the nickel-steel is analogous to the heat produced (heat of formation of the crystals) when the benzophenone crystallizes. The molecular changes are of the same nature in the two cases; the only difference is in the character of the magnetic property possessed by the molecules and by which such molecular changes are indicated.§

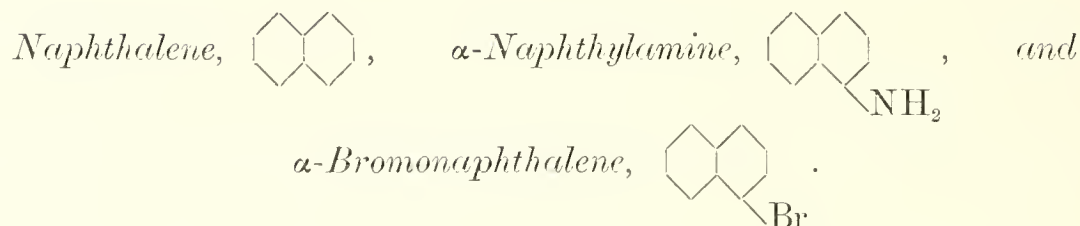
* ‘Roy. Soc. Proc.’ vol. XLVIII., p. 1, 1890; or EWING’S ‘Magnetic Induction in Iron and other Metals,’ p. 184, *et seq.* HOPKINSON referred to the paramagnetic state above the critical temperature as the non-magnetizable state.

[† At least we may regard it as an allotropic modification of the alloy. See the article by GUILLAUME, ‘Rapports du Congrès Int. Paris,’ 1900, vol. 1; also DUMAS, ‘Journ. of the Iron and Steel Institute,’ 68, p. 255, 1905. It should be mentioned that the results described in this communication concerning change of state are in accordance with the work of TAMMANN, ‘WIED. Ann.,’ LXII., p. 285, 1897. See also the article by SPRING, ‘Paris Reports,’ 1900, and WHETHAM’S ‘Theory of Solution,’ Chap. II., where additional references to TAMMANN’S work are given.—*Note added March, 1914.*]

‡ It would be more correct to speak of the “range of temperature of transformation” instead of “critical temperature.”

§ Mr. M. OWEN has observed that the specific susceptibility of super-cooled *gallium*, between 30° C. and 16° C., is constant and equal to that of the liquid above the normal fusion point. The crystalline state is much more diamagnetic and χ_c remains nearly constant until the fusion point, 30° C., is reached.

My attention was drawn to this point of priority, after my work had been completed, by the curve given in OWEN’S paper, ‘Ann. der Phys.,’ IV., 37, p. 693 (fig. 4F), 1912. The English translation of this paper, ‘Versl. Kon. Ak. v. Wetensch.,’ Amsterdam, XIV., p. 637, 1912, which first came to my notice, contains no diagrams and I had overlooked the brief reference there made, at the foot of p. 642, to the behaviour of *gallium*.—*Note added March, 1914.*

(b) *A Study of Naphthalene Compounds.*

In each case the liquid state is more diamagnetic than the crystalline state. The change of susceptibility on crystallization was 4.5 per cent. for each substance, and took place at 80° C. with naphthalene, at 40° C. with α -naphthylamine (fig. 9*k*), and at 4° C. with α -bromonaphthalene.

The rather low value of $\partial\chi$ for naphthalene and its derivatives may be due to the two carbon atoms "condensed" in the naphthalene nucleus, which differ considerably in their chemical nature from the remaining carbon atoms of the naphthalene system.

(c) *Mercury.*

The liquid state was found to be more diamagnetic than the crystalline state, the value of $\partial\chi/\chi$ amounting to 14 per cent. The curves are shown in fig. 14. These were obtained by first cooling the mercury with liquid air and observing the susceptibility as the temperature gradually rose. In the curve ABCD, when the

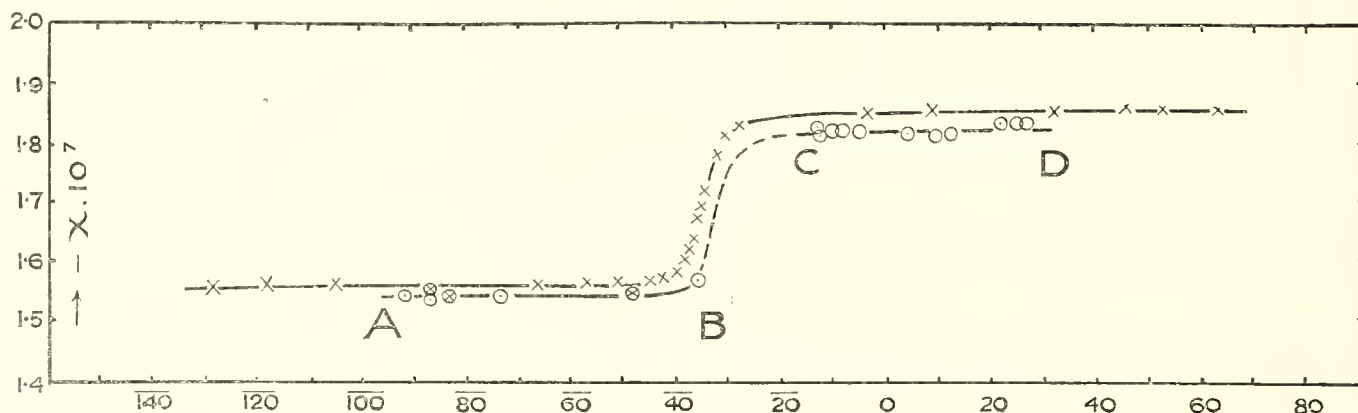


FIG. 14

point C had been reached, the mercury was re-cooled to a temperature just below that of the freezing-point (−37° C., as determined from the position of the dip) and afterwards to −85° C., and the points marked ⊗ were obtained. Between −37° C. and 70° C. the diamagnetic property remained constant.

Experiments on Substances which show a Liquid Crystal State.

Cholesteryl Chloride, $C_{27}H_{45}Cl$, and Para-azoxyanisole, $CH_3O \cdot \langle \text{hexagon} \rangle \cdot N \text{---} N \cdot \langle \text{hexagon} \rangle \cdot O \cdot CH_3$.

In each case the diamagnetic property decreased during the transition from the

true crystalline to the true liquid state. The value of $\partial\chi/\chi$ for para-azoxyanisol was 6 per cent. A smaller and less definite value was obtained with cholesteryl chloride, and after the liquid state had appeared the susceptibility continued to decrease, probably owing to further dissociation of molecular complexes. M. PASCAL* has recently investigated para-azoxyphenetol, $\text{C}_2\text{H}_5\cdot\text{O}\cdot\langle\bigcirc\rangle\cdot\text{N}-\text{N}\cdot\langle\bigcirc\rangle\cdot\text{O}\cdot\text{C}_2\text{H}_5$, and has

discovered that the red liquid form is less diamagnetic than the yellow crystalline form, the value of $\partial\chi/\chi$ being 18 per cent. This substance appears to behave like para-azoxyanisol, but gives a larger value of $\partial\chi$.

In the case of para-azoxyanisol, the $\chi-\vartheta$ curve for ascending temperature was similar to that for descending temperature, but the two curves were slightly displaced owing to thermal lag. Fusion took place at 120°C .

The sign of $\partial\chi$ for each of these liquid crystal compounds is the same as that for benzoyl chloride and phenylhydrazine. Through the kindness of Prof. POPE, I have been able to examine some crystals of the last substance under the crystallization microscope, but no trace whatever of a doubly refracting layer at the boundaries of the crystals could be discerned.

It may perhaps be supposed that the transition of all substances, from the true liquid to the true crystalline state, takes place over a finite though small interval of temperature. This does not imply that all substances should show the liquid crystal state, but merely that a labile state exists, which may or may not be doubly refracting, in which the molecular grouping is determining how the substance will crystallize on further cooling.

(7) GENERAL DISCUSSION OF THE EXPERIMENTAL RESULTS.

The dependence of diamagnetic property on crystallization readily follows as a result of the production of homogeneously close-packed assemblages of atomic spheres of influence.† The difference of structure of the crystalline and amorphous states of a substance is due to the distribution of the restraining forces acting on the molecules of the crystalline structure, which will necessarily produce a small distortion of each molecule; while in the amorphous or gelatinous structure, the molecules will possess a more symmetrical shape on account of the withdrawal of the restraining forces. This deformation effect in every molecule of the crystalline structure is in accordance with the hypothesis stated at the beginning of this communication—a deformation of the atom presumably producing a modification of the internal structure which will in turn modify the diamagnetic property. In Part II., an attempt will be made to interpret this change of diamagnetism in terms of the change of self induction of the electron orbits contained in the atom. The inappreciable deformation of the molecules when

* 'Ann. de Chim. et de Phys.,' VIII., vol. 25, p. 375, 1912.

† BARLOW and POPE, 'Trans. Chem. Soc.,' vol. LXXXIX., p. 1675, 1906.

the substance assumes a gelatinous or liquid form is in accordance with the constancy of the magnetic property which is observed when the substance super-cools or passes into a jelly. Until we postulate some definite internal structure of the atom or molecule, it is impossible to say what the precise effect of the adjustment, from the symmetrical shape of the molecule of the gelatinous state to the non-symmetrical shape of the crystalline state, will be, either as regards magnitude or sign.

Providing the molecular complexity does not change, the diamagnetic property does not depend upon the temperature, and it can therefore be represented by a series of straight lines parallel to the temperature axis. Continuity between these straight lines is represented by the equation

$$\chi_D = \frac{1}{H} \sum n_p \cdot \Delta M_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

During the transition from one state to another, different types of particles co-exist, n_p characterizing the growth of the number of particles of type p . ΔM_p , the diamagnetic moment produced by the applied field H , in a complex of type p , is dependent upon the molecular distortion of the complex.

In the crystalline and liquid states,

$$\chi_{D_c} = \frac{N}{H} \cdot \Delta M_c, \quad \chi_{D_l} = \frac{N}{H} \cdot \Delta M_l,$$

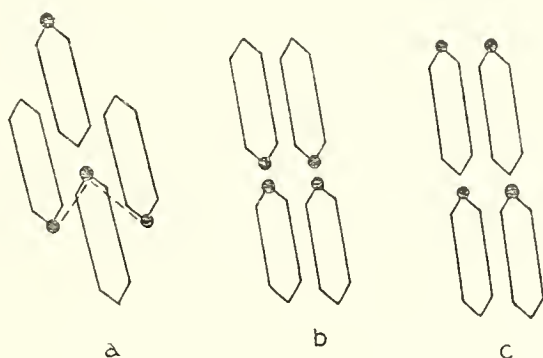
where ΔM_c and ΔM_l are the diamagnetic moments produced in a distorted molecule of the crystalline and liquid states respectively, and N is the number of molecules per gramme of the substance. The magnitude and sign of the change of susceptibility will be determined by the nature of the crystalline structure, while the shape of the curve of transition* depends on the variation of n_p with temperature.

Below the melting points of benzaldehyde, benzophenone, and phenylhydrazine, two distinct χ - ϑ curves have been obtained. In each case the velocity of crystallization of the super-cooled liquid was small, for observations showed that, several hours after cooling, the susceptibility of the gelatinous state was indistinguishable from that of the true liquid.

For benzene derivatives, the diamagnetic property of the crystalline state is in general less than that of the liquid state; but in addition to those compounds which show a liquid crystal state, there are two important exceptions, viz., benzoyl chloride and phenylhydrazine, in which the crystals are more diamagnetic than the liquid. It is interesting to notice that there exists for benzene and its derivatives an alternative kind of packing which is found in the crystalline structure of some of the halogen derivatives. This alternative or cubic system is highly symmetrical for

* In the experiments which have been described the shape of this curve is masked by the effect of low thermal conductivity of the substance.

benzene itself, but the presence of a halogen atom destroys the symmetry of the complex, the latter being sheared in various directions. It is by a process of shearing, similar to that which corresponds to the change from the cubically to the hexagonally derived complex, that we may account for the different diamagnetic properties accompanying polymorphism and by which we may, at any rate provisionally, explain the abnormal results obtained with benzoyl chloride and phenylhydrazine. Diagrammatically, we may represent such a shear by the accompanying figures (a) and (b), where the black dot represents the substituent of the benzene complex. During crystallization it frequently happens that pairs of molecules assume mirror-image positions—(a) and (b) are types of this arrangement. In the case shown at (c), the mirror-image formation is absent. The forces of crystallization will be different in the three cases. On the whole, the resultant of the forces between the molecules will act inwards on each ring system in case (a), and outwards in case (b). Each of these types differs from (c) where the forces on the whole may act neither inwards nor outwards on any given ring system.



Hence we may regard the substituent as a directive agent, and in so far as it influences its own complex and the neighbouring complexes, there is a reversal of the effects produced by the forces of crystallization in case (b) as compared with case (a). This reversal of the interaction of substituent and nucleus with respect to any particular molecule will produce a distortion of the internal structure of an opposite kind.

For convenience and clearness the disposition of the six carbon atoms of the benzene ring is represented as uniplanar. This need not be the case. It is possible now to explain the two abnormal cases of benzoyl chloride and phenylhydrazine without having to assume that the liquid state possesses a greater molecular complexity or a closer packing of molecules than the corresponding crystalline state. I have had the opportunity of discussing, with Prof. POPE, this question of a reversal of the change in magnetic property due to crystallization, and he informed me that the demands of this explanation are in accordance with the modern views of chemists and crystallographers.

The nature of the irregularity of the χ - θ curves does not appear to be directly

connected with the change of volume on crystallization; for in the case of water* the effect is small, and is normal as far as sign is concerned, while a large expansion takes place on crystallization. In the case of benzene there is a fall and a rise of χ on crystallization, while the change of state involves a contraction simply.

With regard to the crystalline forms of the elements, BARLOW and POPE† point out that those elements which show allotropic modifications crystallize in forms of low symmetry, while other elements not so easily obtainable in allotropic forms crystallize in forms of high symmetry. It seems necessary, therefore, to distinguish between the forces holding the atoms in position in the molecule and the forces holding the molecules together in the crystal structure. The atoms of an element are assumed to be identical, and the molecular aggregate to which they give rise and the accompanying distortion depend upon the kind of packing which these equal atomic spheres of influence assume. The only element which has been investigated in this research is mercury. The large change of χ on freezing indicates a large change of molecular configuration.

The allotropic forms of an element, which possess characteristic crystalline structures, will have different molecular distortions and consequently different magnetic properties. Moreover, as these forms are of different degrees of stability, the disintegration of the structure with change of temperature will in general be different for each form. The χ - ϑ curves may therefore be expected to show irregularities. Reference has already been made to the work of DU BOIS and HONDA‡ on the magnetic properties of the allotropic modifications of phosphorus, antimony, carbon, and tin. But these authors used the irregularities which their observations disclosed as an argument against the truth of the foundations of LANGEVIN'S electron theory of diamagnetism instead of interpreting them as due to the variation of molecular complexity. The experiments described above show that the peculiarities of the χ - ϑ curves can in all cases examined be traced to the variation of molecular complexity. In the liquid state most of the compounds that have been investigated are unassociated, and the absence of any variation of χ with ϑ is conclusive proof of the truth of the Curie law of diamagnetism when the molecules are not appreciably distorted. This law is true for distorted molecules, if the distortion is not a function of the temperature, as the parallelism of the curves for the various crystals and for the empty phial shows.

It is clear from the experimental results that diamagnetism is not wholly an atomic property, for the near approach of the molecules, which takes place on crystallization,

* PICCARD, 'Comptes Rendus,' vol. 155, p. 1497, December 23, 1912, and WEISS and PICCARD, 'Comptes Rendus,' vol. 155, p. 1234, December 9, 1912. These authors find that the variation of the diamagnetic property of water with temperature can be interpreted as due to the co-existence of two types of complexes, one type growing at the expense of the other as the temperature is varied. This explanation is in accordance with equation (1) which I had published earlier.

† 'Trans. Chem. Soc.,' vol. LXXXIX., p. 1741, 1906.

‡ 'Versl. Kon. Ak. v. Wetensch.,' Amsterdam, XII., p. 601, 1910.

is able to modify the susceptibility appreciably. From the decrease of diamagnetism accompanying the crystallization of benzene derivatives, it appears that the process of crystallization implies the utilization of the residual forces between the molecules which tends to make them more normally saturated. The value of $\partial\chi$ is not constant for the different members of the same class of compounds, and in benzene derivatives it is a function not only of the ring system but also of the substituent.

Before the extent of this adjustment of the diamagnetic property—due to the juxtaposition of unsaturated groups—can be of value, it is necessary to examine different classes of compounds each containing a large number of members.

It will have been seen that a satisfactory interpretation of the variations of the diamagnetic property found in this research can be given in terms of molecular theory. All that has been assumed concerning the structure of the molecule is that it is a unit which can be distorted by the near approach of other units. In order that we may inquire more fully into the nature of the distortion and the forces which cause it, we must postulate some definite structure of the molecule. We shall assume, in the investigation which follows (Part II.), that the molecule consists of a congeries of electrons; and that the positive charges, which keep the electrons whirling in their orbits, are fixed in the molecule.

For the present, looking at the problem from a purely molecular point of view, the hypothesis stated at the beginning concerning the molecular distortion which accompanies crystallization, has been found justifiable, and has led to results in harmony with the views of chemists and crystallographers.

Little has been said in this communication concerning the Curie law of paramagnetism, which may be written in the form

$$\chi_P = \sum \frac{n_p \cdot c_p}{\mathfrak{J}}.$$

The recent work of Prof. KAMERLINGH ONNES,* on the variation of paramagnetism at low temperatures, shows how we may interpret departures from the simple Curie law, $\chi_P = c/\mathfrak{J}$, by a modification of the Curie constant. The above expression for χ_P shows that if the change of aggregation or the process of crystallization modifies the structure of the molecule, then for every type of complex p , we shall have a new Curie constant c_p per particle of this type. In addition to this, the above expression provides a continuity of the magnetic property during the transition stage where there will be a co-existence of various types of particles. The passage from the liquid to the crystalline state modifies the paramagnetic as well as the diamagnetic property. ONNES† has shown that there is a sudden change of the value of χ_P during the crystallization of liquid oxygen.

* 'Versl. Kon. Ak. v. Wetensch.,' Amsterdam, XV., p. 322, 1912.

† 'Versl. Kon. Ak. v. Wetensch.,' Amsterdam, XIV., p. 674, 1912.

It appears therefore that these two expressions for the diamagnetic and paramagnetic properties are generally applicable and explain the variation of susceptibility with temperature in terms of two Curie laws. These laws are exact to within the limits of experimental accuracy, and any variation of the magnetic property which is apparently not in accordance with them can be attributed to variation of the molecular complexity.

PART II.

(1) A THEORY OF THE PHENOMENA FOUND IN THE ABOVE EXPERIMENTAL RESEARCHES IN TERMS OF THE ELECTRON THEORY OF MAGNETISM.

The theory of LORENTZ, which postulates no free magnetism, but attributes magnetic phenomena to the motion of electric charges, accounts admirably and in a simple way for the main characteristics of the Zeeman effect and for the rotation of the plane of polarisation in a magnetic field. Each of the latter phenomena appears to be closely related to the diamagnetic property of a substance; for LANGEVIN has shown that the magnetic moment produced in a molecule by the application of an external magnetic field is such as would lead to diamagnetism by a process which is compatible with the simple explanation of the Zeeman effect given by LORENTZ. Further, the fact that the magnetic rotation produced by the ferromagnetic elements is proportional to the intensity of magnetization, and not simply to the intensity of the applied magnetic field, shows that we must take into consideration the magnetic property in this case also.

No theory has yet been devised, however, which is completely consistent with the principles of mechanics and the phenomena—optical and electric—of the luminiferous medium, and in adopting the theory of LORENTZ, we are merely seeking to obtain a fuller interpretation, on certain hypotheses, of the experimental facts obtained in Part I. But it is evident that whatever theory be adopted with regard to the origin of the diamagnetic property, any distortion of the molecular system, such as is produced by crystallization, must be accompanied by an adjustment of the magnitude of that property.

To obtain a satisfactory interpretation of the phenomena which have been observed in the experiments of Part I., we must take into consideration the forces which the systems of rotating electrons in different molecules exert on each other. LORENTZ has shown that the force exerted on an electron (surrounded by neighbouring systems) by an electric force E is not simply ϵE , where ϵ is the charge on the electron. A term $\epsilon \cdot f(P)$ must be added, where P is the electric polarisation of the medium, and the function f characterizes the grouping of the molecules for any particular state of the substance. The total mechanical force acting on an electron is therefore

$$\epsilon [E + f(P)].$$

We may regard the crystalline state as isotropic to a first approximation, since the crystals formed in the experiments have all kinds of orientations. The introduction of the term $f(P)$ will then represent the effect due to the modification of the internal motions of an atom or molecule, by the process of crystallization, and will no longer characterize the different properties which the substance really possesses along its crystalline axes.

We shall follow the electron theory of M. LANGEVIN* and refer the electron system of any molecule to axes fixed in direction relative to the æther, but whose origin always coincides with the centre of gravity of the whole system of electrons. Let x, y, z be the co-ordinates of a particular electron. Then $\Sigma x = \Sigma y = \Sigma z = 0$, and since the medium is assumed to be isotropic,

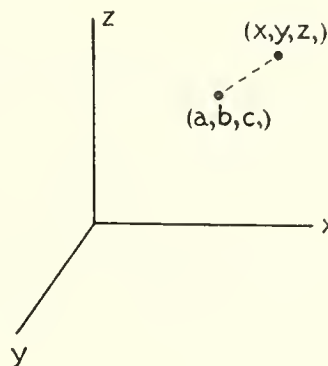
$$\Sigma xy = \Sigma yz = \Sigma zx = 0.$$

Let a, b, c be the co-ordinates of the centre of gravity of a molecule. Then, since the medium is isotropic,

$$\Sigma a = \Sigma b = \Sigma c = 0$$

and

$$\Sigma ab = \Sigma bc = \Sigma ca = 0.$$



The co-ordinates of an electron, with respect to the centre of the molecule in which it is situated, will be denoted by

$$(\xi, \eta, \zeta) = (x-a, y-b, z-c).$$

The above equations give

$$\Sigma \xi = \Sigma \eta = \Sigma \zeta = \Sigma \xi \eta = \Sigma \eta \zeta = \Sigma \zeta \xi = 0.$$

The areal velocity of an electron, with reference to the centre of gravity of the molecule, will have a component along Oz given by

$$\Omega_z = \frac{1}{2} (\xi \dot{\eta} - \eta \dot{\xi}), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and, assuming that we can interpret the magnetic effect of the electrons spinning in their orbits by the magnetic moment M_z where

$$M_z = \Sigma e . \Omega_z, \dagger . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

we may write from (1) and (2)

$$\dot{M}_z = \frac{e}{2} \Sigma (\xi \ddot{\eta} - \eta \ddot{\xi}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

* 'Ann. de Chim. et de Phys.,' VIII., vol. 5, p. 70, 1905. For clearness I have borrowed the treatment up to equations (4) directly from M. LANGEVIN's paper.

† Sir JOSEPH LARMOR ('Phil. Trans.,' 1897, A, p. 230), has shown that this relation implies the existence of a constraint to prevent the accumulation of displacement during the action of a steady magnetic field on a system of spinning electrons. See also LEATHEM, 'Roy. Soc. Proc.,' vol. 89A p. 31, 1913.

In addition to the forces due to external sources acting on the electrons contained in a given molecule, there will be internal forces having components X, Y, Z , which are determined by the configuration of the molecule. If the origin move with velocity (u, v, w) the velocity of any electron will be $(u + \dot{x}, v + \dot{y}, w + \dot{z})$, and LANGEVIN writes the equations of motion in the form

$$\left. \begin{aligned} m\ddot{\xi} &= X + \epsilon E_x + \epsilon H_z(v + \dot{y}) - \epsilon H_y(w + \dot{z}) - m\ddot{a} - m\dot{u} \\ m\ddot{\eta} &= Y + \epsilon E_y + \epsilon H_x(w + \dot{z}) - \epsilon H_z(u + \dot{x}) - m\ddot{b} - m\dot{v} \end{aligned} \right\}, \quad \dots \quad (4)$$

where E and H are the electric force and magnetic force and m is the mass of an electron.

If we wish to take into account the forces due to neighbouring molecules, when crystallization takes place, we must add the term $\epsilon f(P)$ to these equations. Now, in a crystalline structure, the motion of a molecule will consist of oscillations about a centre, and, therefore, if we take a mean value of this polarisation term and add it to the equations (4) we shall include the effect of the presence of neighbouring molecules. Assuming that the medium is isotropic (which is permissible since the crystals will have all kinds of orientations, and we are seeking the effect of close approach of the molecules only and not any particular property in a specified direction) the new equations are

$$\left. \begin{aligned} m\ddot{\xi} &= X + \epsilon [E_x + f(P_x)] + \epsilon H_z(v + \dot{y}) - \epsilon H_y(w + \dot{z}) - m\ddot{a} - m\dot{u} \\ m\ddot{\eta} &= Y + \epsilon [E_y + f(P_y)] + \epsilon H_x(w + \dot{z}) - \epsilon H_z(u + \dot{x}) - m\ddot{b} - m\dot{v} \end{aligned} \right\}. \quad \dots \quad (4')$$

On account of the small dimensions of the elementary system considered, the electric force E and the electric polarisation term $f(P)$ will be nearly constant throughout its extent, so that denoting the value of E and $f(P)$ at the centre of the elementary system by E_0 and $f(P_0)$, we have, expanding and neglecting higher powers than the first,

$$E_x = E_{0x} + x \left(\frac{\partial E_x}{\partial x} \right)_0 + y \left(\frac{\partial E_x}{\partial y} \right)_0 + z \left(\frac{\partial E_x}{\partial z} \right)_0 \quad \dots \quad (5)$$

and

$$f(P_x) = f(P_{0x}) + x \left[\frac{\partial f(P_x)}{\partial x} \right]_0 + y \left[\frac{\partial f(P_x)}{\partial y} \right]_0 + z \left[\frac{\partial f(P_x)}{\partial z} \right]_0$$

Calculating $\dot{M}_z$ from (3), (4)', and (5), and using the relations given on p. 137 for an isotropic medium, we find (writing $\Sigma \xi^2 = \Sigma \eta^2 = \Sigma \zeta^2 = A/2$)

$$\begin{aligned} \dot{M}_z &= \frac{\epsilon^2}{4m} \left[A \left\{ \left(\frac{\partial E_y}{\partial x} \right)_0 - \left(\frac{\partial E_x}{\partial y} \right)_0 \right\} + wA \left\{ \left(\frac{\partial H_x}{\partial x} \right)_0 + \left(\frac{\partial H_y}{\partial y} \right)_0 \right\} \right. \\ &\quad \left. - A \left\{ u \left(\frac{\partial H_z}{\partial x} \right)_0 + v \left(\frac{\partial H_z}{\partial y} \right)_0 \right\} - H_{0z} \frac{dA}{dt} \right] \\ &\quad + \frac{\epsilon^2}{4m} \left[A \left\{ \left(\frac{\partial f(P_y)}{\partial x} \right)_0 - \left(\frac{\partial f(P_x)}{\partial y} \right)_0 \right\} \right] + \frac{\epsilon^2}{2m} \Sigma (\xi Y - \eta X) \end{aligned}$$

The last term is zero if each molecule has no initial moment, as LANGEVIN's theory of diamagnetism requires. Leaving aside the subscript 0 and using the equations of the electromagnetic field

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial H_z}{\partial t}, \quad \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0,$$

and

$$\frac{\partial H_z}{\partial t} + u \cdot \frac{\partial H_z}{\partial x} + v \cdot \frac{\partial H_z}{\partial y} + w \cdot \frac{\partial H_z}{\partial z} = \frac{dH_z}{dt},$$

we get, following LANGEVIN,

$$\dot{M}_z = -\frac{\epsilon^2}{4m} \cdot \frac{d}{dt} (H_z A) + \frac{\epsilon^2}{4m} \left\{ A \left[\frac{\partial f(P_y)}{\partial x} - \frac{\partial f(P_x)}{\partial y} \right] \right\}.$$

Integrating this equation from time 0 ($H = 0$) to time τ ($H = H_z$) we find

$$\Delta M_z = -\frac{\epsilon^2}{4m} \cdot H_z A + \frac{\epsilon^2 A}{4m} \int_0^\tau \left[\frac{\partial}{\partial x} f(P_y) - \frac{\partial}{\partial y} f(P_x) \right] dt,$$

where ΔM_z is the magnetic moment produced in the molecule during this interval. The second term on the R.H.S. of (6) will depend upon the molecular configuration of the substance and implies a modification of the electron circuits which will change their self-induction. Now any change of the self-induction may be represented by a small change of the intensity of the applied magnetic field.

We may write

$$f(P) = \alpha \cdot P,*$$

where " α " characterizes the grouping of the molecules and therefore

$$\left[\frac{\partial}{\partial x} f(P_y) - \frac{\partial}{\partial y} f(P_x) \right] = \alpha \left(\frac{\partial P_y}{\partial x} - \frac{\partial P_x}{\partial y} \right) = -\alpha \frac{\partial}{\partial t} (\delta H_z),$$

where $\alpha \cdot \delta H_z$ is the elementary change in the external field during a small interval of time δt .

Therefore

$$\begin{aligned} \Delta M_z &= -\frac{\epsilon^2}{4m} H_z A - \frac{\epsilon^2 A \alpha}{4m} \cdot \int_0^\tau \frac{\partial}{\partial t} (\delta H_z) \cdot \partial t \\ &= -\frac{\epsilon^2 H_z A}{4m} \cdot \left[1 + \frac{\alpha \cdot \Delta H_z}{H_z} \right], \quad \dots \dots \dots (6) \end{aligned}$$

where $\alpha \cdot \Delta H_z$ is the total variation of H_z due to establishing this magnetic field when the electron orbits are modified by the presence of neighbouring molecules.

* LORENTZ, 'Theory of Electrons,' p. 137.

If N be the number of molecules in one gramme of the substance we may write the specific susceptibility

$$\chi = \frac{N\Delta M_z}{H_z} = - \frac{N\epsilon^2 A}{4m} \cdot \left[1 + \frac{\alpha \cdot \Delta H_z}{H_z} \right] \dots \dots \dots (7)$$

and it is clear that the value of χ is modified by crystallization, for the polarisation term αP varies with the state of the substance. If $\alpha = 0$, (7) reduces to the Langevin expression for diamagnetic susceptibility in which the mutual influences of the molecules are neglected.

The variation of χ on crystallization may be written

$$\frac{\partial \chi}{\chi} = (\alpha_c - \alpha_l) \cdot \frac{\Delta H_z}{H_z} \dots \dots \dots (8)$$

where α_c and α_l are the electric polarisation constants for the crystals and liquid respectively; α_l is of the order $1/3$ for most liquids.* It now remains for us to examine the amount of this change and to do this we must determine α_c . The value of the polarisation constant for any substance is positive and may be very large compared with the value $1/3$ which it usually possesses for the liquid state; but its exact determination in any particular case is difficult since it depends upon the actual distribution of the molecules, about which we know very little.

It appears, however, that we can deduce an approximate value of the magnitude of α_c from CHAUDIER's researches on the change of magnetic rotatory power with change of state. This deduction leads to a reasonable value.

It has been shown by BECQUEREL, BICHAT, and later by CHAUDIER,† that the magnetic rotatory power of the liquid and vapour states is not generally the same, and it is reasonable to suppose that a similar but more pronounced effect will be found in the passage from the liquid to the crystalline state, but the phenomenon will usually be complicated in the latter case owing to the appearance of double refraction. The researches of GOUY‡ and CHAUVIN§ show that in the case of Iceland spar the effects of double refraction and magnetic rotation are simply superposed. Now CHAUDIER|| has established that the magnetic rotatory power disappears completely during the crystallization of aniline, benzene, and nitrobenzene (all of which have been investigated in Part I.). If this is a true discontinuity of the magnetic rotatory power, apart from any effect due to double refraction, then we can use this fact to obtain an estimate of

* *Loc. cit.*, p. 138, or LARMOR, 'Phil. Trans. Roy. Soc.,' 1897, A, p. 233.

† 'Comptes Rendus,' vol. 156, p. 1008, 1913.

‡ 'Journ. de Phys.,' sér. II., vol. 4, p. 149, 1885.

§ 'Journ. de Phys.,' sér. II., vol. 9, p. 5, 1890.

|| 'Comptes Rendus,' vol. 156, p. 1529, 1913.

the value of α_c for the crystalline forms of these substances. The value deduced* (see footnote) on these assumptions must be at least of the order 10^2 and may be larger. Now reverting to equation (8), we see that the experimental value of $\partial\chi/\chi$, approximately 5 per cent., will be in accordance with the above theory of the effect of crystallization if the change of self-induction of an electron orbit of a substance in the liquid state, due to the presence of neighbouring molecules, be such that

$$\Delta H_z/H_z = 5 \times 10^{-4}.\dagger$$

The value of this ratio will be still smaller if α_c possesses a greater value than 10^2 . In the crystalline state, the modification of the electron orbit will be determined by $\alpha_c \Delta H_z/H_z$ and since α_c is large, this term accounts for the variation, $\partial\chi$, found in the experimental investigation of Part I. In the case of liquids, where $\alpha_l = 1/3$, this term produces no appreciable effect on the value of χ , as equation (7) shows.

(2) THE MEAN MOLECULAR FIELD OF A DIAMAGNETIC CRYSTALLINE SUBSTANCE.

The large value of the electric polarisation constant α_c is compatible with the considerable rigidity possessed by crystalline structures. The term $\alpha_c \cdot \Delta H_z/H_z$ will therefore be important in the crystalline state. For a super-cooled liquid we may write

$$\chi_l = -\frac{N\epsilon^2 A}{4m},\ddagger$$

* An extension of the theory of magnetic rotation has just been published by G. H. LIVENS ('Phil. Mag.,' vol. 26, p. 362, 1913), in which the influence of the electric polarisation due to neighbouring molecules is taken into consideration. LIVENS shows that the rate of rotation of the plane of polarisation by a solution may be written

$$w = \frac{\epsilon H n^2}{\mu c} \cdot \sum \frac{\epsilon^2}{m^2 (n_0^2 - n^2)^2} \left/ \left(1 - \sum \frac{a \epsilon^2}{m (n_0^2 - n^2)} \right)^2 \right.,$$

where n is the frequency of the light wave, n_0 the restoring coefficient due to displacement of an electron, a the constant of electric polarisation, and ϵ , H , μ , and c have their usual interpretation. The Σ extends to all the electrons in unit volume.

If $a = 0$ we get the result of DRUDE'S theory, where the mutual influences of the molecules are neglected; w is of the order 10^{-2} for the liquid state, and if w is reduced to 1 per cent. of this value when crystallization sets in, the value of

$$\sum \frac{a_c \epsilon^2}{m (n_0^2 - n^2)}$$

must be at least of the order 10. Now $\epsilon \doteq 10^{-10}$ E.S.U., $m \doteq 10^{-27}$, and $n^2 \doteq n_0^2 \doteq 10^{30}$. As there are about 10^{22} electrons per cubic centimetre, a_c must be of the order 10^2 at least.

We have assumed that the medium is isotropic, but as the crystals are differently orientated this assumption will not change the order of a_c .

† This ratio corresponds to a small alteration of the area of an electron orbit to the maximum extent of 1 in 2,000, a magnitude which is quite permissible.

‡ This equation is in accordance with the observations of M. CHAUDIER, who has shown that, if a substance super-cools or assumes a gelatinous form, the magnetic rotation is the same as for the normal liquid state (*i.e.*, $a \doteq \frac{1}{3}$ for the gelatinous state).

neglecting the term $\alpha_l \cdot \Delta H_z / H_z$ since $\alpha_l \doteq 1/3$. For the crystals, at the same temperature, we have

$$\chi_c = -\frac{N\epsilon^2 A}{4m} \cdot \left[1 + \alpha_c \cdot \frac{\Delta H_z}{H_z} \right].$$

Hence

$$\chi_c = \chi_l \cdot \left[1 + \alpha_c \cdot \frac{\Delta H_z}{H_z} \right],$$

or

$$\chi_c \cdot H_z = \chi_l \cdot [H_z + \alpha_c \cdot \Delta H_z].$$

I shall call $\alpha_c \cdot \Delta H_z$ the mean molecular field of the diamagnetic crystals. Now for a given change of the electron orbits due to crystallization, ΔH_z will be proportional to H_z . Hence we may neglect the mutual forces between the diamagnetic molecules when the substance crystallizes providing we supply a molecular field which is proportional to H_z , and therefore to ΔM_z , or to the intensity of magnetization of the substance (equation (6)). For the liquid and gelatinous states this molecular field is negligibly small, because α_l is small, but for the crystalline state α_c is large, and the molecular field is comparable with the external field H_z .

This is exactly analogous to the theory of ferro-magnetism developed by M. WEISS, in which the molecular field is $\mathbf{Nl}$, where $\mathbf{l}$ is the magnetic moment produced by the external field and $\mathbf{N}$ is a constant determined by the intermolecular forces of the crystalline ferro-magnetic material. $\mathbf{N}$ corresponds to the polarisation constant α_c of the diamagnetic theory. We ought, however, to compare $\mathbf{N}$ with α'_c , where the molecular field

$$\alpha_c \cdot \Delta H_z = \alpha'_c \cdot \mathbf{N} \cdot \Delta M_z \rho,$$

ρ , the density of the substance, is of the order 1.

If $\alpha_c = 10^2$, this gives (since $\Delta H_z = H_z/2 \times 10^3 = \mathbf{N} \cdot \Delta M_z / \chi \times 2 \times 10^3$)

$$\alpha'_c = \alpha_c / \chi \times 2 \times 10^3.$$

Assuming $\chi = 5 \times 10^{-7}$, we find $\alpha'_c = 10^3 \alpha_c$. Hence the constant of the diamagnetic field is 10^5 . The corresponding values of $\mathbf{N}$ given by WEISS and BECK* are 0.38×10^4 for iron, 1.27×10^4 for nickel, and 3.32×10^4 for magnetite. The mean molecular fields are enormously greater in the cases of ferro-magnetic substances on account of the large value of the intensity of magnetization as compared with that for diamagnetic substances. The values of $\mathbf{Nl}$ (the saturation molecular field) are 6, 530, 000 gauss for iron, 6, 350, 000 gauss for nickel, and 14, 300, 000 gauss for magnetite, while the corresponding value of $\alpha'_c \cdot \Delta M_z$ for a diamagnetic substance is of the order 150 gauss for an applied field of 3000 gauss, the field used in the above experiments.

To account for the phenomena of the Zeeman effect, magnetic rotation of the plane of polarisation, and diamagnetism, it is necessary to assume that an atom contains

* 'Journ. de Phys.,' sér. IV., vol. 7, p. 249, 1908.

oppositely rotating systems of electrons. A diamagnetic molecule, on LANGEVIN's theory, has no initial magnetic moment but there can be complete compensation of the electro-magnetic forces due to the spinning electrons only at points considerably removed from the molecule. Although these forces are compensated at an external point, and the magnetic property observed corresponds to diamagnetism, yet between the molecules the forces will be large and the local intensity of magnetization may be comparable with that of ferro-magnetic substances.* It is interesting that the value of α'_c , which defines the crystal structure of a diamagnetic substance, should be at least of as high an order of magnitude as the constant **N** of the ferro-magnetic field. It is probable that these large values of the coefficients of the molecular fields are the origin of the stability and rigidity of crystal structure in general.

Since we may interpret the effect of the surrounding molecules by means of the term αP , it follows that if from any cause the polarisation **P** suffers a reversal of sign; then the sign of ΔH_z will change, *i.e.*, the variation of χ on crystallization will correspond to the abnormal cases referred to in Part I. The mechanism by which this reversal of $\partial\chi$ is brought about may be attributed to a directive influence of the substituent (or other dissymmetry of the complex molecule) which will produce an electric polarisation effect of opposite kind in a neighbouring nucleus, according to the direction of the force between the substituent and the nucleus.

The liquid state will be more diamagnetic than the crystalline state if $\Delta H_z/H_z$ is negative, *i.e.*, if the process of crystallization involves a small reduction of the self-induction of the electron orbits. This corresponds to the majority of cases investigated in Part I.

In the above theory no account has been taken of the small reaction forces† which will arise from the production of the diamagnetic moment ΔM_z . Since, however, χ (and therefore ΔM_z) does not vary by more than a few per cent. as the substance passes from the liquid to the crystal state, these small reaction effects will appear in both states approximately to the same extent and therefore the value of $\partial\chi$ will not be affected on their account.

SUMMARY OF CONCLUSIONS.

The experimental results described in Part I. have established an almost complete parallelism between the dependence of diamagnetic and ferro-magnetic susceptibilities upon molecular constitution. The molecular changes are of the same nature and the only difference between the two classes of magnetic phenomena lies in the character of the magnetic property possessed by the molecules and by which the variation of molecular complexity is disclosed.

* See Note added, p. 145.

† LANGEVIN, 'Ann. de Chim. et de Phys.,' VIII., vol. 5, p. 88, 1905, concludes that such effects would not produce an appreciable change in the magnitude of χ .

In the liquid and crystalline states, providing there are no molecular complexes whose constitutions vary with the temperature, the diamagnetic property involves no temperature co-ordinate and is represented by a series of straight lines parallel to the temperature axis. The work of MM. WEISS and KAMERLINGH ONNES has shown that ferro-magnetic and paramagnetic properties can be represented by a series of hyperbolæ. In either case, I have shown that if the substance forms aggregates, or assumes a crystalline structure, the transition stage is represented by the equations

$$\chi_D = \frac{1}{H} \sum n_p \cdot \Delta M_p \text{ (diamag.)}, \quad \chi_P = \sum \frac{n_p \cdot c_p}{g} \text{ (paramag.)}$$

which take into account the coexistence of molecular complexes of different types.

These two expressions are generally applicable and represent the variation of magnetic property with temperature in terms of the two Curie laws, and the superposed effect of the change of molecular complexity with temperature.

For most of the substances examined, the liquid state is more diamagnetic than the crystals. The cases of benzoyl chloride, phenylhydrazine and the liquid crystal compounds, cholesteryl chloride and para-azoxyanisol, are abnormal. Nearly all benzene derivatives show a change of susceptibility on crystallization, amounting to 5 per cent. (approximate). The sign of this change is attributed to the directive action of the substituent.

The experiments show that diamagnetism is not wholly an atomic property; for the process of crystallization, involving simply a closer approach of the atomic spheres of influence, is able to modify the susceptibility.

The existence of a gelatinous or super-cooled liquid state, possessing in all the cases examined a diamagnetic property equal to that of the liquid above the normal fusion point, corresponds to the existence of a supercooled paramagnetic state of nickel-steels below the critical temperature. Definite hysteresis loops with respect to temperature have been obtained, similar to those discovered by the late Prof. HOPKINSON for nickel-steels. The heat of formation of the diamagnetic crystals corresponds to the heat evolved at the critical temperature of the nickel-steel.

The constancy of the diamagnetic property when the substance supercools or passes into a jelly is in harmony with the constancy of the closely related phenomenon of the magnetic rotation of the plane of polarisation by that substance when a gelatinous form is assumed. The process of crystallization is accompanied by a modification of the susceptibility and of the magnetic rotation.

The hypothesis of molecular distortion stated at the beginning of this communication has been justified, and the interpretation of the effect of crystallization as due to a slight modification of the internal structure of the molecule is in agreement with the conceptions of chemists and crystallographers concerning the nature of crystal structure.

In Part II., the extension of LANGEVIN'S theory shows that an exceedingly small and reasonable deformation of the molecules in the liquid state is sufficient, owing to the large value of the constant α_c , to account for the change of susceptibility actually observed when the substance crystallizes.

Moreover, the introduction of the idea of a molecular field of the diamagnetic crystals allows us to neglect the mutual influences of the diamagnetic molecules in exactly the same way as the molecular field of WEISS allows us to express the phenomena of ferro-magnetism, without bringing into consideration the mutual influences of the molecules. Both the molecular field of WEISS and the molecular field of the diamagnetic crystals are proportional to the intensity of magnetization, in the respective cases, and the minimum value of the constant of the latter field, deduced from the variation of magnetic rotatory power when crystallization sets in, is of the same order as that of the ferro-magnetic field.

Although on account of a particular structure of the molecules the magnetic properties of different substances, observed externally, may vary enormously, yet, within the substances, the local intensities of magnetization and the forces between the molecules will be large in all crystalline substances. These forces determine the rigidity of the crystalline structures.

These investigations have been made in the Cavendish Laboratory and I wish to thank Prof. Sir JOSEPH THOMSON for placing the necessary apparatus at my disposal and for the interest he has shown in the work.

I am indebted to Prof. POPE for supplying me with many substances for investigation and also for a valuable discussion on the subject of crystal structure.

[It will have been observed that the theoretical treatment given in Part II. consists of an interpretation of the polarisation term, $\frac{e^2 A}{4m} \cdot \int_0^r \left[\frac{\partial}{\partial x} f(P_y) - \frac{\partial}{\partial y} f(P_x) \right] dt$, (which represents that part of the diamagnetic moment due to the distortion of a molecule by the influences of neighbouring molecules of the crystalline structure) by means of a magnetic field. This field is superposed upon the applied field and it has been shown that we may disregard the mutual influences of the molecules forming the crystalline structure if the resultant of these fields is substituted for the applied field.

This corresponds to the interpretation, given by M. WEISS, of the mutual influences of the molecules of ferro-magnetic substances by a uniform magnetic field, $\mathbf{NI}$, superposed upon the applied field.

Since the work described in Parts I. and II. was communicated to the Royal Society, a paper has appeared by WEISS ('Comptes Rendus,' vol. 157, No. 25, p. 1405, December 22, 1913) which deals more fully with the nature of the molecular field in ferro-magnetic substances. It is there shown that the large forces which act between the molecules are not necessarily magnetic but may be of any other nature. A

consideration of the continuity of magnetic flux has led WEISS to abandon the idea that the magnetic field is actually a *uniform* magnetic field. The difficulty may be surmounted by assuming the molecular field to be localized. This localized nature is a necessary property of the molecular field in diamagnetic crystalline substances. But, as WEISS points out, it is rather difficult to see how the molecular magnets, or the electron orbits, in adjacent molecules, can approach one another so closely that, in spite of thermal agitation, the local field between the molecules should amount to the enormous value of 10^7 gauss.

In any case, whatever may be the nature of the local forces which hold the molecules of a crystalline structure together, we may interpret the accompanying distortion by means of a magnetic field of such magnitude that it produces in the molecules the same change of moment as the actual forces within the crystalline structure will produce. This statement holds whether the substance be ferro-magnetic or diamagnetic.

The large value of this local magnetic field in diamagnetic substances has been briefly mentioned at the end of Part II. It can be shown that a local field, comparable in intensity with the ferro-magnetic field of WEISS, will produce, according to LANGEVIN's theory, a modification of the electron orbits in the molecules of a diamagnetic substance such as would account for the change of χ observed on crystallization in the experiments of Part I. In a subsequent paper I hope to describe in detail several extensions of the work contained in Parts I. and II., and I should like to make a brief reference here to one or two points because they furnish additional support to the conclusions which have already been mentioned. Further experiments have been made, similar to those described in Part I., and the change of susceptibility on crystallization has been correlated with the magnetic double refraction of the same substance observed by MM. COTTON and MOUTON. The large double refraction of a crystalline structure, as compared with that of a liquid subjected to the strongest magnetic field which we can produce in the laboratory, is compatible with the relatively large local magnetic field by which we may express the forces which hold the molecules in position in the crystal.

Perhaps these large local fields between the molecules of a crystalline structure may be identified with the intense intra-atomic fields of RITZ ('Ann. der Phys.,' IV., vol. 25, p. 660, 1908) which give us a direct interpretation of BALMER's and RYDBERG's expressions for spectral series, or with the magneton of WEISS. This would imply that the molecules in the crystal structure are exceedingly close together and that their oscillations are of extremely small amplitude. It is possible that the magneton will prove to be a constituent of diamagnetic as well as of paramagnetic and ferro-magnetic matter.—*Note added March, 1914.*]

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[PLATES 1-2.]

A DETERMINATION OF THE ELECTROMOTIVE FORCE OF THE
WESTON NORMAL CELL IN SEMI-ABSOLUTE VOLTS.

BY

A. NORMAN SHAW, M.Sc.,

1851 EXHIBITIONER, AND KING RESEARCH FELLOW, MCGILL UNIVERSITY, MONTREAL.

WITH A PREFACE

BY

PROF. H. L. CALLENDAR, F.R.S.



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By A. NORMAN SHAW, *M.Sc.*, 1851 *Exhibitioner*, and *King Research Fellow*,
McGill University, Montreal.

With a Preface by Prof. H. L. CALLENDAR, F.R.S.

Communicated by Prof. H. L. CALLENDAR, F.R.S.

Received November 1,—Read December 11, 1913.

[PLATES 1-2.]

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PREFACE BY PROF. H. L. CALLENDAR.

THE measurement of the E.M.F. of the Weston cell affords the best means of comparing the performances of different methods and instruments for the absolute determination of the ampere. Great progress has been made in the last six years, but the most recent determinations by independent methods, giving equal promise of accuracy, still show discrepancies covering a range of 2 parts in 10,000, which must be debited for the most part to the difficulty of the absolute determination of current. Each method in itself appears to give an order of accuracy of repetition approaching, or even exceeding, 1 in 100,000. It is therefore of special interest and importance to compare the results of methods differing as widely as possible in experimental details in endeavouring to arrive at a value comparatively free from the constant errors which may beset any particular type of method.

The measurements described by Mr. SHAW in the following paper were made by the method of the Weber bifilar electrodynamicometer, as modified by CLERK MAXWELL and LATIMER CLARK, which has not hitherto been employed for work of the highest accuracy, and which merits attention on account of its many fundamental points of difference from recent methods. The instrument originally supplied to McGill College for this purpose was a faithful copy of CLERK MAXWELL'S instrument at Cambridge, of which the theory is given together with a figure and description in his 'Electricity and Magnetism,' vol, 2, p. 367. The chief sources of error in this instrument were (1) the uncertainty of insulation of the coils, which proved to be of the

order of nearly one half of 1 per cent.; (2) the difficulty of determining the mean radii of the coils, which were wound with silk-covered wire; (3) the want of rigidity of the pulley arrangement for equalising the tensions of the suspending wires, and the imperfect elasticity of the control, which depended too much on torsion, and made it impossible to obtain readings consistent to 1 in 1000 for the deflections or the times of oscillation. These defects were so fatal to accurate work even of the order of 1 in 10,000, which was all that it was originally contemplated, that it was found necessary to reconstruct the instrument entirely until nothing remained of the original except the frame, and even that required stiffening to a material extent.

The arduous work of reconstructing the instrument was undertaken by Mr. R. O. KING as part of his work during the tenure of an 1851 Exhibition Scholarship during the years 1897–8. The coils were re-wound with a double winding to give a perfect check on the insulation. The large coils were wound with a carefully measured length of hard rolled copper tape, which gave a very high order of accuracy in the determination of the mean radius—this device possesses the great advantage of avoiding the excessive refinement necessary in measuring the linear dimensions of the coils in the majority of other methods. The coils were also made reversible and interchangeable to eliminate possible errors of symmetry, especially in the measurement of the distance between their planes.

The dimensions of the suspended coils were determined by an electrical method of comparison with the fixed coils, which proved to be one of the most difficult measurements owing to extraneous magnetic disturbances. Mr. KING's results for this comparison showed extreme differences amounting to nearly 1 in 5000. This led me, in revising his work, to put the limit of accuracy at 1 in 10,000, although all the fundamental quantities could be measured to 1 in 100,000 or better. I have since observed that the largest discrepancy in this comparison was undoubtedly due to a slight failure of insulation duly noted by Mr. KING at the time. Another series was interfered with by an inopportune magnetic storm. Mr. SHAW employing the identical tube made by Mr. KING for supporting the small coils in a fixed position relative to the large coils, has succeeded after 15 years in recovering Mr. KING's value for the ratio of the currents required for balance to 1 in 30,000. I am now of opinion that I had greatly under-estimated the probable accuracy of this comparison, which Mr. SHAW appears to have carried out to an order of accuracy little if at all inferior to 1 in 100,000.

A weak point in many of the methods of absolute measurement is the relative smallness of the electrical force to be measured. The arrangement of the coils in parallel, rendered possible by rewinding the coils, permitted the attainment of a forty times larger force for the same current in the small coils, *i.e.*, without introducing any unsteadiness or uncertainty of dimensions due to the heating effect of the current on the small coils. The relative importance of the gravity control in the bifilar suspension could thus be increased, with a corresponding improvement in the steadiness

and accuracy of the deflection observations. There still remained, however, an appreciable effect due to imperfect elasticity of the wires, and a good deal of time was spent in selecting a suitable suspension. With the round wires ultimately adopted, and subsequently employed by Mr. SHAW, the effect amounted, as nearly as I could estimate, to 3 parts in 10,000 under the conditions of the deflection experiments. It appeared to me at the time that the effect was largely due to the sharp bending of the wire at the four points where it was rigidly clamped. I accordingly prepared a rolled strip of equal carrying capacity to take the place of the round wire with the object of reducing the creep. This would probably have proved effective, but it was considered inadvisable to disturb the suspension at the time, when half the observations were completed, seeing that the correction was so small in relation to the order of accuracy then contemplated. The exact study of this suspension since carried out by Mr. SHAW, has made it possible to determine the correction to a much higher order of accuracy, so that this last objection to the deflection method has been removed.

Now that Mr. SHAW has repeated the observations and revised the whole theory of the experiment with such minute care and accuracy, I feel sure that the results should take rank among the best determinations of recent years, and should possess special interest as an independent determination on account of the many radical differences of procedure involved. At the same time I hope that Mr. KING's share in the research as a student in reconstructing the apparatus and getting it into working order, and his generosity in providing the means for the ultimate completion of the observations will not be forgotten.

I. INTRODUCTION.

ACCOUNTS of several absolute determinations of current have been published in recent years, and it does not, therefore, appear necessary to present an extended introduction to the subject. The results of the most recent determinations of the electromotive force of the Mean Weston Normal Cell are as follows :—

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1908, PELLAT‡		1.01831	„ „ „
1908–10, JANET, LAPORTE, and JOUANST§	. .	1.01836	„ „ „
1910, HAGA and BOEREMA		1.01825	„ „ „
1912, ROSA, DORSEY, and MILLER¶		1.01822	„ „ „

* AYRTON, MATHER, and SMITH, 'Phil. Trans.,' A, vol. 207, pp. 463–544, 1908.

† GUILLET, 'Bull. de la Soc. Int. des Élect.,' 2, vol. 8, pp. 535–561, 1908.

‡ PELLAT, 'Bull. de la Soc. Int. des Élect.,' 2, vol. 8, pp. 573–633, 1908.

§ JANET, LAPORTE, and JOUANST, 'Bull. de la Soc. Int. des Élect.,' 2, vol. 8, pp. 459–522, 1908; also (II.), vol. 10, p. 482, 1910.

|| HAGA and BOEREMA, 'Kon. Akad. Wetensch. Amsterdam, Proc.,' p. 587, 1910.

¶ ROSA, DORSEY, and MILLER, 'Bull. Bur. of Stan.,' vol. 8, p. 269, 1912.

where the semi-absolute volt is taken as the potential difference between the terminals of an international ohm (Britain, America, and Germany) when it is traversed by a current of one absolute ampere.

The writer desires as a result of the investigation recorded in this paper to add to the above list the value **1.01831** semi-absolute volts at 20° C., which has been determined with the aid of a Weber electro-dynamometer in the possession of McGill University, Montreal.

This instrument was purchased by the University as a part of the general equipment of the Macdonald Physics Building about 1893. It was carefully made by NALDER, and was an exact copy of the dynamometer constructed by the Electrical Committee of the British Association, and described by MAXWELL.* The instrument was set up by Mr. R. O. KING in 1895 under the direction of Prof. H. L. CALLENDAR. The work by KING has been fully described by CALLENDAR† who made some important changes in the design of the instrument. Preliminary observations were obtained by KING for the electromotive force of the old Board of Trade form of crystal Clark cell, and it was intended that more accurate work should be obtained later. This work was not, however, carried out, and the dynamometer remained in disuse for over ten years. It was suggested by Prof. BARNES that the writer should complete this investigation, and through the generosity of Mr. R. O. KING, a Fellowship was provided, which enabled him to devote a year almost without interruption to the work.

Several sources of error which appeared in the early use of electro-dynamometers have been eliminated, and it is considered that an absolute accuracy closer than 1 part in 40,000 has been obtained in this investigation. Measurements can be repeated with the instrument to a much higher order of accuracy, but it was estimated that there might possibly be a constant error of 2 or 3 parts in 100,000.

The deflection measurements, the determination of the ratio of the radii of the two sets of coils, and the evaluation of the controlling couple formed the main parts of the investigation. For the convenience of comparison all the observations on each factor are grouped together, and the final calculation for the mean electromotive force of the standard cell is calculated from the mean values for the various factors under known conditions. An illustration of the dynamometer is shown in Plate 1. Descriptive details of the various parts of the instrument appear in the respective sections of the paper.

II. THE THEORY OF THE WEBER ELECTRODYNAMOMETER.

(a) *The Expression for the Magnetic Couple Acting on the Suspended System when Currents are Flowing in the Coils.*

If W is the mutual potential energy of the two systems when currents i_1 and i_2 flow through the fixed and suspended coils respectively, and if ϕ is equal to $\frac{\pi}{2} - \theta$,

* MAXWELL, 'Electricity and Magnetism' (third edition), vol. 2, p. 367.

† CALLENDAR, 'Phil. Trans.,' A, vol. 199, p. 55, 1902.

where θ is the angle of deflection from the position in which the suspended coils are at right angles to the fixed coils, then the couple acting on the suspended coils and tending to increase θ is $\frac{dW}{d\phi}$.

The mutual potential energy, W , is equal to the sum of the four mutual potential energies of each fixed coil with each suspended coil. In fig. 1 a section of the coils is represented for $\phi = 0$. If we call the coils I, II, III, IV, as marked in the figure, then

$$W = W_{(I, III)} + W_{(I, IV)} + W_{(II, III)} + W_{(II, IV)}, \quad (1)$$

where the suffixes indicate the pairs of coils considered.

If we take O as the origin, α, α as the radii of the large and small coils, and d, δ their distances from the origin respectively, then it can be shown that

$$W_{(II, IV)} = i_1 i_2 [G_1 g_1 P_1(\cos \phi) + G_2 g_2 P_2(\cos \phi) + \dots + G_n g_n P_n(\cos \phi) + \dots],$$

where $G_1, G_2, G_3, \dots$ are constants depending on " α ," " d ," and the sectional dimensions of the winding, $g_1, g_2, g_3, \dots$ depending on α, δ (and the sectional dimensions of the winding), and $P_1(\cos \phi), P_2(\cos \phi), P_3(\cos \phi), \dots$ are surface Zonal Harmonics of orders 1, 2, 3, ...

It is obvious from the symmetry of the dynamometer that

$$W_{(II, IV)} = W_{(I, III)}$$

and also

$$W_{(I, IV)} = W_{(II, III)},$$

but in the latter case we must have $\pi + \phi$ for ϕ , and the sign of either i_1 or i_2 must be reversed, hence

$$W_{(I, IV)} = W_{(II, III)} = i_1 i_2 [G_1 g_1 P_1(\cos \phi) - G_2 g_2 P_2(\cos \phi) + G_3 g_3 P_3(\cos \phi) - G_4 g_4 P_4(\cos \phi) + \dots].$$

It follows, therefore, from (1), that

$$W = 4i_1 i_2 [G_1 g_1 P_1(\cos \phi) + G_3 g_3 P_3(\cos \phi) + G_5 g_5 P_5(\cos \phi) + \dots], \quad (2)$$

and hence the required couple $\frac{dW}{d\phi}$ is equal to

$$-4i_1 i_2 \sin \phi [G_1 g_1 P'_1 + G_3 g_3 P'_3 + G_5 g_5 P'_5 + \dots], \quad (3)$$

where

$$P'_n = \frac{d[P_n(\cos \phi)]}{d(\cos \phi)}.$$

We shall require to evaluate this couple in terms of $i_1, i_2, \alpha, d, \alpha, \delta$, and ϕ .

It can be shown that

$$\begin{aligned} P'_1 &= 1; \quad P'_3 = \frac{3}{2}(5 \cos^2 \phi - 1); \quad P'_5 = \frac{15}{8}(21 \cos^4 \phi - 14 \cos^2 \phi + 1); \\ P'_7 &= \frac{1}{16}(3003 \cos^6 \phi - 3465 \cos^4 \phi + 945 \cos^2 \phi - 35); \quad \&c. \end{aligned}$$

If we neglect temporarily the corrections due to the breadth and depth of the windings, it can be shown that the constants G_n and g_n are given by

$$G_n = \frac{2\pi N_1}{(\alpha^2 + d^2)^{\frac{n}{2}}} [\cos \overline{AOC} \cdot P_n(\cos \overline{AOC}) - P_{n-1}(\cos \overline{AOC})]$$

and

$$g_n = 2\pi N_2 (\alpha^2 + \delta^2)^{\frac{n+1}{2}} [\cos \overline{BOD} \cdot P_n(\cos \overline{BOD}) - P_{n+1}(\cos \overline{BOD})],$$

where N_1 and N_2 are the number of turns on each of the large and small coils respectively, and the angles are those shown in fig. 1. It will be seen that

$$\cos \overline{AOC} = \frac{d}{\sqrt{\alpha^2 + d^2}} \text{ and } \cos \overline{BOD} = \frac{\delta}{\sqrt{\alpha^2 + \delta^2}}.$$

We have, by substitution in the above expressions,

$$\begin{aligned} G_1 &= -\frac{2\pi N_1 \alpha^2}{(\alpha^2 + d^2)^{3/2}}, & g_1 &= \pi N_2 \alpha^2, \\ G_3 &= -\frac{4\pi N_1 \alpha^2 \left(d^2 - \frac{\alpha^2}{4} \right)}{(\alpha^2 + d^2)^{7/2}}, & g_3 &= 3\pi N_2 \alpha^2 \left(\delta^2 - \frac{\alpha^2}{4} \right), \\ G_5 &= -\frac{6\pi N_1 \alpha^2 \left(d^4 - \frac{3}{2} \alpha^2 d^2 + \frac{\alpha^4}{8} \right)}{(\alpha^2 + d^2)^{11/2}}, & g_5 &= 5\pi N_2 \alpha^2 \left(\delta^4 - \frac{3}{2} \alpha^2 \delta^2 + \frac{\alpha^4}{8} \right), \\ G_7 &= -\frac{8\pi N_1 \alpha^2 \left(d^6 - \frac{15}{4} \alpha^2 d^4 + \frac{15}{8} \alpha^4 d^2 - \frac{5}{64} \alpha^6 \right)}{(\alpha^2 + d^2)^{15/2}}, & g_7 &= 7\pi N_2 \alpha^2 \left(\delta^6 - \frac{15}{4} \alpha^2 \delta^4 + \frac{15}{8} \alpha^4 \delta^2 - \frac{5}{64} \alpha^6 \right), \\ &\&c., & &\&c. \end{aligned}$$

Substitutions of the values for α , d , α and δ for this instrument showed that $\alpha = 2d$ and $\alpha = 2\delta$ to a degree of approximation such that the term $G_3 g_3 P'_3$ could be neglected as well as all the terms where $n > 7$.

It remains to find the corrections due to the breadth and depth of the windings. (A short method of calculation is given in MAXWELL'S 'Electricity and Magnetism,' 3rd edition, § 700, Vol. II., p. 337). If ξ_1 , η_1 , ξ_2 , η_2 represent the depths and breadths of the large and small coils respectively, it can be shown that we must multiply the value of G_1 recorded above by

$$1 + \frac{1}{24} \left(\frac{2}{\alpha^2} - 15 \frac{d^2}{(\alpha^2 + d^2)^2} \right) \xi_1^2 + \frac{1}{8} \frac{4d^2 - \alpha^2}{(\alpha^2 + d^2)^2} \eta_1^2 + \text{negligible terms},$$

but if $\alpha = 2d$ approximately, this reduces to

$$1 - \frac{1}{60} \frac{\xi_1^2}{\alpha^2}.$$

We may take, therefore,

$$G_1 = -\frac{2\pi\alpha^2 N_1}{(\alpha^2 + d^2)^{3/2}} \left(1 - \frac{1}{60} \frac{\xi_1^2}{\alpha^2}\right)$$

and in a similar manner we have

$$g_1 = \pi N_2 \alpha^2 + \frac{1}{12} \pi N_2 \xi_2^2.$$

In the case of G_5 , G_7 , g_5 , g_7 we may neglect the corrections due to the depth and breadth of the windings.

(b) *The Expression for the Current Flowing through the Suspended Coils.*

Let the ratio of the current i_1 through the fixed coils, to the current i_2 through the suspended coils be r . If we assume that the magnetic azimuth of the planes of the fixed coils is zero, and that the angle which the axis of the suspended coils (in the undeflected position) makes with the planes of the fixed coils is also equal to zero,* then it can be shown that

$$i_2 = \sqrt{\frac{B \tan \theta}{r(-4G_1g_1 - 4G_5g_5P'_5 - 4G_7g_7P'_7)}} \quad \dots \quad (4),$$

where $B \sin \theta$ is the controlling couple due to the bifilar suspension, and $\tan \theta$ is the mean of four values $\tan \theta_1$, $\tan \theta_2$, $\tan \theta_3$, $\tan \theta_4$ where

θ_1	is the deflection when i_1 is + and i_2 is +
θ_2	„ „ „ — „ —
θ_3	„ „ „ + „ —
θ_4	„ „ „ — „ +.

In this way the effect of the earth's magnetic field is eliminated to a sufficient degree of accuracy.

III. THE DIMENSIONS OF THE FIXED COILS.

(a) *The Mean Radius of the Large Coils.*

The radii of the fixed coils had been determined by Mr. R. O. KING. The following summary is taken from Prof. CALLENDAR's account of the measurement:—

“The mean radius of the pair of large coils was determined from the length of the copper tape with which they were wound. This method is not satisfactory with soft

* If the axis of the suspended coils (in the undeflected position) makes an angle Ψ with the planes of the fixed coils, the above expression for i_2 is reduced to $\frac{i_2}{\sqrt{\cos \Psi}}$. The coils could, however, be adjusted by optical methods with sufficient accuracy for us to neglect this factor. The choice of θ_1 , θ_2 , θ_3 , θ_4 eliminates the effect of small magnetic azimuth of the planes of the fixed coils.

annealed copper wire owing to stretching, but the hard rolled copper tape could be wound without any tension, and did not undergo any change of length. This was verified by graduating the tape itself on a 50-foot comparator, the errors of which were known; then winding the coil for trial, and unwinding and measuring the tape again, which was found not to have changed in length by more than a tenth of a millimetre in each 50 feet. The tape was supported horizontally on the polished surface of the comparator, and measured under a tension of 6 kgr. YOUNG'S modulus was determined for each section. The tape was wound on each coil in two lengths of 19 turns each, starting at opposite ends of a diameter, with two thicknesses of paraffined paper between each turn, so that the insulation could be tested with absolute certainty at any time. There were 38 complete turns, and nearly 6000 cm. of tape, on each coil. The probable error of the measurement was less than 1 mm. on the whole length, *i.e.*, less than 1 part in 100,000. The coils could not be boiled in paraffin after winding, as this would subject the tape to uncertain strains owing to the contraction of the wax. It was found necessary to re-wind the coils two or three times with minor improvements before the insulation proved to be perfect. Finally, silk ribbon was adopted in place of paper."

The following is a quotation from the notes of Mr. R. O. KING, containing the summary of his measurements :—

"We have the following data :—

Coil I.

1st winding. Total length = 2970·17 cm.

Corrected for tension, it = 2969·90 cm.*

2nd winding. Total length = 2969·97 cm.

Corrected for tension, it = 2969·69 cm.

There are nineteen double turns so that the mean radius

$$= \frac{2969\cdot90 + 2969\cdot69}{2 \times 19 \times 2\pi} = 24\cdot8767 \text{ cm.}$$

Coil II.

1st winding. Total length = 2972·88 cm.

Corrected for tension, it = 2972·60 cm.

2nd winding. Total length = 2973·20 cm.

Corrected for tension, it = 2972·92 cm.

* These tension corrections reduce the value of the length measured under a tension of 6 kilos. to its value when wound under no tension.

There are nineteen double turns so that the mean radius

$$= \frac{2972.60 + 2972.92}{2 \times 19 \times 2\pi} = 24.9016 \text{ cm.}$$

The mean radius of both coils is therefore 24.8892 cm.

(A rough verification of these measurements was made by measuring the channel before and after with a steel tape.)"

The measurements were taken at an average temperature of about 17° C., hence correcting for the temperature change in the coil frames and tape we have the mean radius

$$a = 24.8905 \text{ cm. at } 20^\circ \text{ C.}$$

The mean depth of the winding ξ_1 was approximately 1.72 cm.

It will be noted that the determination of this radius was made by Mr. KING some time before the present work. The constancy of the dimensions of the coils could, it was thought, be assumed without chance of error, as the coils had not been used at all in the interval and the insulation (silk ribbon in the case of the fixed coils) was not of such a nature that strains could be caused in the winding by temperature or humidity variations in the laboratory. A check on the validity of this assumption can, however, be found in § IV. (c), p. 167. It will be seen from the value of I_2/I_1 that Mr. KING's value for the ratio of the dimensional constants for the two systems of coils differed from the present one by less than 3 parts in 100,000. The construction and the arrangement of insulation in the two systems was so different that this constancy in ratio can be taken as evidence of their individual constancy.

The system of interchanges adopted in the series of observations recorded later made it possible to use the mean value of the radii without the introduction of any further correction on account of their slight difference.

(b) *The Determination of the Distance between the Large Coils.*

The distance between the planes of the large coils, $2d$, was determined by measuring the distance between the external edges at five evenly distributed positions round the coils, and subtracting the mean thickness of the channel-frames at these points from the mean of these measurements. The reversing of the coils during a series of observations eliminated errors due to any asymmetry in the position of the copper tape in its channel. An extension micrometer gauge of special size and shape for adjustment on the dynamometer was constructed by attaching a sensitive screw gauge to one end of a thick brass tube and a suitably shaped end-piece and stud at the other end. The brass part was wrapped in asbestos and a thermometer inserted in the tube. It was found that the variations due to the temperature changes while handling could be followed with sufficient closeness to avoid any possibility of error.

These calipers were calibrated by means of rectangular blocks with sharp edges

which were clamped on to the comparator (see § VI. (c)) at a distance approximately equal to the distance between the fixed coils. Comparisons of the comparator readings with the caliper measurements gave an accurate calibration. As it was found difficult to make the blocks absolutely rectangular the mean of the two measurements with the blocks placed on their opposite faces was taken in every case. The following is a summary of the calibration in which the values are reduced to those corresponding to a setting of the micrometer at 0.8000 inches and a temperature of 25° 0 C. Each value is the mean of three repetitions.

TABLE I.

	Blocks in one position.	Blocks in reversed position.	Mean.
	cm.	cm.	cm.
Using first pair of faces . . . {	27.4106	27.4112	27.4109
	27.4103	27.4124	27.4114
	27.4085	27.4113	27.4099
	27.4100	27.4114	27.4107
Using second pair of faces . . {	27.4088	27.4132	27.4110
	27.4093	27.4131	27.4112
	27.4087	27.4103	27.4095
	27.4083	27.4111	27.4097
			27.4105

A second calibration was made at the end of the investigation to ascertain whether the gauge had been disturbed in any way by usage. This was done by means of a standardized pair of steel calipers. The results are tabulated below. Each figure represents the mean of five settings. Although this method was not capable of the same accuracy as the first, it served as a very satisfactory check.

TABLE II.

Reading of the gauge in inches.	Temperature of the gauge.	Standard caliper reading for the same measurement.	Calculated value of gauge set at 0.8000 inch, at 25° 0 C.
	° C.	cm.	cm.
0.8146	29.0	27.450	27.411 ₀
0.7805	29.0	27.363	27.410 ₅
0.8042	29.1	27.424	27.411 ₆
0.8191	29.0	27.460	27.409 ₆
0.8097	29.1	27.437	27.410 ₄
Mean			27.410 ₆

We may therefore take 27.4105 cm. as the equivalent of a reading 0.8000 at 25°0 C.

A sample set of measurements for determining the distance between the fixed coils is given below. In every case where d is recorded in this paper the value is the mean of a set of observations taken in this way. The bracketed figures give the temperature of the extension calipers.

TABLE III.—Micrometer readings in inches. Dynamometer at 20°0 C.

At 1st point.	At 2nd point.	At 3rd point.	At 4th point.	At 5th point.
0.8074 (25.1)	0.8255 (24.6)	0.8215 (24.2)	0.8125 (23.6)	0.8168 (23.0)
76 (25.0)	45 (25.3)	24 (25.4)	25 (25.4)	72 (25.4)
80 (25.0)	45 (25.0)	22 (24.0)	26 (24.4)	65 (24.8)
80 (24.3)	48 (24.8)	13 (25.3)	22 (25.2)	72 (25.0)
81 (24.3)	57 (24.8)	27 (25.3)	20 (25.2)	75 (25.0)
Mean 0.8078 (24.7)	Mean 0.8250 (24.9)	Mean 0.8220 (24.8)	Mean 0.8124 (24.8)	Mean 0.8170 (24.6)

The mean for the five points is **0.8168 at 24°8 C.**

Now since we know that a reading of 0.8000 inches corresponds to 27.4105 cm. at 25°0 C. it follows that 0.8168 at 24°8 C. corresponds to a length of 27.4530 cm. and the mean thickness of a coil is 2.5556 cm., hence $2d = 24.8974$ cm. and we have $d = 12.4487$ cm. in this case.

Five measurements were taken with micrometer calipers at each of the five points on each of the fixed coils to determine the thickness of the coil frames, and a value of 2.5556 cm. was obtained for the average thickness of a coil frame at the points used for measurement.

All the deflection observations were reduced to the value for

$$d = 12.4448 \text{ cm. at } 20^\circ \text{ C.}$$

IV. THE RATIO OF THE RADII OF THE FIXED AND SUSPENDED COILS.

(a) *The Method.*

The method employed for the determination of the ratio of the radii is similar to that outlined by CALLENDAR, and is a development of BOSSCHA'S method. A thick brass tube carrying a delicately suspended series of magnets and a mirror mounted at its centre was rigidly attached to the framework of the large coils. The small coils could be mounted co-axially on this tube against accurately turned shoulders at a distance of about 30 cm. on either side of the centre of the fixed coils (see Plate 2).

The large and the small coils were arranged in parallel, currents were passed through in opposite directions and adjusted until the balance point for equal magnetic forces at the centre could be determined by interpolation from the observations on the deflections of the small magnetic system. Readings were taken with the small coils in the four possible positions (viz., at either side of the dynamometer and then reversed in each case) for each of the four positions of the fixed coils (as enumerated in the account of the deflection measurements), making sixteen positions in all. In this way the effects of small asymmetries in the coils were eliminated and the mean equivalent value of the constants could be calculated. It is pointed out later that four positions are sufficient to determine the mean to a high degree of approximation. Further data concerning the arrangements, the observations, and the avoidance of temperature variations in the coils under the necessary experimental conditions, are given below. It is first necessary to determine the expression for calculating the constants of the small coils from such measurements.

(b) *The Theory of the Method.*

Fig. 2 represents the two sets of coils arranged co-axially with their centres at a distance x apart, a , d , α and δ have the same significance as before, and at o the

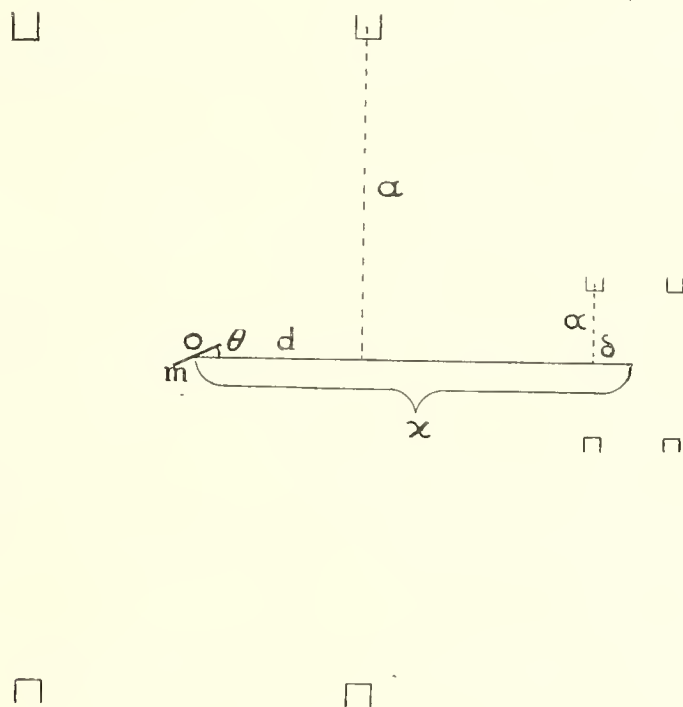


FIG. 2

magnetic system of strength m is represented as being deflected through an angle θ which should of course be in a plane at right angles to the paper. When no current is flowing, and also when a perfect balance is obtained, $\theta = \frac{\pi}{2}$.

Let us first find the couple on the magnets tending to decrease θ , due to a current I_1 in the large coils. If the magnetic system is small, it can be shown that this couple is given by

$$-2I_1 \sin \theta [\gamma_1 G_1 P'_1 + \gamma_3 G_3 P'_3 + \gamma_5 G_5 P'_5 + \dots]$$

where G_1, G_2, G_3 , &c., have the same significance as before;

$$P'_n = \frac{d[P_n(\cos \theta)]}{d(\cos \theta)}, \text{ and } \gamma_1 = M; \quad \gamma_3 = ML^2; \quad \gamma_5 = M\lambda^4 \dots,$$

where M is the magnetic moment of the suspended magnets, L is their "equivalent length," and λ is a length differing slightly from L . Now we have $\theta = \frac{\pi}{2}$ and $G_3 = 0$, hence our required couple can be taken as

$$-2I_1 [\gamma_1 G_1 + \frac{1}{8} \gamma_5 G_5].$$

In the same way it can be shown that the couple due to a current I_2 (in the opposite direction) in the small coils with their centres at a distance x from the centre of the magnetic system, is given by

$$-I_2 [\gamma_1 \Gamma_1 + \frac{1}{8} \gamma_5 \Gamma_5],$$

where Γ_1, Γ_5 , are constants for the two small coils added together. It should be noted that Γ_1 and Γ_5 are not derived in the same way as g_1 and g_5 . In this case, the calculation of γ corresponds to that for g , and the method of calculating Γ is analogous to that for determining $2G$ for the large coils. It can be shown that

$$\Gamma_1 = -2\pi N_2 \alpha^2 \left\{ \frac{A}{\{\alpha^2 + (x-\delta)^2\}^{3/2}} + \frac{B}{\{\alpha^2 + (x+\delta)^2\}^{3/2}} \right\},$$

where

$$A = 1 + \frac{1}{24} \left\{ \frac{2}{\alpha^2} - 15 \frac{(x-\delta)^2}{\{\alpha^2 + (x-\delta)^2\}^2} \right\} \xi^2 + \frac{1}{8} \left\{ \frac{4(x-\delta)^2 - \alpha^2}{\{\alpha^2 + (x-\delta)^2\}^2} \right\} \eta^2,$$

and

$$B = 1 + \frac{1}{24} \left\{ \frac{2}{\alpha^2} - 15 \frac{(x+\delta)^2}{\{\alpha^2 + (x+\delta)^2\}^2} \right\} \xi^2 + \frac{1}{8} \left\{ \frac{4(x+\delta)^2 - \alpha^2}{\{\alpha^2 + (x+\delta)^2\}^2} \right\} \eta^2.$$

When the currents are such that there is no deflection in the magnetometer (that is, when $\theta = \frac{\pi}{2}$) we have

$$2I_1 [\gamma_1 G_1 + \frac{1}{8} \gamma_5 G_5] = I_2 [\gamma_1 \Gamma_1 + \frac{1}{8} \gamma_5 \Gamma_5],$$

and therefore

$$\frac{I_2}{I_1} = \frac{2G_1}{\Gamma_1} \left\{ 1 + \frac{1}{8} \frac{\gamma_5}{\gamma_1} \left(\frac{G_5}{G_1} - \frac{\Gamma_5}{\Gamma_1} \right) \right\}. \quad \dots \dots \dots (5)$$

The magnetic system consisted of fourteen small magnets made from hair springs and each about 1 cm. in length, seven being placed on either side of a mica strip such that the system of magnets occupied a space about 1 cm. broad and 1.5 cm. high. It

The method of determination finally adopted consisted of a set of observations

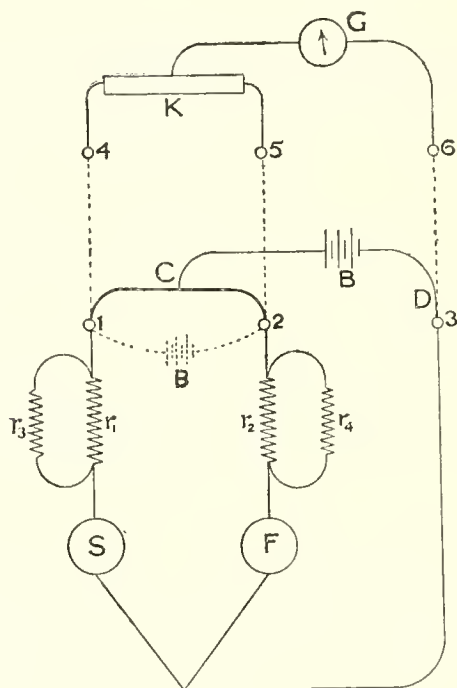


FIG. 3

taken in the following order. It is thought that by this system of measurement we have been able to eliminate or correct for all appreciable extraneous effects.

Order of Procedure.

(i) A current equal in magnitude to that used in the deflection measurements (approximately an eighth of an ampere in each of the four windings, in parallel as in use) is passed through the suspended coils.

(ii) r_2 and r_4 are adjusted until the magnetometer gives only a small deflection on the simultaneous reversal of both currents. This deflection is recorded.

(iii) The magnetometer scale is calibrated in terms of a small variation in the shunt r_4 , and from the subsequent operation (xi), this can be expressed in terms of the Kelvin-Varley slide.

(iv) The connection C is replaced rapidly by the connections shown in dotted lines in the figure. Previous to the observations, the Kelvin-Varley slide has been set at the reading nearest to the correct ratio; hence if the galvanometer circuit be connected immediately after, it will show only a small deflection. The current now flowing through the coils is smaller than in the first connection, and hence there is a slow change corresponding to the cooling of the suspended coils. The change in the fixed coils by themselves was found to be quite negligible. Galvanometer readings are taken 15, 20, 25, and 30 seconds after making the second connection.

(v) The current is reversed and another galvanometer reading taken at 60 seconds; this will be in the opposite direction.

(vi) The current is reversed back to the original direction and the galvanometer reading at 90 seconds is noted.

(vii) The Kelvin-Varley slide is reversed (*i.e.*, the terminals are interchanged) and

FIG. 4

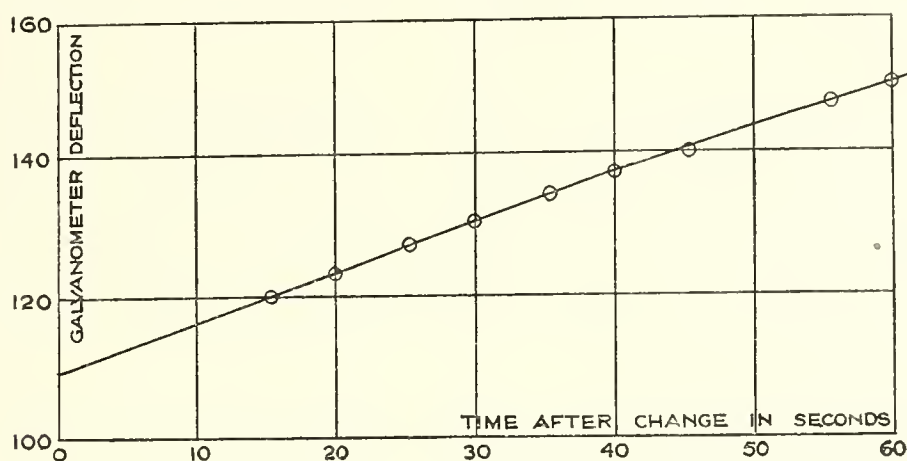
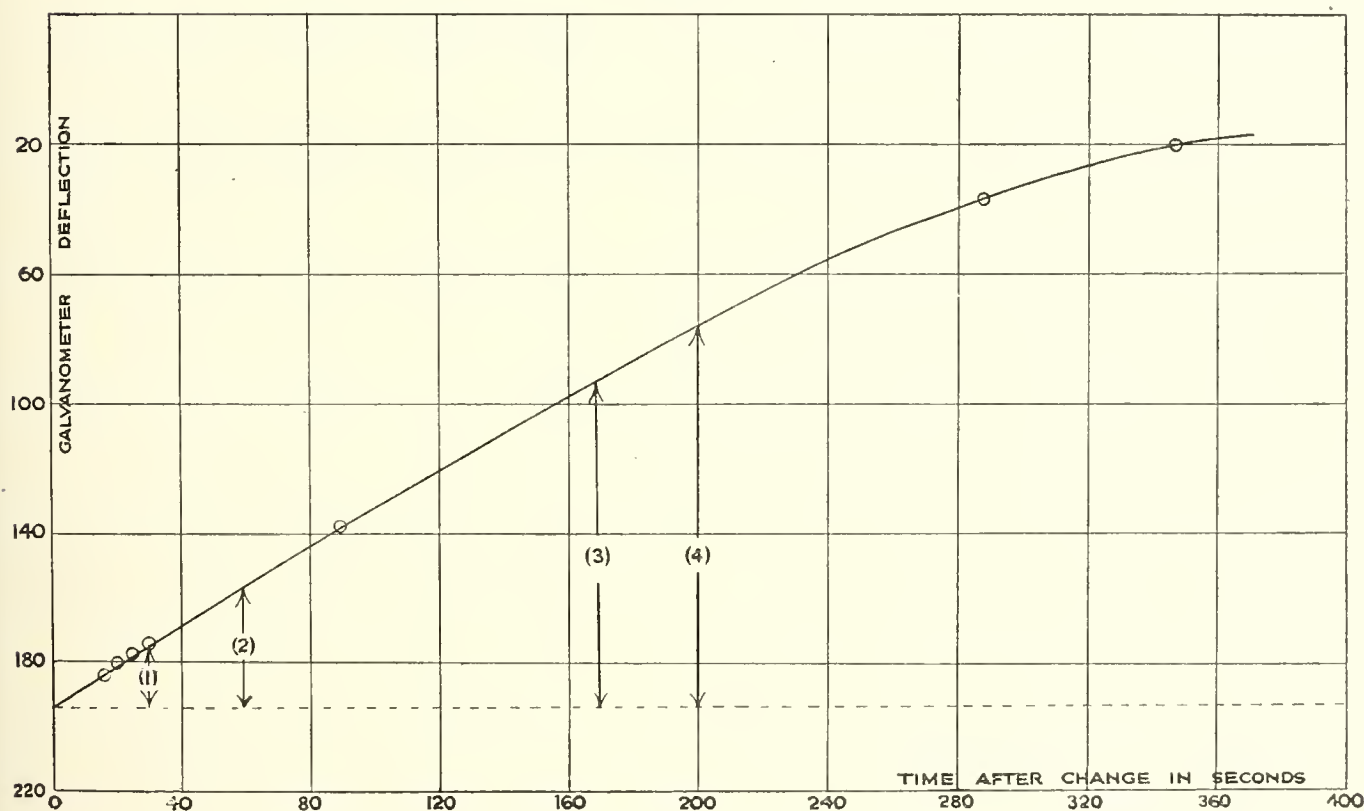


FIG. 5



the new reading and accompanying galvanometer deflection are noted at 210* seconds after the change.

(viii) The current is reversed, and the galvanometer read at 240* seconds.

* These intervals were decreased when proficiency in performing the operations rapidly had been attained.

(ix) The Kelvin-Varley is placed at the initial reading, and the galvanometer read at 330 seconds, and later at 390 seconds again.

(x) The value of a galvanometer scale division is determined in terms of the Kelvin-Varley slide.

(xi) The value of a small change in the shunt r_4 is determined in terms of the Kelvin-Varley slide.

We have thus obtained four measurements of the ratio for a given position of the coils chosen in such a way as to eliminate thermal and contact effects, and if the galvanometer deflections for 15, 20, 25, 30, 90, 330, and 390 seconds (which are all for the same arrangement of the circuit) are plotted, we can find by extrapolation the correction necessary to reduce the Kelvin-Varley readings taken 30, 60, 310, and 240 seconds after the change, to the value for the ratio under the conditions of the operation (ii).

In fig. 4 the type of behaviour at the commencement of the curve is shown, and in fig. 5 a curve is given which is typical of the curves taken with every set of operations. It will be seen how the corrections in column 8 of the table below can be determined with sufficient accuracy from such curves.

Table IV. (p. 165) gives a summary of all the observations on the determination of the ratio of these currents.

The first column gives the position of the coils. These are defined by A, B, C, D, E, F, G, H, I, J, K, L, M, N, O and P:—

In positions of—	Fixed coils are in position—	Suspended coils are placed—		Remarks.
		With marked face to the—	On—side of fixed coils—	
A, B, C, D	II.	—	—	} As defined for the main deflection experiments (see § VII. (e)).
E, F, G, H	I.	—	—	
I, J, K, L	III.	—	—	
M, N, O, P	IV.	—	—	
A, E, I, M	—	West	East	
B, F, J, N	—	„	West	
C, G, K, O	—	East	East	
D, H, L, P	—	„	West	

The second column gives the arrangement of the circuit for the four readings on the Kelvin-Varley slide. (1) corresponds to the reading in operation (iv) above; (2) to that in (v); (3) to that in (vii); and (4) to that in (viii).

The slide reading is given in the third column; the calibration correction has already been applied.

The headings for columns (4) to (12) explain themselves.

TABLE IV.—Observations in the Ratio Determinations.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)
Position of the coils.	Arrangement of the circuit.	Kelvin-Varley slide reading (with calibration correction).	The change in the magnetometer reading for a reversal of the current.	The value of 1 magnetometer scale division in Kelvin-Varley ohms.	The magnetometer correction for the Kelvin-Varley reading.	The galvanometer deflections.	The deflection correction for the cooling of the small coils after the change of circuit.	The value 1 galvanometer scale division in Kelvin-Varley ohms.	The galvanometer correction for the Kelvin-Varley reading.	The corrected Kelvin-Varley reading.	The temperature of the dynamometer, t .	d at the temperature, t .	The correction reducing the Kelvin-Varley reading to that for $d = 12.4448 \{1 + a'(t - 20)\}$.	The equivalent reading for the true balance point.	
{ (A)	(1)	8311.2	+4.2	3.30	+13.8	+28	+20	0.128	+6.2	8331.2	°C.	12.4357	+4.0	8335.2	{ 8334.9
	(2)	8311.2	+4.2		+13.8	+11	+37		+6.2	8331.2	26.8		+4.0	8335.2	
	(3)	91690.6	-4.2		-13.8	+55	-112		-7.3	91669.5			-4.0	8334.5	
	(4)	91690.6	-4.2		-13.8	+67	-125		-7.4	91669.4			-4.0	8334.6	
{ (A)	(1)	8311.2	+3.1	3.14	+9.7	+54	+22	0.124	+9.4	8330.3	26.9	12.4357	+4.0	8334.3	{ 8334.3
	(2)	8311.2	+3.1		+9.7	+38	+42		+9.9	8330.8			+4.0	8334.8	
	(3)	91690.6	-3.1		-9.7	+28	-117		-11.0	91669.9			-4.0	8334.1	
	(4)	91690.6	-3.1		-9.7	+40	-129		-11.0	91669.9			-4.0	8334.1	
{ (A)	(1)	8311.2	+5.1	3.15	+16.0	+4	+20	0.125	+3.0	8330.2	26.5	12.4356	+4.0	8334.2	{ 8334.4
	(2)	8311.2	+5.1		+16.0	9	+39		+3.7	8330.9			+4.0	8334.9	
	(3)	91690.6	-5.1		-16.0	+72	-110		-4.7	91669.9			-4.0	8334.1	
	(4)	91690.6	-5.1		-16.0	+83	-112		-4.9	91669.7			-4.0	8334.3	
{ (A)	(1)	8301.2	+0.9	3.15	+2.8	+169	+21	0.120	+22.8	8326.8	25.4	12.4223	+8.9	8335.7	{ 8335.4
	(2)	8301.2	+0.9		+2.8	+151	+43		+23.2	8327.2	25.4		+8.9	8336.1	
	(3)	91700.6	-0.9		-2.8	-82	-118		-24.0	91673.8			-8.9	8335.1	
	(4)	91700.6	-0.9		-2.8	-67	-130		-23.6	91674.2			-8.9	8334.7	

The following values for the other positions were obtained in the same manner as those in Table IV. :—

(B) 8317·9	(F) 8306·8
(B) 8318·9	(G) 8299·3
(B) 8320·3	(G) 8298·7
(B) 8319·4	(H) 8329·0
(C) 8312·0	(I) 8334·6
(C) 8310·0	(J) 8318·6
(C) 8311·2	(K) 8310·1
(D) 8341·9	(L) 8341·8
(D) 8342·1	(M) 8325·6
(D) 8346·0	(N) 8307·8
(D) 8345·3	(O) 8298·8
(E) 8322·5	(P) 8334·1
(F) 8305·0	

In column (13) the values for d , half the mean distance between the planes of the large coils, are given. Each of these determinations was obtained from a full set of observations similar to those outlined in Section III. (b).

It is necessary to express the ratio of the currents in terms of d where d would equal 12·4448 cm. at 20° C., the figure used for the mean as explained in the section on the main deflection observations. The slide reading must therefore be reduced to its value for $d = 12·4448 \{1 + \alpha'(t-20)\}$ where " α' " is the co-efficient of expansion, taken as 0·000018. It can easily be shown that for small differences, an increase of x per cent. in d would mean a decrease of 0·60 x per cent. in the ratio of the currents, and hence an increase of 0·56 x per cent. in the Kelvin-Varley reading. This correction is given in column (14).

Columns (15) and (16) contain the corrected means. The readings at the upper end of the Kelvin-Varley scale are subtracted from 100,000. (The slide had been calibrated in terms of the exact value of 100,000 for the whole slide. See Appendix.)

The sixteen positions include two equally reliable groups of eight, namely, positions A, F, I, N, C, H, K, P and B, E, J, M, D, G, L, O. It will also be seen that there are a number of suitably chosen groups of four whose means should approximate very closely to the final mean.

TABLE V.—Summary of Mean Values.

The mean value for positions A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, is 8321·0₄.

Position A gives 8334·8	Position B gives 8319·1
" F " 8305·9	" E " 8322·5
" I " 8334·6	" J " 8318·6
" N " 8307·8	" M " 8325·6
" C " 8311·1	" D " 8343·8
" H " 8329·0	" G " 8299·0
" K " 8310·1	" L " 8341·8
" P " 8334·1	" O " 8298·8
Mean = 8320·9	Mean = 8321·1

It will be seen from the table below that the means of the groups AFIN, CHKP, BEJM, DGLO, and also ABEF, IJMN, KLOP, CDGH have an average deviation from 8321.0 of less than 4 parts in 100,000.

TABLE VI.

AFIN	give	8320.8	ABEF	give	8320.6
CHKF	„	8321.1	IJMN	„	8321.6
BEJM	„	8321.4	KLOP	„	8321.2
DGLO	„	8320.8	CDGH	„	8320.8

We have for the mean value of our ratio with the four windings of the small coil in parallel

$$\frac{4I_2}{I_1} = \frac{100000 - 8321.04}{8321.04} = 11.01773,$$

and hence $I_2/I_1 = 2.75443$.

It is important to record that Mr. R. O. KING also made an accurate determination of this ratio, but unfortunately the notes containing his observations on “ d ” have been lost. However, assuming that the average d in his case was the same as that found for the dynamometer as it was set up when the writer first examined it in 1910, we calculated from his notes the mean value, $2I_2/I_1 = 5.5087$ where one suspended coil is in parallel with the other, and hence

$$I_2/I_1 = 2.75435.$$

This figure was the mean for the eight positions which we have called I, J, K, L, M, N, O, P.

The value of I_2/I_1 is independent of the temperature of the dynamometer.* Hence if the ratio is expressed for the value

$$d_t = d_{20}\{1 + a'(t-20)\},$$

where d_t is the length of d at the temperature of observation t , and “ a' ” is the coefficient of expansion, we can use d_{20} , x_{20} , δ_{20} , and α_{20} in our general formula instead of d_t , x_t , δ_t , and α_t .

* The current through the fixed coils produces no perceptible heating (tested by resistance measurements), and that through the suspended coils during the magnetometer readings is the same as in the deflection experiments, hence any error due to the extra heating of the suspended coils is eliminated in the final results.

(d) *The Determination of x , the Mean Distance between the Centres of Symmetry of the Large and the Small Coils.*

The distance x can be most accurately determined by measuring the total distance between the shoulders at each end of the supporting tube, and also the thickness of the frame of the small coils where it fits against the shoulders. The reversing and replacing of the small coils during a complete series of observations for I_2/I_1 eliminated errors due to any asymmetry (in the direction of the axis) in the position of the frame or of the supporting tube with regard to the centres of the two systems of coils.

The following series of twenty readings taken at regular intervals round the shoulders was obtained with large calibrated vernier calipers* :—

TABLE VII.

Distance between shoulders.	Temperature of tube.
cm.	° C.
60·720	24·5
20	
22	
20	
20	
22	
22	
20	
20	
22	
22	
20	
20	
22	
22	
20	
20	
22	24·9
Mean . . . 60·7209	24·7

Hence at 20° C. the distance reduces to **60·7158 cm.**

This number was checked by two similar sets taken at 24°·0 C. and 22°·0 C.

The thickness of the coil frame where it fitted against the shoulders was determined by means of a Sweet Measuring Instrument and standard distance pieces. The following figures are each a mean value of a series of ten observations taken round the parts which fit against the shoulders :—

* The mean of the observations, rather than the maximum, is chosen because this variation depended on the setting of the calipers.

TABLE VIII.

Thickness.	Mean temperature of observation.	Value at 20° C.
cm. 4·9786 4·9789 4·9790	° C. 22·0 24·0 25·3	cm. 4·9784 4·9785 4·9785
Mean		4·9785

We have therefore at 20° C.

$$x = \frac{60\cdot7158 + 4\cdot9785}{2} = 32\cdot8472 \text{ cm.}$$

(e) *The Determination of “ δ ,” the Mean Distance between the Planes of the Small Coils.*

The distance between the planes of the small coils was determined by measuring the distance between the external edges at ten evenly distributed positions round the coils, and subtracting the mean thickness of the channel frames at these points from the mean of these measurements.

The measurement was taken with calibrated vernier calipers.

Mean of first set of measurements.	. . .	7·4261 cm. at 20° C.
„ second „	„ . . .	7·4261 „ „

The mean thickness of the channel frames at the points used was measured with micrometer calipers. Twenty settings were made and the value for the mean thickness of a coil was found to be 1·4426 cm. Therefore, at 20° C.,

$$\delta = \frac{7\cdot4261 - 1\cdot4426}{2} = 2\cdot9917 \text{ cm.}$$

(f) *The Calculation of the Mean Radius of the Small Coils.*

We have from the above

$$x = 32\cdot8472 \text{ at } 20^\circ \text{ C.}$$

$$\delta = 2\cdot9917 \quad ,,$$

and

$$I_2/I_1 = 2\cdot75443 \quad ,,$$

for the mean complete balance. ξ_2 was 0·75 cm. and η_2 was 1·00 cm.

$2G_1 = -13\cdot7268$, see § V.

Substituting these values in expression (7) and solving for α , we get

$$\alpha = 5.96076 \text{ cm. at } 20^\circ \text{ C.,}$$

and hence

$$2g_1 = 2(\pi N_2 \alpha^2 + \frac{1}{12} \pi N_2 \xi_2^2) = 88521.8 \text{ at } 20^\circ \text{ C.}$$

V. THE CALCULATION OF THE DYNAMOMETER CONSTANTS.

We have to evaluate

$$(-4G_1g_1 - 4G_5g_5P'_5 - 4G_7g_7P'_7).$$

The following data are given, elsewhere, in this paper:—

At 20° C. :—

$$\text{The mean radius “} \alpha \text{” of the large coils} = 24.8905 \text{ cm.}$$

$$,, \quad \text{distance “} d \text{” of the large coils} = 12.4448 \quad ,,$$

$$,, \quad \text{channel depth } \xi_1 \text{ of the large coils} = 1.72 \quad ,,$$

$$\text{The number of turns } N_1 \text{ on each large coil} = 38.$$

Hence we find by substitution that

$$2G_1 = -\frac{4\pi N_1 \alpha^2}{(\alpha^2 + d^2)^{3/2}} \left(1 - \frac{1}{6} \frac{\xi_1^2}{\alpha^2}\right) = -13.7268 \text{ at } 20^\circ \text{ C.}$$

$$2G_5 = -\frac{12\pi N_1 \alpha^2 \left(d^4 - \frac{3}{2} \alpha^2 d^2 + \frac{\alpha^4}{8}\right)}{(\alpha^2 + d^2)^{11/2}} = +0.000008242 \text{ at } 20^\circ \text{ C.}$$

$$2G_7 = -\frac{16\pi N_1 \alpha^2 (d^6 - \frac{15}{4} \alpha^2 d^4 + \frac{15}{8} \alpha^4 d^2 - \frac{5}{64} \alpha^6)}{(\alpha^2 + d^2)^{15/2}} = -0.0000000104 \text{ at } 20^\circ \text{ C.}$$

We have also from § IV. (f)

$$2g_1 = 88521.8 \text{ at } 20^\circ \text{ C.,}$$

and since

$$\alpha = 5.96076, \quad \delta = 2.9917 \quad \text{and} \quad N_2 = 396,$$

we have at 20° C.

$$2g_5 = 10\pi N_2 \alpha^2 \left(\delta^4 - \frac{3}{2} \alpha^2 \delta^2 + \frac{\alpha^4}{8}\right) = -106000000,$$

and

$$2g_7 = 14\pi N_2 \alpha^2 \left(\delta^6 - \frac{15}{4} \alpha^2 \delta^4 + \frac{15}{8} \alpha^4 \delta^2 - \frac{5}{64} \alpha^6\right) = +4830000000.$$

If $\theta = 7^\circ 55' 29.1''$ (see § VII. (i)) then $\phi = 82^\circ 4' 30.9''$ and therefore $P'_5 = +1.3902$ and $P'_7 = -1.14$.

Hence substituting these values we get

$$\begin{array}{rcl}
 -4G_1g_1 & = & +1215122 \\
 -4G_5g_5P'_5 & = & + \quad 1215 \\
 -4G_7g_7P'_7 & = & - \quad 57 \\
 \hline
 \text{and the total} & & = +1216280 \\
 \hline
 \end{array}$$

VI. THE BIFILAR SUSPENSIONS AND THE DETERMINATION OF THEIR DIRECTIVE FORCE.

(a) *The Suspensions and the Theory of the Method.*

The bifilar suspensions were made of hard drawn copper wire, diameter 0.0450 cm., length 80 cm., and were rigidly clamped 3.1 cm. apart. They had been hanging for over ten years under the tension of the suspended coils arranged as in actual use, and it was thought that the wires would be exceptionally satisfactory in their behaviour. An account of the influence of elastic changes in the part of the controlling couple, due to torsion, is presented in the section on the deflection measurements. It should be noted here that the examination and treatment of this problem is of fundamental importance to the whole investigation. A full discussion of the matter is given by the writer in a paper on "Increased Accuracy in the Use of Bifilar Suspensions" in the 'Philosophical Magazine' for September, 1911.

The suspended system was placed with its centre coincident with that of the fixed coils, and the wires were adjusted symmetrically for use according to the "method of tilting" described by GRAY.* The position of the coils could be easily checked to within the desired limits of accuracy by simple optical methods. Each wire was finally clamped at both the top and the bottom.

If the controlling couple due to the bifilar suspensions is taken as $B \sin \theta$ for a deflection θ then it can be shown that

$$B = \frac{W a^2}{4(l-x-z)} + \frac{2\mu d^4}{l} \frac{\theta}{\sin \theta} \dots \dots \dots (8)$$

for symmetrical suspensions, where W is the weight of suspended system, " a " is the distance between the bifilars, " l " is their length, " x " is the constant correction for the effect of rigidity at the ends, " z " is the distance through which the suspended system is raised for a deflection θ , " d " is the diameter of the wire, and μ is COULOMB'S torsional coefficient.

Under the conditions of our investigation it was found that an error of less than 5 parts in 1,000,000 in B would be introduced by taking $z = 0$ and $\frac{\theta}{\sin \theta} = 1$. If we

* GRAY, 'Absolute Measurements in Electricity and Magnetism,' vol. 1, p. 242.

take B as the directive force *at the instant of deflection* we may, therefore, assume that B is a constant quantity for values of θ from zero up to the deflections considered in this investigation. As μ for our suspensions decreased rapidly with time after deflecting the system it was always necessary to apply a correction if the deflection was observed an appreciable time after it was produced. The calculation of this correction under the condition of continued reversals of θ is treated fully in the place quoted above.

B can be most accurately calculated from determinations of moments of inertia and times of oscillation of the suspended coils. The following method was adopted. An accurately constructed movable tube 45 cm. in length was placed coaxial with the coils, and into this a pair of similar cylindrical weights could be fitted either at the centre or at the extremities of the tube. It was thus possible to vary the moment of inertia of the suspended system without altering the total mass. A sliding support was inserted in the dynamometer to hold the suspended coils when it was necessary to handle the system.

If T_1 is the time of a complete infinitesimal oscillation (two transits), and K_1 is the moment of inertia with the cylinders at the outside position, and T_2 and K_2 the period and moment of inertia when they are at the internal position, then we have

$$T_1 = 2\pi \sqrt{\frac{K_1}{B}} \quad \text{and} \quad T_2 = 2\pi \sqrt{\frac{K_2}{B}},$$

and hence

$$B = \frac{4\pi^2}{T_1^2 - T_2^2} (K_1 - K_2),$$

and it can easily be shown that

$$K_1 - K_2 = M \left\{ \left(\frac{1}{2}d_1 \right)^2 - \left(\frac{1}{2}d_2 \right)^2 \right\}$$

where d_1 and d_2 are the distances between the centres of mass of the two cylindrical weights and M is the total mass of the two cylinders.

Thus

$$B = \frac{4\pi^2 M \left\{ \left(\frac{1}{2}d_1 \right)^2 - \left(\frac{1}{2}d_2 \right)^2 \right\}}{T_1^2 - T_2^2} \dots \dots \dots (9)$$

It was important that the tube and especially the cylinders should be very carefully constructed in order that the above formula should hold to the desired limits of accuracy. After a trial of several pairs two very satisfactory solid brass cylinders were made, and the series of tests outlined below will indicate that the necessary refinement was obtained.

(b) *The Cylindricity of the Weights.*

Forty diameters of each cylinder were measured with micrometer calipers. Each column in the table below represents a set taken at regular intervals from end to end with the cylinder in each of four equally rotated positions.

TABLE IX.

Diameter of Cylinder I. in Centimetres.			
2·0180	2·0180	2·0182	2·0183
81	80	82	81
80	79	80	81
80	81	81	81
85	83	81	84
85	82	82	85
82	83	82	83
82	81	80	83
84	81	80	85
81	81	80	82
Means . . 2·0182 ₀	2·0181 ₁	2·0181 ₀	2·0183 ₀
Mean diameter of Cylinder I. = 2·0181 ₈ cm.			
Diameter of Cylinder II. in Centimetres.			
2·0180	2·0180	2·0181	2·0180
80	80	80	79
80	80	80	80
79	79	77	75
75	81	83	83
79	80	80	80
79	81	81	81
80	81	82	81
82	84	84	82
80	81	81	80
Means . . 2·0179 ₄	2·0180 ₇	2·0180 ₉	2·0180 ₁
Mean diameter of Cylinder II. = 2·0180 ₃ cm.			

(c) The Linear Measurements.

Before proceeding with the records of results involving linear and time determinations it is perhaps necessary to describe briefly the methods used. Linear measurements were made on a steel Queen Company comparator fitted with high power microscopes. Experiment showed that with care it was possible to transfer the tube from the dynamometer to the comparator without jarring the cylinders apart to any perceptible amount. A Geneva brass metre with inlaid silver scale was used as the standard for comparison. This involved careful attention to temperature, but a calibration of this scale showed such fine marking and accurate sub-division that it was used in preference to two Konstat metres which has been certified at the National Physical Laboratory, Teddington, for the full length, but which were not very accurately sub-divided. A comparison of the brass standard with the Konstat

standard at the certified marks gave, at different temperatures, the following results which are reduced in each case to their value at $19^{\circ}\cdot 0$ C. :—

TABLE X.

						cm.
Absolute length of 100 cm. on standard brass scale at $19^{\circ}\cdot 0$ C. . . .						100·0297
"	"	"	"	"	"	100·0293
"	"	"	"	"	"	100·0297
"	"	"	"	"	"	100·0290
"	"	"	"	"	"	100·0286
"	"	"	"	"	"	100·0294
"	"	"	"	"	"	100·0293
Mean						100·0293

It is necessary to test the assumption that the edges of the cylinders as seen in the comparator microscope lay quite in the plane of their ends. Comparisons were made by placing them end to end and pressing together tightly, then (1) measuring the distance between marks made on each cylinder, at this position; (2) placing the cylinders at the extreme position and measuring the distance between the marks, and (3) measuring the distance between the inner edges at the external position. The difference of the first two readings should equal the latter measurement if the sighted edge is quite at the extremity of the cylinder. Measurements of this kind were made with the weights in various rotated positions in the tube and a very good agreement was found. The variations between the results of the two methods were only of the same order as those obtained for different determinations of the distance between the marks at the inside position when the cylinders were placed end to end. The variation in this case was caused mainly by the difficulty of keeping them quite together without maintaining pressure. As the method with marks involves two measurements it will be seen from the figures below that it is more satisfactory to take the observations by sighting on the edges of the cylinders. The best and worst agreements of the two methods which were obtained were :—

TABLE XI.

Distance by method with marks.	Distance by sighting on edges.
cm.	cm.
(1) 30·2878	30·2877
(2) 30·2889	30·2878

Great care was taken to maintain constancy of temperature during all the linear measurements and in every case the lengths are reduced to their value at $20^{\circ}\cdot 0$ C. in standard centimetres.

The following small variations were obtained for the distances between the edges at the external position, with the cylinders in various rotated positions. The third column is the difference of the two measurements and represents the lengths of the two cylinders :—

TABLE XII.

Position of cylinders.	Distance between inner edges.	Distance between outer edges.	Sum of lengths.
	cm.	cm.	cm.
Initial position	30·2879	44·7850	14·4971
Rotated 45 degrees	79	47	68
„ 90 „	81	53	72
„ 135 „	77	59	82
„ 180 „	65	44	79
„ 225 „	59	42	83
„ 270 „	66	43	77
„ 315 „	76	43	67
Initial position	76	48	72
Means . . .	30·2874	44·7848	14·4974

Each cylinder was reversed in direction, and rotated haphazard into different positions, giving for the distances between the inner edges in the different positions—

TABLE XIII.

cm.	cm.	cm.	cm.
30·2875	30·2868	30·2886	30·2872
30·2875	30·2866	30·2881	30·2869
Mean . . . 30·2873 cm.			

The cylinders now placed end to end at the interior position gave the following values for the distance between the outer edges :—

TABLE XIV.

Position of the cylinders.	Distance between the outer edges.	Cylinders reversed in direction.
	cm.	cm.
Initial position	14·4978	14·4986
Rotated 90 degrees	81	82
„ 180 „	80	73
„ 270 „	77	78
Means . . .	14·4979	14·4980

By maintaining strong pressure it was possible to reduce these values to about 14.4974 as obtained at the outer position. This difference is due to the small distance apart at which the cylinders remained under ordinary circumstances. These observations show that this distance is approximately 0.0005 cm. Direct measurement on the distance between the edges, when no pressure was exerted to keep the cylinders together, gave from 0.0003 to 0.0006 cm.

It was advisable to check the apparent possibility of the influence of these small variations by some observations on T_1 and T_2 with the cylinders in the different positions.

TABLE XV.

Position of the cylinders.	T_1 .
Cylinders in initial external position	11.6566
Reversed in direction	11.6563
Rotated 90 degrees	11.6564
" 180 "	11.6564
" 270 "	11.6565
	T_2 .
Cylinders at the initial internal position	6.5621
Rotated through 90 degrees	6.5620
" " 180 "	6.5622
" " 270 "	6.5619
Reversed in direction	6.5620

(d) *The Time Measurements.*

The observations for T were obtained by recording the transits on an electric chronograph controlled from the McGill Observatory. It was necessary to take the time of a large number of swings in order to get the required accuracy. When the period T is determined in this way it is necessary to apply a correction to the calculated value in order to reduce it to the value T_0 , for infinitely small oscillations. In the experiment we really obtain the average period T' . Now if the amplitude " α " is very small, we have, by a well-known calculation,

$$T = T_0 \left(1 + \frac{\alpha^2}{16} \right),$$

and hence

$$T' = T_0 \left(1 + \frac{\alpha_1^2 + \alpha_2^2 + \dots + \alpha_{2N}^2}{32N} \right),$$

where α_r is the amplitude (*i.e.*, the arc described in $\frac{T}{4}$) for the r^{th} transit, and $2N$ is the number of transits considered.

Now the amplitudes for successive transits are in geometrical progression such that $\alpha_1/\alpha_2 = e^{(n-1)\lambda}$ where λ is the logarithmic decrement; hence we have

$$T_0 = T' \left\{ 1 - \frac{\alpha_1^2}{32N} \left(\frac{1 - e^{-4N\lambda}}{1 - e^{-2\lambda}} \right) \right\},$$

and therefore approximately

$$T_0 = T' \left\{ 1 - \frac{\alpha_1^2 - \alpha_{2N}^2}{64N\lambda} \right\}.$$

If α_n is the distance between extreme scale readings for the n^{th} period or $2n^{\text{th}}$ transit and D is the distance from the axis of rotation to the scale we have $\tan 2\alpha = a/2D$ or approximately, $\alpha = a/4D$ and therefore

$$T_0 = T' \left\{ 1 - \frac{\alpha_1^2 - \alpha_{2N}^2}{1024N\lambda D^2} \right\} \dots \dots \dots (10)$$

If the correction due to the resistance of the air had not been neglected there would have been a further factor, $(1-b)$. It can be shown that $b < (k\lambda)^2$ where k is a constant less than unity, and since λ^2 is quite negligible in our case we need not consider b .

Since the value of the correction in (10) was usually less than 2 parts in 100,000, the various figures recorded below for T_1 and T_2 have already been corrected when necessary.

In the case when the cylindrical weights were in the exterior position it was found experimentally that $\lambda = 0.00090$. It was convenient to start observations with an oscillation of about 5 cm. on the scale. If the observations extended over 1500 periods the correction for T would be less than 1 part in 1,000,000. Starting at 5 cm. it took over 3000 periods before the oscillations were too small to be observed. When the weights were at the internal position $\lambda = 0.0016$, and if desired it was again possible conveniently to choose α_1 and N so that the correction for T would be negligible.

The elastic fatigue produced by the continuous torsional oscillations was not of a sufficient magnitude appreciably to affect these observations in any way.

The transits were observed through a telescope and recorded electrically on the chronograph with the aid of a tapping key. With a little practice the time between transits could be repeated to within a tenth of a second, but the period as deduced from a full set of oscillations could be repeated to within 1 part in 100,000.

(e) *The Masses of the Cylinders.*

The weighings were made on a Troemner balance sensitive to 0.0001 gr. with the given mass of the cylinders, and using a set of standardized weights with corrections obtained at the Bureau of Standards in Washington. The readings were taken by the usual method of oscillations, and repeated with the weights and cylinders

interchanged. The following mean values for the two cylinders were obtained and repeated without a variation of more than three in the last figure:—

Mass of cylinder I.	196·1797 gr.
„ „ II.	196·1711 „
	392·3508 „

(f) *Further Remarks on the Accuracy of the Method.*

An examination of the possible effects of small asymmetries appears to show conclusively that no error as large as 1 part in 100,000 could occur in these measurements for the determination of B. Let us consider—

(1) The effect produced by a small deviation in the position of the tube and cylinders as a whole, which might occur initially or vary during readjustment. (It must be noted, however, that this is quite a different matter from a variation of the cylinder with respect to the tube. In the final observations the tube was never readjusted between determinations for a “T₁” and a “T₂” which were to be taken together.)

(2) The effect produced by the deviation of the centre of mass of the two cylinders, in either the internal or external position, from the desired position on the axis of rotation of the suspended system. This effect might be produced by the effect mentioned in (1) and also by a variation of the position of the cylinder with respect to the tube.

(3) The effect produced by the slight difference in weight.

Suppose (a) that the centre of the tube is at a small distance x from the axis of the rotation; (b) that the centre of mass of the cylinders, at the external position, is at a small distance y_1 from *the centre of the tube*, and, at the internal position, at a small distance y_2 ; (c) that $m_1 - m_2$ is not zero, where m_1 is the weight of one cylinder and m_2 that of the other.

Let

$$x + y_1 = z_1 \quad \text{and} \quad x + y_2 = z_2,$$

then it can be shown that

$$K_1 - K_2 = M \left\{ \left(\frac{d_1}{2} \right)^2 - \left(\frac{d_2}{2} \right)^2 \right\} + M (z_1^2 - z_2^2) + (m_1 - m_2) (z_1 d_1 - z_2 d_2),$$

provided that we take z as lying in the direction of the axis of the tube. The possible displacements in other directions would be far less. Now d_1 and d_2 , involving y_1 and y_2 , could be reset to within a few thousandths of a millimetre; the centre of mass of the two cylinders could be reset to within a few hundredths of a millimetre; and $m_1 - m_2$ was 0·0086 gr. Substitution in the above formula shows that $K_1 - K_2$

would be changed by less than 5 parts in 1,000,000 through neglecting the last two terms, if we assumed, for example, that x was as much as half a millimetre, y_1 a twentieth of a millimetre, and y_2 a tenth of a millimetre.

It is thus quite apparent, considering this discussion in conjunction with the experimental tests on the variations of T for resettings of the cylinders (both in rotated and reversed positions), that B can be determined satisfactorily in this way.

During the deflection experiments the suspensions carried a current of half an ampere. The following observations on the period illustrate the constancy under these conditions.

The figures below marked with an asterisk represent the mean of the two values obtained when the current flowed through the suspended system, first in one direction and then in the other.

TABLE XVI.

	Period.
	seconds.
I. No current flowing	11·6672
Current of half an ampere	*11·6669
No current flowing	11·6669
Current of half an ampere	*11·6669
II. (Slight change in weight of system.)	
No current flowing	11·6571
Current of half an ampere	*11·6574
No current flowing	11·6575
Current of half an ampere	*11·6573
III. May 5	11·6566
„ 28	11·6564
June 3	11·6564
„ 13	11·6564
IV. (Moment of inertia changed.)	
August 2	6·5659
November 14	6·5652

The temperature of the dynamometer was about 20° C. during all recorded observations for T , but the clamping connections at the supports were of such a nature that B was nearly compensated for temperature changes.

(g) *The Value of B during the Deflection Observations.*

Observations for T_1 , T_2 , d_1 , and d_2 , were taken immediately before and after the set of deflection readings for the current measurement. As the repetition of the various sets of deflections took several weeks, it was necessary to test the constancy during these observations. This was done by several determinations of T_1 at a temperature of 20° C.

TABLE XVII.

Mean value of T_1 before taking deflection set on May 28	seconds 11·6564
„ „ during „ „ „ June 3	11·6564
„ „ „ „ „ June 13	11·6564
„ „ after „ „ „ June 28	11·6564
Mean value of T_1	11·6564 ₀
Mean value of T_2 before taking deflection set	6·5620
„ „ after „ „ „	6·5621
Mean value of T_2	6·5620 ₅

The slight variation in T_2 is probably due to a variation in the very small distance which the cylinders remain apart, at the inside position. This distance was approximately 0·0004 cm. at the time of the final observations on d_2 . The cylinders after careful polishing had been placed in positions giving values of d_1 and d_2 most closely corresponding to the mean of those recorded in Tables XII., XIII., and XIV.

The distance between the inner edges at the external position
at 20° C. = 30·2874 cm.

The distance between the outer edges at the external position = 44·7848 cm.

Hence

$$d_1 = \frac{1}{2}(30·2874 + 44·7848) = 37·5361 \text{ cm.}$$

The distance between the inner edges at the internal position = 0·0004 cm.

The distance between the outer edges at the internal position = 14·4978 cm.

Hence

$$d_2 = \frac{1}{2}(14·4978 + 0·0004) = 7·2491 \text{ cm.}$$

Substituting for d_1 , d_2 , T_1 , T_2 , and M in formula (9) we have

$$B = \frac{4\pi^2 \left\{ \left(\frac{37·5361}{2} \right)^2 - \left(\frac{7·2491}{2} \right)^2 \right\} 392·351}{11·6564^2 - 6·56205^2}$$

$$= 56593·2.$$

This is the value of B during the deflection observations, and it is important to note again that it represents the directive force of the suspensions only at the instant of the initial deflection after a long period of rest.

VII. THE DEFLECTION OBSERVATIONS.

(a) Method of Observation.

The duplex system which had been installed by Prof. CALLENDAR was used for the observation of the deflections.* The suspended coils were fitted with a plane parallel mirror 0.3584 cm. thick, 5 cm. in diameter, and silvered on both sides. Two metre scales, accurately divided on plane milk-glass, were suitably mounted on a rigid frame of copper pipe at distances of approximately a metre and a half east and west of the mirrors respectively. The parts of the scales where the readings occurred were illuminated by strong reflectors, and the rest was wrapped in asbestos to preserve an even temperature. A pair of two-foot telescopes with two-inch apertures were mounted on the same frame immediately below and at right angles to the centres of the scales. Both the mirrors and the telescopes had been specially constructed by BRASHEAR for this work. The deflections were taken simultaneously through each of the telescopes which were provided with filar micrometers. It was found, however, that after some practice, readings could be estimated through the telescope directly to within three hundredths of a millimetre. (As the time for taking a reading was limited, this plan of estimation was adopted and proved to be of quite sufficient accuracy for the mean results.) A constant circulation of water was maintained in the supporting pipes, and the distances between the scales could be measured with great accuracy. The dynamometer was protected by non-magnetic covers which were provided with mica windows.

In these observations the value of $\tan \theta$ was deduced from readings taken first in one direction and then in the other. It is not necessary to introduce the zero positions; for if we let S equal the mean of the values obtained for the distances between the readings for opposite deflections, and if β is the average distance between the scales and m the thickness of the mirror, then

$$\tan 2\theta = \frac{S}{\beta - m},$$

and $\tan \theta$ can at once be deduced. This duplex method of reading eliminates small errors in the setting of the telescopes and scales.

Separate series of observations were taken with the fixed coils both *reversed* and *interchanged* in position. This eliminated errors introduced by any asymmetry in the position of the copper tape within its channel.

(b) The Correction for Torsional Variations in the Controlling Couple.

Preliminary observations showed slight variations which depended on the time elapsing between the reversing of the deflections and the taking of a reading. There

* CALLENDAR, *loc. cit.*

was a gradual increase of the deflection which reached a limiting value according to an expression of the type

$$y = a + b \log (1 + ct),$$

where y represents the distance from an arbitrary origin at the time t after producing a given deflection and a , b , and c are constants. Deflections were observed only when the temperature of the whole circuit was quite constant, and it was found that the variation was due to a decrease in the controlling couple while the system was maintained in a deflected position. As the measurement of this couple depends on observing small oscillations of the suspended system, it is necessary to obtain the instantaneous value for a deflection reading. An investigation was undertaken by the writer for the purpose of examining this effect in bifilar suspensions, and the results, which have special application to the present work, have been published in the article already quoted. It is shown there how the behaviour of these suspensions may be subjected to accurate calculation. The direction of the deflection was reversed every five minutes throughout all the series of final observations and the readings were made in each case three and a half minutes after producing a reversal. All auxiliary observations were performed between each reversal and it was planned that any desired series of readings could be performed continuously in this way. A manipulation of the reversing commutators made it possible to bring the suspended system to rest in a few seconds. The object of this method was to maintain the wires in a known elastic state which could be referred to the initial one. The *problem* thus arose to find the equation representing the variation of the deflection after its direction had been reversed every T minutes for a comparatively long period, and to express this with reference to the instantaneous position of the first deflection. The general solution to this was shown to be

$$x = b \log (1 + ct + nT) - 2b \log (1 + ct + (n-1)T) \\ + 2b \log (1 + ct + (n-2)T) - \dots + (-1)^n 2b \log (1 + ct),$$

where x is the distance from the initial instantaneous deflection, t is the time after the last reversal, n is the number of previous reversals, and b and c are constants. The application of this formula is not practicable, but when $T < 15$ and n very large, it was found that a much simpler formula could be deduced which was tested experimentally and found to be perfectly satisfactory. The formula obtained was

$$x = D - D \frac{2}{1 + e^{-kT}} e^{-kt} \dots \dots \dots (11)$$

where D was shown to be the distance of the asymptotic limit of the creep from the position of the instantaneous deflection and k a constant depending on the rigidity of the suspensions and the dimensions of the system. D depends on the magnitude of the deflection. The formula was tested for different values of T , and curves were

given showing a splendid agreement between the calculated and observed results for x . It was thus possible to calculate accurately the correction for readings, but it was not always practicable to wait until the wires had made a complete recovery from any long period of distortion. Any further superimposed effects were eliminated, therefore, by (1) reversing several times at intervals of five minutes; (2) taking readings at the usual interval after the last reversal; (3) reversing and reading again at the right intervals; and (4) reversing and reading for the first position again. The mean of the first and third readings were taken with the second, and the mean of these for both scales gave a mean value of S , the double deflection on the scales, which could now be corrected according to the formula (11).

(c) *The Electrical Connections.*

Fig. 6 shows diagrammatically the arrangement of the circuit for the deflection experiments.

AB represents two ohm standards in series (2.00368 international ohms, see § VIII.) and AD represents two-tenth ohms standards in series (0.199625 international ohms, see § IX.); S and F represent the suspended and the fixed coils respectively; r_1, r_2, r_3, r_4, r_5 , are suitable non-inductive resistances and shunts capable of very delicate

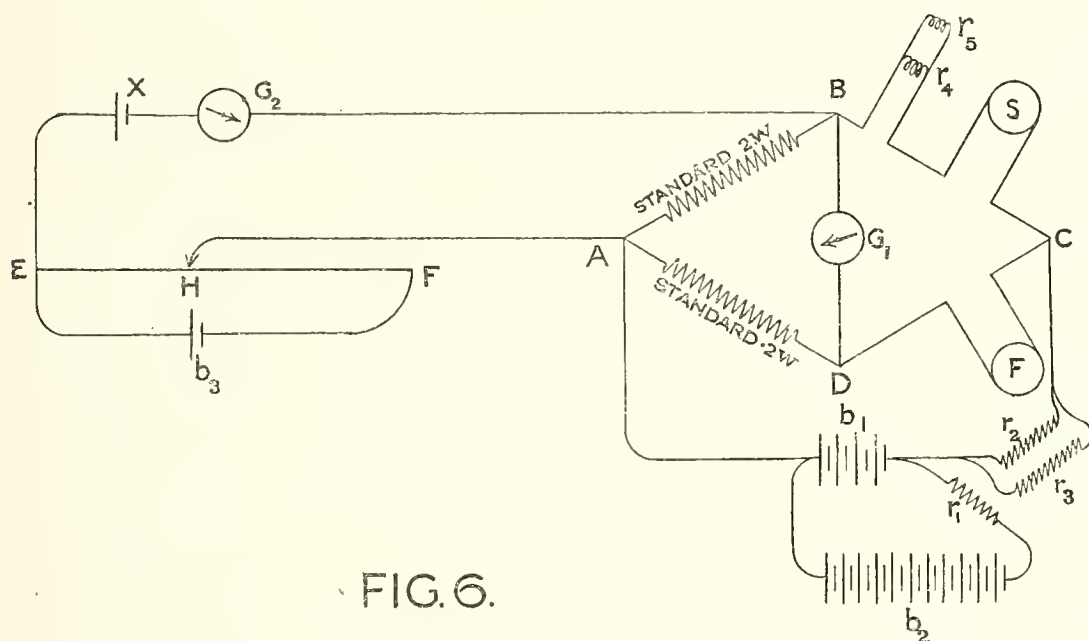


FIG. 6.

adjustment; G_1 is a sensitive low resistance galvanometer, and G_2 a sensitive high resistance galvanometer; EF is the potentiometer wire; b_1, b_2, b_3 , are batteries; and X is the standard cell. Suitable commutators, not shown in the diagram, are arranged so that the current may be reversed (1) between A and C ; (2) through the suspended coils; (3) through the fixed coils; (4) through G_1 ; (5) through G_2 , (6) through EF .

The battery b_1 was composed of sixteen storage cells (each forty-ampere hour capacity) arranged in parallel four by four, to give an electromotive force of between

eight and nine volts. b_2 represents a large main battery of 100 volts. By proper adjustment of r_1 , it was possible to supply a current of between five and six amperes for several hours with a constancy of better than 1 part in 100,000. It was arranged that b_2 supplied the larger part of the current, while b_1 served as a means of maintaining a constant voltage.

While taking deflection measurements r_4 and r_5 were kept adjusted so that there was only a small deflection in the galvanometer G_1 . A change of 1 part in 100,000 was represented by seven scale divisions; r_2 and r_3 were kept adjusted so that the deflection in G_2 would be small, and this deflection could be eliminated and interpreted by the proper choice of the point H (see below).

It was arranged that there was a drop of a millionth of a volt per millimetre along EF; one scale division of G_2 corresponded to one millimetre of the wire.

(d) *The Potentiometer Correction.*

A complete record of the deflection measurements for this investigation is given in Table XVIII. The electrical connections are explained in detail above, and it will be seen that the current which passed through the suspended coils also passed through a standard resistance in an oil bath. It was arranged that the drop in potential across this differed from the electromotive force of a normal Weston cell at 25° C. by only a small number of microvolts. When opposing these two potential differences and measuring their difference on an ordinary potentiometer with a sensitive galvanometer in the circuit, it was possible to adjust the current to a constant value. It was not, however, necessary always to adjust the current in order to obtain a given reading, for if the difference on the potentiometer was read, the value of the current could be easily expressed in terms of that current which would produce a zero difference between the E.M.F. of the standard cell and the drop in potential across the resistance. Observations of the deflection were taken simultaneously with a potentiometer reading which never exceeded 700 microvolts and averaged about 250 microvolts. As this variation is so small and was taken to within five microvolts of the true balance, it was always possible to apply to the deflection a correction which could not be in error by as much as 0.001 cm. on the scale. (For example if the drop in potential across the standard resistance was 100 microvolts greater than the electromotive force of the Weston cell, then the current would have to be cut down by only 1 part in 10,180 to get the value which would produce the same drop in potential as the cell. As the circuits connecting the fixed and suspended coils of the dynamometer were in parallel and as the deflection varies with the product of the two currents which are kept in a fixed ratio we see that the deflection must be reduced by 2 parts in 10,180 or, in our case, where the readings lie between 42.3 and 42.4 cm. the correction would be -0.0083 . We could thus determine this more closely than we could read the deflection and were able to save considerable time and much

trouble in regard to keeping the currents absolutely constant. It should be noted that the potentiometer corrections to the scale readings can be at once obtained from column four in the tables by dividing by 12'000.)

(e) *The Deflection Tables.*

In the first column an indication is given of the arrangement of the circuit. The different positions of the fixed coils are referred to as Positions I., II., III., and IV. where

In Position I. we have		{ Coil I. on west side of dynamometer with marked face out.							
			„ II. „ east	„	„	„	„	„	„
„	II.	„	{ „ I. „ west „ „ „ „ in.						
			„ II. „ east	„	„	„	„	„	„
„	III.	„	{ „ I. „ „ „ „ „ „ „						
			„ II. „ west	„	„	„	„	„	„
„	IV.	„	{ „ I. „ east „ „ „ „ out.						
			„ II. „ west	„	„	„	„	„	„

A+ and A− corresponds to a reversal of the current through the whole circuit. (The A represents the point so marked in fig. 6.) Changes from (1) to (3) or from (2) to (4) correspond to a reversal of the current through the fixed coils only, while a change from (1) and (3) readings to (2) and (4) means a reversal of the current through the suspended coils only if the current has not been reversed at A also. Observations (1) and (4) are always at the zero end of the scales, and (2) and (3) are always at the “hundred” end. In the reading marked A+(1), A+(3), A−(2), A−(4), the earth's magnetic field acts to decrease S, while in A−(1), A−(3), A+(2), and A+(4) it acts to increase S. Wherever two consecutive readings (between groups) are not in opposite directions, it is due to the omission of a whole five-minute period in which it had not been possible to get all the auxiliary observations performed in time.

In the second and third columns, the uncorrected telescope readings are given, and the fourth contains the potentiometer readings, a + value meaning that the drop in potential across the standard resistance was greater than the E.M.F. of the cell by the amount given.

The fifth column contains the various temperatures t_1 of the standard two ohm coils at the time of the corresponding deflection; and t_2 represents the temperature of the two-tenths standard resistance in the circuit of the fixed coils.

The value of half the average distance between the centres of the fixed coils “ d ,” is recorded in the next column. (See § III. (b) for details of measurement), and the distance between the scales, β in the eighth column (see § VII. (h)).

TABLE XVIII.—The Deflection Tables.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)
Arrangement of circuit.	East telescope reading.	West telescope reading.	Potentiometer reading.	t_1 .	t_2 .	d .	β .	Mean of east and west telescope readings reduced to values for zero potentiometer readings.	S .	Correction reducing S to value for $t_1 = 23^{\circ}.12$ C.	Correction reducing S to value for $t_2 = 28^{\circ}.0$ C.	Correction reducing S to value for $d = 12.4448$ cm.	Correction reducing S to value for $\beta = 298.450$ cm.	S reduced to standard conditions.	Mean S .
Position I. $\left\{ \begin{array}{l} A + (1) \\ (3) \\ (1) \end{array} \right.$	7.810 92.215 7.840	7.812 92.258 7.830	+ 110 + 60 - 10	$^{\circ}$ C. 23.62 23.67 23.72	$^{\circ}$ C. 27.8	cm. 12.4487	cm. 298.449	7.820 92.232 7.834	84.405	+ 0.012	+ 0.000	+ 0.016	+ 0.000	84.433	84.708
Position I. $\left\{ \begin{array}{l} A + (3) \\ (1) \\ (3) \end{array} \right.$	92.215 7.860 92.195	92.250 7.855 92.238	- 40 - 230 - 370	23.77 23.77 23.77	27.8	12.4487	298.449	92.236 7.838 92.247	84.404	+ 0.014	+ 0.000	+ 0.016	+ 0.000	84.434	
Position I. $\left\{ \begin{array}{l} A + (2) \\ (4) \\ (2) \end{array} \right.$	92.429 7.485 92.420	92.480 7.482 92.468	+ 110 + 120 - 10	23.77 23.77 23.77	27.8	12.4487	298.449	92.445 7.494 92.445	84.951	+ 0.014	+ 0.000	+ 0.016	+ 0.000	84.981	
Position I. $\left\{ \begin{array}{l} A - (1) \\ (3) \\ (1) \end{array} \right.$	7.515 92.427 7.525	7.508 92.477 7.523	- 80 - 70 - 180	23.82 23.82 23.82	27.8	12.4487	298.499	7.505 92.458 7.509	84.951	+ 0.015	+ 0.000	+ 0.016	+ 0.000	84.982	84.705
Position I. $\left\{ \begin{array}{l} A - (2) \\ (4) \\ (2) \end{array} \right.$	92.202 7.885 92.205	92.245 7.878 92.248	- 250 - 320 - 410	23.82 23.87 23.87	27.8 28.3	12.4487	298.449	92.245 7.855 92.261	84.398	+ 0.016	+ 0.000	+ 0.016	+ 0.000	84.428	

The following values for columns 15 and 16 were obtained in the same manner as those in Table XVIII. :—

(Column 15.) S.	(Column 16.) Mean S.
Position II. $\left\{ \begin{array}{l} 84.844 \\ 84.327 \end{array} \right\}$	84.586
$\left\{ \begin{array}{l} 84.848 \\ 84.327 \end{array} \right\}$	84.587
$\left\{ \begin{array}{l} 84.329 \\ 84.844 \end{array} \right\}$	84.586
Position III. $\left\{ \begin{array}{l} 84.343 \\ 84.874 \end{array} \right\}$	84.608
$\left\{ \begin{array}{l} 84.863 \\ 84.340 \end{array} \right\}$	84.602
$\left\{ \begin{array}{l} 84.346 \\ 84.875 \end{array} \right\}$	84.610
Position IV. $\left\{ \begin{array}{l} 84.986 \\ 84.454 \end{array} \right\}$	84.720
$\left\{ \begin{array}{l} 84.456 \\ 84.997 \end{array} \right\}$	84.726
$\left\{ \begin{array}{l} 84.984 \\ 84.459 \end{array} \right\}$	84.722
Position I. $\left\{ \begin{array}{l} 84.963 \\ 84.441 \end{array} \right\}$	84.702
$\left\{ \begin{array}{l} 84.457 \\ 84.960 \end{array} \right\}$	84.708
$\left\{ \begin{array}{l} 84.981 \\ 84.452 \end{array} \right\}$	84.716
$\left\{ \begin{array}{l} 84.449 \\ 84.980 \end{array} \right\}$	84.714
$\left\{ \begin{array}{l} 84.970 \\ 84.436 \end{array} \right\}$	84.703
Position II. $\left\{ \begin{array}{l} 84.323 \\ 84.850 \end{array} \right\}$	84.586
$\left\{ \begin{array}{l} 84.869 \\ 84.320 \end{array} \right\}$	84.594
$\left\{ \begin{array}{l} 84.323 \\ 84.861 \end{array} \right\}$	84.592

(Column 15.) S.	(Column 16.) Mean S.
Position III. $\left\{ \begin{array}{l} 84.866 \\ 84.349 \end{array} \right\}$	84.608
$\left\{ \begin{array}{l} 84.356 \\ 84.861 \end{array} \right\}$	84.608
Position IV. $\left\{ \begin{array}{l} 84.450 \\ 84.988 \end{array} \right\}$	84.720
$\left\{ \begin{array}{l} 84.981 \\ 84.448 \end{array} \right\}$	84.714

The figures in the ninth column are obtained by applying the potentiometer correction to the mean of the two telescope readings; and the next column contains S which is obtained in the manner outlined above, by taking the difference of the middle reading from the mean of the first and last readings in each group.

To facilitate comparison and to illustrate the accuracy in repetition of readings, the next four columns contain corrections for reducing S to its corresponding value for the following standard conditions $t_1 = 23^{\circ}.12$ C., $t_2 = 28^{\circ}.0$ C., $d = 12.4448$ cm., and $\beta = 298.450$ cm., these numbers being chosen because they differed but slightly from the average values and agreed with those occurring in the various auxiliary calibrations or standardizations. The fifteenth column gives S reduced to the value corresponding to these standard conditions, and the last column contains the figures for the mean S, from which the effect of H, the earth's field, is eliminated.

The temperature coefficient of 0.00025 is taken for the standard two ohms in calculating the correction due to t_1 , and 0.000018 is used for the two-tenths in correcting for t_2 . It is sufficiently accurate to assume that S is changed by the same percentage as the resistance in each case, but in the opposite direction.

To obtain the correction due to d , it can easily be shown that for small differences a change of X per cent. in d means a change of 0.60 X per cent. in S in the opposite direction.

A change in β means a proportional change in S.

A summary of the value for the mean S for each of the different positions of the fixed coils is given below. The letters (a), (b), and (c), &c., represent a readjustment of the dynamometer and circuit. This involves a resetting of the fixed coils and general connections as well as the minor variations of condition recorded in the previous tables. As each position involves a resetting it will be seen that there are thus nine readjustments of the general conditions of the experiment, and it should be noted that the final mean involves 290 deflection readings. The controlling force of the suspensions and the position of the suspended coils were constant throughout the measurements.

TABLE XIX.

Position I.	Position II.	Position III.	Position IV.	Mean S.
(a) 84·708 ·705	(b) 84·586 ·587	(c) 84·608 ·602	(d) 84·720 ·726	84·656 84·655
(b) 84·702 ·708	} ·586	·610	·722	84·656
(i) 84·716 ·714 ·703	(f) 84·586 ·594 ·592	(g) 84·608 ·608 —	(h) 84·720 ·714 —	84·657 84·657 84·656*
84·708 ₀	84·588 ₅	84·607 ₂	84·720 ₄	84·656 ₀

(f) The Evaluation of Corrections.

It is necessary to apply several further corrections to this value of S.

The following table contains data omitted above:—

TABLE XX.

The positions in the order given in Table XVIII.	Average temperature of the scales.	Average temperature of the dynamometer.
Position I.	° C. 20·3	° C. 21·5
„ II.	19·5	20·9
„ III.	20·0	20·3
„ IV.	20·8	20·6
„ I.	20·7	20·7
„ I.	20·2	20·8
„ II.	20·5	20·0
„ III.	20·2	20·2
„ IV.	20·4	20·9
Means	20·3	20·7

(1) *Temperature Correction.*—The average dynamometer temperature was 20°·7 C. If d were kept at 12·4448 cm. and the temperature reduced to 20°·0 C. the radius would be decreased 0·0013 per cent. and a single deflection would be increased, therefore, by 0·005 per cent. which would amount to a correction of +0·0008 cm. in S.

(2) *Calibration for the Scales.*—Each scale was compared with the standard scale described in § VI. (c) at the actual marks where deflection observations were taken.

* The mean values for III. and IV. are used in calculating this figure.

TABLE XXI.

Measurement { between	7.5 and 92.0.	7.6 and 92.1.	7.7 and 92.2.	7.8 and 92.3.	7.9 and 92.4.	8.0 and 92.5.	Mean.	Tempera- ture of scale.
East scale . . .	84.5042	84.5036	84.5015	84.5034	84.5028	84.5039	84.5032	° C. 20.8
West scale . . .	84.5001	84.5002	84.5000	84.5007	84.5003	84.4999	84.5002	20.6
Mean							84.5017	20.7

The average temperature of the scales during the deflection observations was 20°·3 C. (see Table XX.). Hence using the temperature coefficient 0.0000085 for milk-glass we see that the mean value for this distance at 20°·3 C. is 84.5014 cm. and a correction of +0.0014 cm. should therefore be added to our deflection mean in order to express it in true centimetres.

(3) *Correction for the Mica Windows.*—A very small displacement of the reflected scale reading is produced by the presence of the thin mica windows through which the suspended mirror is viewed. These windows are parallel to the scales and the telescope is at right angles to each, hence it can be shown that telescope readings for deflection measurements are each decreased by an amount x given by

$$x = z (\tan 2\theta - \tan \phi),$$

where z is the thickness of the mica, θ is the angle through which the suspended coils and mirror are deflected, and ϕ is the angle of refraction within the mica. In order to eliminate this deviation it is therefore necessary to add $2x$ to the mean value for S.

The mean thickness of the mica windows was 0.0061 cm.; 2θ was approximately 15° 50' and ϕ can at once be deduced from $\sin \phi = \frac{\sin 2\theta}{\mu}$ where μ is the refractive index of mica and was taken equal to 1.58.

Substitution in the above equation gave $x = +0.00066$ cm. and hence the desired correction $2x = +0.0013$ cm.

(4) *Correction Due to the Change in the Controlling Couple.*—The last and most important correction is that required to reduce the deflection to its initial instantaneous value. It was pointed out above that this was equal to

$$\left(D - D \frac{2}{1 + e^{-kt}} e^{-kt} \right).$$

The experimental determination of D and k for these wires and for this magnitude of deflection is given in the paper on "Bifilar Suspensions," quoted before. We have

in this case $t = 3\frac{1}{2}$ minutes, $T = 5$ minutes, and it was found that $D = 0.049$ cm. and $k = 0.22$. Hence $x = 0.0149$ cm. This is for a single deflection, and therefore for S we must subtract 0.0298 cm.

(g) *The Final Value of S.*

We have to apply the above corrections.

(1)	+0.0008	cm.
(2)	+0.0014	„
(3)	+0.0013	„
(4)	-0.0298	„
Total	-0.0263	„

The final value of S was therefore

$$84.629_7 \text{ cm.}$$

(h) *The Determinations of β , the Distance between the Scales.*

This distance was measured by means of a rigid brass tube which was fitted with micrometer calipers at one end, and with an end stud at the other. The distance between the faces of the scales was determined directly by placing the end-stud against one scale, and the micrometer was so attached that it could be screwed against the face of the other scale. The temperature could be obtained from two thermometers which were inserted about 50 cm. from each end, respectively, and the tube was bound with thick asbestos cord to provide handling places.

The tube was calibrated on the standardized fifty-foot comparator at the Macdonald Engineering Building, McGill University. The method adopted was similar to that described in § III. (b), for the calibration of the smaller instrument used for determining the distance between the fixed coils. The following results were obtained for different calibrations. The figures given are reduced to the value of the length with the micrometer set at 0.300 cm. at a temperature of 20.0°C. ,

	117.503	inches,
	117.505	„
	117.505	„
	117.504	„
Mean . . .	117.504	„ i.e., 298.460 cm.

The scale distance was determined before and after each deflection series. It was found that the water circulation through the supporting pipes kept this quite constant.

Measurements were made at both the north and south ends of the scales *where the actual deflection readings occurred*. In order to insure delicacy of adjustment the tube could be supported from the ceiling in the correct position for measurement. A sample set is given to show the type of observations taken.

TABLE XXII.

Micrometer readings at north ends—	Micrometer readings at south ends—	Temperature of tube.
cm. 0·313 0·309 0·311 0·314 — — — —	cm. — — — — 0·312 0·315 0·315 0·314	° C. 18·8 18·8 18·8 18·9 19·0 19·0 19·0 19·0
Means . . . 0·312	0·314	18·9

Mean micrometer reading = 0·313 cm.

At 20°·0 C. a reading of 0·300 corresponds to a total length of 298·460 cm., hence at 18°·9 C. this becomes 298·454 cm., and a value for a reading of 0·313 is, therefore, 298·441 cm.

Each determination of the mean distance between the scales which is recorded in the deflection tables was obtained in this manner.

It is necessary to subtract the correction m for the thickness of the mirror from 298·450, the standard value for β . A series of micrometer caliper measurements for m gave 0·3584 cm. We have therefore, $\beta - m = 298·092$ cm.

(i) *Calculation of the Mean $\tan \theta$.*

Under the standard conditions enumerated, we have

$$\tan 2\theta = \frac{S}{\beta - m} = \frac{84·6297}{298·092} = 0·2839046 ;$$

hence

$$2\theta = 15^\circ 50' 58·14'',$$

and

$$\theta = 7^\circ 55' 29·1'',$$

therefore

$$\tan \theta = 0·139201_6.$$

VIII. THE RESISTANCES.

The two ohms used as the standard in this investigation were made of platinum-silver wire wound on mica frames, annealed after winding, and immersed in an oil bath. Prof. H. T. BARNES, who constructed them in 1898, gives a detailed account of their special design on p. 173 of his paper "On the Capacity for Heat of Water between the Freezing and Boiling Points."* At that time their resistance in series was found to be 2.00218 true ohms at 20° C. with a temperature coefficient of +0.000248.

The present value of their resistance was obtained from a comparison with a manganin standard, No. 4086, which was sent to the Bureau of Standards for calibration. It was certified that its resistance could be expressed to within 0.005 per cent. between 15° C., and 35° C. by the formula

$$R_t = 1.00032 + 0.000018(t - 20) - 0.0000005(t - 20)^2.$$

Each of the two platinum-silver ohms was compared with No. 4086 on a Carey-Foster bridge,† and the comparisons gave the following mean values (reduced to 20° C.) at different times during the investigation:—

The resistance of

(a)	Platinum-silver Coil No. I.	=	that of No. 4086	+0.00138	ohms.
(b)	"	"	"	=	" " +0.00140 "
(c)	"	"	"	=	" " +0.00137 "
(d)	"	"	"	=	" " +0.00138 "
Mean	"	"	"	=	" " <u>+0.00138</u> "

and the resistance of

(a)	Platinum-silver Coil No. II.	=	that of No. 4086	+0.00011	ohms.
(b)	"	"	"	=	" " +0.00007 "
(c)	"	"	"	=	" " +0.00014 "
(d)	"	"	"	=	" " +0.00013 "
Mean	"	"	"	=	" " <u>+0.00011</u> "

Hence

Coil I. = 1.00170 international ohms at 20° C.

" II. = 1.00043 " " "

Coil I. + Coil II. = 2.00213 " " "

* BARNES, 'Phil. Trans.,' A, vol. 199, p. 173, 1902.

† The same bridge is described and its accuracy indicated on p. 175 of the paper quoted above.

It will be seen that this differs from the earlier value of the resistance by only 5 parts in 200,000.

In this investigation the two ohms were used at $23^{\circ}\cdot 12$ C. At this temperature their resistance, obtained by using the temperature coefficient $+0\cdot 000248$, is

$2\cdot 00368$ international ohms.

During the deflection observations a current of about half an ampere was flowing through these coils. If the oil was kept well stirred it was found that the temperature of the coils was given to a sufficient degree of accuracy by the temperature of the oil.

IX. THE "RATIO OF THE CURRENTS" IN THE TWO SYSTEMS OF COILS DURING THE DEFLECTION EXPERIMENTS.

In § VII. (c) it will be seen that the ratio of the currents is determined by the ratio of the standard two ohms to a resistance composed of two standard manganin tenth ohms (Nalder, Nos. 4389 and 4388). This ratio was measured by means of the Kelvin-Varley slide (calibrated for refined measurements and used as described in the appendix). In each case the measurements were taken with a current of about half an ampere flowing through the coils to be compared; and a current of five amperes had been allowed to flow through the tenth ohms for some time preceding the measurement. In this way the actual conditions of the deflection experiment were closely approached, and any error due to the inaccuracy of the manganin temperature coefficient was eliminated.

Taking the temperature coefficient of $0\cdot 000248$ for the two ohms and $0\cdot 000018$ for the manganin resistances, and reducing the value of the ratio to that for the condition when the two ohms were at a temperature of $23^{\circ}\cdot 12$ C. and the tenths at a temperature of $28^{\circ}\cdot 0$ C. we got the following mean results for the ratio in two separate series of measurements:—

$$(1) 10\cdot 0357, \quad (2) 10\cdot 0357.$$

Mean $10\cdot 0357$.

This result is equivalent to a value of $0\cdot 199625$ international ohms for the two-tenths at 20° C. Thermal or contact effects in the measurement were eliminated in the usual way, and each determination was checked with the Kelvin-Varley slide reversed.

The four windings of the suspended coil were arranged in parallel, and as errors due to any unsimilarity of the windings were eliminated by means of the series of observations involved in the deflection mean, and also by the method of determining the constant " g_1 ," we could assume that the effect was the same as that of a quarter of the current flowing through the four windings in series, and hence the actual ratio of the currents under the conditions of the deflection experiment was

$40\cdot 1428$.

X. THE STANDARD CELLS.

An account of the equipment of the laboratory with regard to standard cells has been given by Dr. H. L. BRONSON and the present writer in a paper entitled: "Clark and Weston Standard Cells," which was published in 'The Electrician,' February 10, 1911.

During the deflection observations the electromotive force of the two cells P_1 and P_2 which were used in the determination, was repeatedly checked by comparison with a set of ten standards of especial constancy and reliability, viz., S 20, M 10, III., IV., A, B, N, O, Q_4 , and Q_5 , all of which are discussed in the paper quoted. S 20 and M 10 were constructed at the National Physical Laboratory, III. and IV. at the Bureau of Standards, and A, B, N, O, Q_4 , and Q_5 at McGill.

Comparisons were made with the Bureau of Standards in 1908 and 1909, and with the National Physical Laboratory in 1909 and 1912. It has, therefore, been possible to express the mean of our cells in terms of the international mean for the Weston normal cell. The electromotive force of each of the cells P_1 and P_2 at the time of the deflection observations was 14 microvolts lower than the mean of the standards enumerated above, and the mean of these standards was found to be between 10 and 20 microvolts lower than the international mean. Calculations based on the exchanges with the National Physical Laboratory gave the larger figure, and those based on the exchanges with the Bureau of Standards gave the lower figure. We have assumed, therefore, that the two cells P_1 and P_2 were each 30 microvolts lower than the international mean value for the Weston normal cell.

XI. INSULATION AND MAGNETIC TESTS.

Both systems of coils were wound with double winding so that it was always possible to perform a complete test on the insulation. In winter the dry Montreal atmosphere rendered insulation precautions unnecessary, but in summer it was found convenient to maintain the room at a higher temperature than the surrounding rooms and passages, in order to keep the connections and the instruments in the same dry condition as in winter. Tests were repeated throughout the investigation, and it was found that an insulation resistance of at least four times the necessary magnitude could always be maintained.

The framework of the dynamometer had been tested for magnetic qualities by Mr. R. O. KING when the instrument was first set up, and found to be quite satisfactory. The influence of the covers, which were made for the sides and the ends of the dynamometer, was determined by comparing deflection observations taken *with* and then *without* the covers on the instrument. In the latter test air-currents were kept out by screens of paper at a distance of about two feet from the dynamometer. No difference in the deflections was observed. In the same way the presence of the

zinc reflectors (1.3 metres from the dynamometer) used for the illumination of the scales, was found to produce no magnetic effect on the deflection.

The "leading in" wires to the fixed coils were all wound non-inductively, and at the entrance to the coils the direction of the currents in the four sets of thicker wires, forming the small loops (visible in Plate 1) were such that their resultant magnetic effect at the centre was completely eliminated. A magnetometer test provided a simple proof of this matter.

The bifilar suspensions which constituted the leading in wires for the suspended coils formed, as will be seen from their dimensions given in § VI. (a), a long rectangular circuit of appreciable dimensions. It is, however, evident that this circuit extends only to the circumference of the suspended coils and is partly *outside* the fixed coils and partly *inside*. A calculation will show that the resultant effect from the suspensions is of a negligible order. The connections on the suspended coils made it possible to test this by reversing the direction of the current in the suspensions without changing its direction in the coils. No effect on our deflection could be detected.

XII. THE ELECTROMOTIVE FORCE OF THE MEAN WESTON NORMAL CELL.

It has been explained that the electromotive force of the standard cell is given by the drop in potential across the two ohm standard when the current $4i_2$ passes through the resistance at a temperature of $23^{\circ}12$ C. We now have all the factors. It has been shown that

$$B = 56593.2, \quad r = 40.1428, \\ \tan \theta = 0.139201_6, \quad -4G_1g_1 - 4G_5g_5P'_5 - 4G_7g_7P'_7 = 1216280.$$

Hence by expression (4), [§ II. (b)], i_2 in absolute amperes is given by

$$i_2 = 10 \sqrt{\frac{56593.2 \times 0.139201_6}{40.1428 \times 1216280}} = 0.127023_3 \text{ absolute amperes.}$$

Hence the drop in potential across the standard two ohms, 2.00368 international ohms, is given by

$$4 \times 0.127023_3 \times 2.00368 = 1.01806 \text{ semi-absolute volts.}$$

This is equivalent to the electromotive force of our cell P_1 or P_2 at $25^{\circ}03$ C. Using the temperature formula

$$E_t = E_{20} - 0.000040(t-20) - 0.0000009(t-20)^2,$$

we see that the electromotive force of our cell at 20° C. is 1.01828 semi-absolute volts. It has been pointed out in § X. that these cells are about 30 microvolts lower than the value of the international "Mean Weston Normal Cell," hence our final result for the electromotive force of the Mean Weston Normal Cell is

$$1.01831 \text{ semi-absolute volts at } 20^{\circ} \text{ C.}$$

In conclusion the writer desires to express his gratitude to Mr. R. O. KING, M.Sc. for providing the fellowship which made this research possible.

The author is especially indebted to Prof. H. T. BARNES, F.R.S., Director of the Macdonald Physics Building, McGill University. The work was undertaken at his suggestion, and throughout the investigation he was a constant source of advice, personal help and kind encouragement.

Many thanks are due also to Dr. H. L. BRONSON and to Mr. F. H. DAY, M.Sc., for assistance in taking simultaneous observations.

APPENDIX.

NOTE ON THE CALIBRATION OF THE KELVIN-VARLEY SLIDE FOR ACCURATE ELECTRICAL MEASUREMENTS.

The Kelvin-Varley slide has been described in many electrical works,* and is well known as a valuable and accurate instrument. It has been found that an exceptional accuracy can be obtained if refined calibrations are occasionally performed. After making several trials of various methods, Prof. CALLENDAR, in connection with his research on "Continuous Electrical Calorimetry,"† briefly outlined the one which he considered to be the most convenient and accurate. This method with some slight modifications was adopted; and the table of corrections given on the next page is analogous to the tables on pp. 69 and 70 of the paper quoted. "*n*" represents the slide reading.

The validity of these corrections was tested both before and after the use of the instrument in the investigation. A comparison of the figures given in this table with those obtained by Prof. CALLENDAR (*loc. cit.*) for the same instrument in 1894 shows that a calibration of this kind must be repeated from time to time in order to maintain the same degree of accuracy in the use of the instrument. The slow alterations in the coils can attain considerable magnitude and ultimately render old calibrations quite useless. The changes are due, usually, to an average effect rather than to especial weakness in some particular coil or coils. It is thus possible to get a good indication of the condition of the whole box by taking and comparing readings for several ratios measured first from one end of the slide and then from the other. At the same time it is at once apparent in such a test, if the variation happens to be a purely local one. For measurements requiring an accuracy of more than 1 part in 10,000 we should not rely upon a calibration which is older than a year.

* PRICE, 'Measurement of Electrical Resistance,' p. 106; FLEMING, 'Handbook for the Electrical Laboratory and Testing Room,' vol. 1, p. 273; MUNRO and JAMIESON, 'Pocket Book of Electrical Rules and Tables,' p. 158; KEMPE, 'Handbook of Electrical Testing,' p. 219; ASPINALL PARR, 'Practical Electrical Testing,' p. 321, &c., &c.

† 'Phil. Trans,' A, vol. 199, 1902, p. 65.

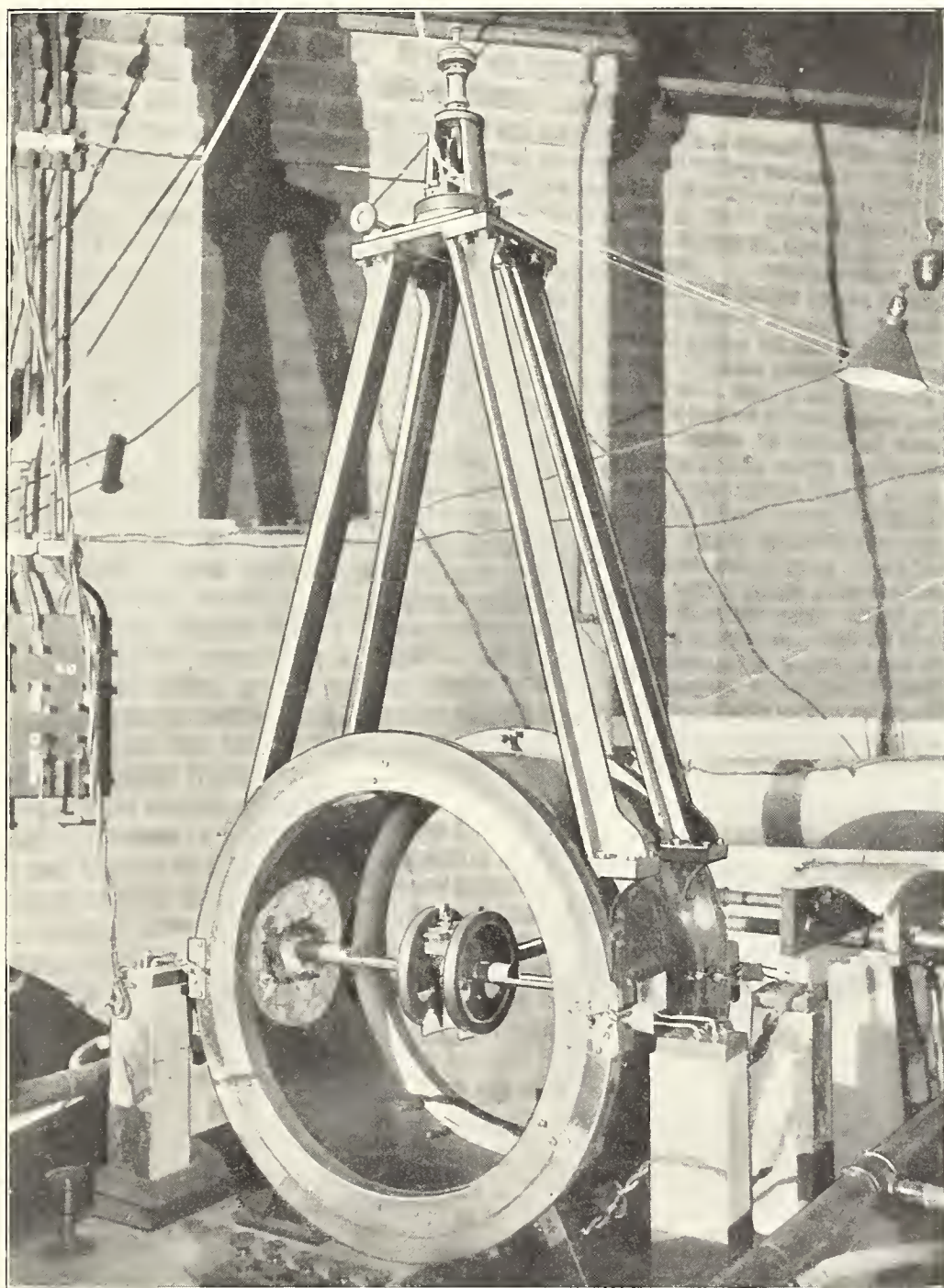
CORRECTIONS for the Kelvin-Varley Slide.

<i>n.</i>	Corrections.	<i>n.</i>	Corrections.	<i>n.</i>	Corrections.
0	0	34	-1.63	68	-2.57
1	+0.94	35	-2.12	69	-2.74
2	+0.74	36	-1.72	70	-2.14
3	+0.45	37	-2.33	71	-2.47
4	-0.07	38	-2.26	72	-2.16
5	+0.31	39	-2.66	73	-2.02
6	+0.43	40	-2.65	74	-1.78
7	+0.27	41	-2.42	75	-1.04
8	+1.19	42	-1.99	76	-0.39
9	+1.29	43	-2.15	77	-0.16
10	+1.16	44	-1.46	78	+0.46
11	+1.00	45	-2.16	79	+0.32
12	+1.20	46	-2.02	80	+0.75
13	+0.88	47	-2.64	81	+0.93
14	+2.33	48	-2.65	82	+1.00
15	+1.97	49	-2.98	83	+0.75
16	+1.87	50	-3.55	84	+0.54
17	+1.65	51	-4.02	85	-0.14
18	+1.44	52	-4.06	86	-0.23
19	+1.15	53	-4.69	87	-0.01
20	+1.19	54	-4.33	88	-0.18
21	+0.89	55	-4.74	89	+0.34
22	+1.33	56	-4.93	90	+0.75
23	+1.31	57	-4.84	91	+0.56
24	+1.43	58	-4.69	92	+0.32
25	+0.74	59	-4.70	93	+0.78
26	+0.51	60	-3.81	94	+1.02
27	-0.04	61	-4.15	95	+0.70
28	+0.07	62	-3.28	96	+0.54
29	-0.27	63	-3.34	97	+0.40
30	-0.11	64	-2.85	98	+0.14
31	-1.27	65	-2.70	99	-0.02
32	-1.14	66	-2.83		
33	-1.59	67	-2.92		

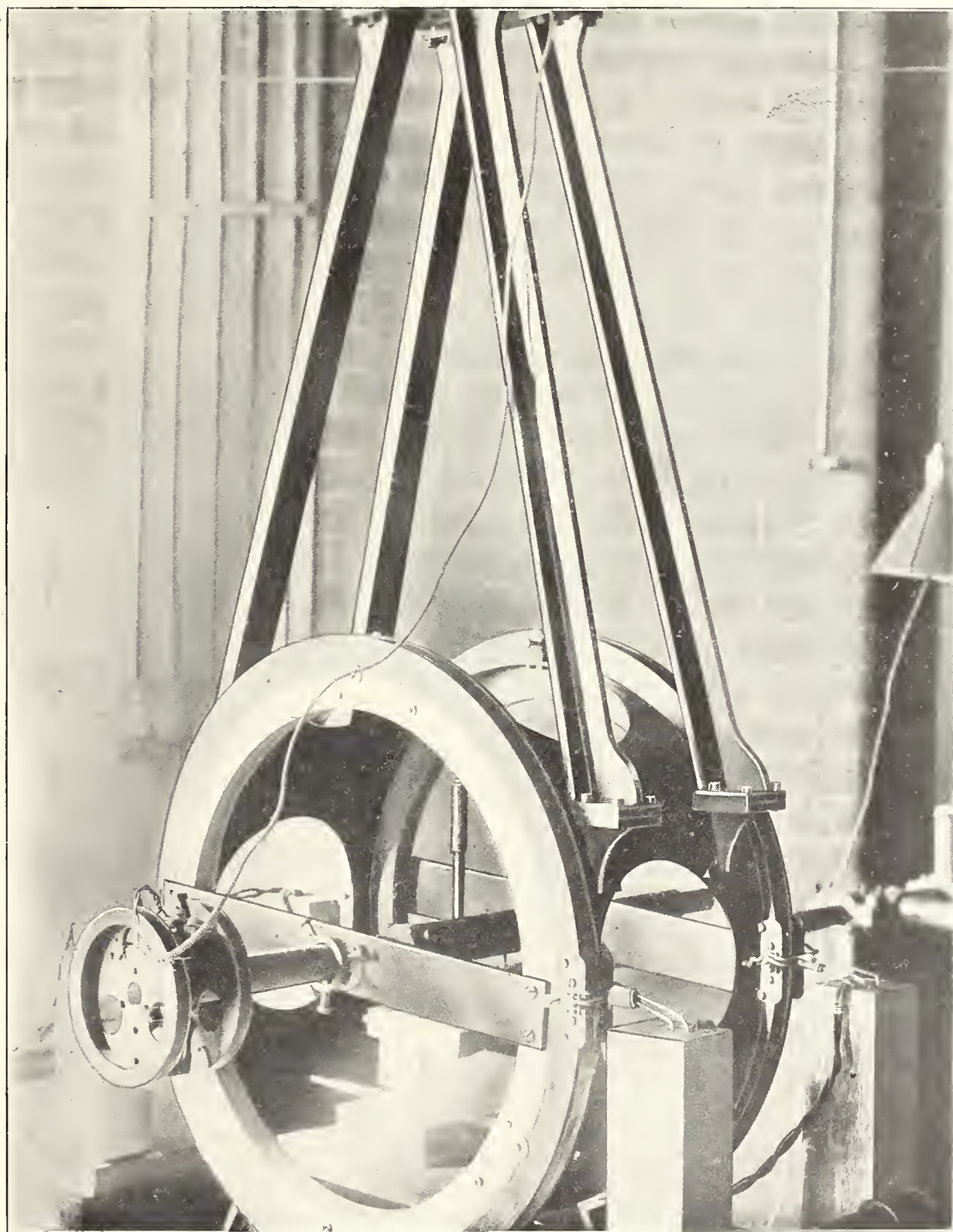
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V. *Similarity of Motion in Relation to the Surface Friction of Fluids.*

By T. E. STANTON and J. R. PANNELL.

Communicated by Dr. R. T. GLAZEBROOK, F.R.S.

(From the National Physical Laboratory.)

Received December 12, 1913,—Read January 29, 1914.

THE laws of the surface friction of fluids have formed the subject of many important investigations during the last 100 years, among which may be mentioned the work of POISEUILLE, DARCY and OSBORNE REYNOLDS on the friction of water flowing in pipes, that of WILLIAM FROUDE on the resistance of thin plates towed in water, and the corresponding experiments of ZAHM on flat plates in a current of air. Researches in this field have also been carried out by BRIX, STOCKALPER, MALLOCK, COKER, GEBERS, BRIGHTMORE, GRINDLEY and GIBSON, and others.

As a result, the effect on the resistance, of the dimensions of the body over whose surface the fluid moves, and of the velocity of flow, are tolerably well known for the particular fluid and character of motion observed. In the case of the surface friction of water in pipes, the researches of OSBORNE REYNOLDS have demonstrated the existence of similar motions in pipes of different dimensions, but, as far as the authors are aware, no systematic series of experiments appears to have been made for the purpose of establishing a general relation which would be applicable to all fluids and conditions of flow, although the existence of such relationships for different aspects of the problem were predicted as a consequence of the laws of motion by STOKES in 1850,* by HELMHOLTZ in 1873,† by OSBORNE REYNOLDS in 1882,‡ by Lord RAYLEIGH§|| in 1899 and 1909, and as has been pointed out by Sir GEORGE GREENHILL, were foreshadowed by NEWTON in Proposition 32, Book II., of the ‘Principia.’

The object of the present paper is to furnish evidence confirming the existence,

* STOKES, ‘Mathematical and Physical Papers,’ vol. III., p. 17.

† HELMHOLTZ, ‘Wissenschaftliche Abhandlungen,’ vol. I., p. 158.

‡ ‘Phil. Trans. Roy. Soc.,’ 1883, p. 935.

§ ‘Phil. Mag.,’ 1899, p. 321.

|| ‘Advisory Committee for Aeronautics, Report,’ 1909–10, p. 38.

under certain conditions, of the similarity in motions of fluids of widely differing viscosities and densities which has been predicted, and further by extending the observations through a range in the velocity of flow which has not hitherto been attempted to investigate the limits of accuracy of the generally accepted formulæ used in calculations of surface friction.

Previous Experimental Investigations.

Apart from the researches on similarity of motion of fluids, which have been in progress in the Aeronautical Department of the National Physical Laboratory during the four last years, the only previous experimental investigation on the subject, as far as the authors are aware, has been that of OSBORNE REYNOLDS, to which a brief reference may be made.

By the introduction of colouring matter into water flowing through glass tubes REYNOLDS showed that the motion was stream-line or lamellar in character at low values of the velocity of flow, and eddying or sinuous at high velocities, and that the change from lamellar motion to eddying motion took place suddenly at a definite value of the velocity (called the critical velocity), the value of which was inversely proportional to the diameter of the tube and directly proportional to the kinematical viscosity of the water.

Expressing this in symbols, if

$$\begin{aligned} d &= \text{diameter of the pipe,} \\ v_c &= \text{the critical velocity,*} \\ \mu &= \text{the coefficient of viscosity of the water,} \\ \rho &= \text{the density of the water,} \\ \nu &= \text{kinematical viscosity of the water } (= \mu/\rho), \\ dp/dx &= \text{rate of fall of pressure along the pipe.} \end{aligned}$$

REYNOLDS'S discovery was that for geometrically similar tubes

$$v_c d / \nu \text{ was constant.}$$

Further, on making a series of observations of the values of dp/dx over as large a range in the velocity of flow as possible in similar pipes of different diameters, REYNOLDS found that for all conditions of flow, stream line or eddying, when the values of vd/ν were identical the corresponding values of $\rho d^3/\mu^2 \cdot dp/dx$ were identical. It appeared, therefore, that the general law of resistance could be expressed by the equation

$$\frac{\rho d^3}{\mu^2} \cdot \frac{dp}{dx} = f\left(\frac{vd}{\nu}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

* Throughout the present paper the symbol v is used to denote the mean velocity of flow through the pipe. Where reference is made to the value of the velocity at the axis of the pipe this is denoted by $v_{\max}$.

Comparing this equation with the theoretical value of dp/dx for stream-line motion, *i.e.*,

$$\frac{dp}{dx} = \frac{32\mu v}{d^2},$$

it will be seen that under conditions of stream-line motion

$$f\left(\frac{vd}{\nu}\right) \propto \frac{vd}{\nu}.$$

For velocities above the critical value, and as high as could be carried in his apparatus, REYNOLDS considered that the function on the right-hand side of the equation could be expressed by the relation

$$f\left(\frac{vd}{\nu}\right) \propto \left(\frac{vd}{\nu}\right)^n,$$

where n has a value which is constant for any given pipe, but may vary from 1.75 to 2 according to the roughness of the surface. As will be seen later, this conclusion has not been verified in the present experiments, which show definitely that when the range in speed is considerable the index law fails to represent the results.

It has been shown by Lord RAYLEIGH that by the Principle of Dynamical Similarity the relation expressed by equation (1) is only a particular case of a general law of resistance of bodies immersed in fluids moving relatively to them, under the assumption that this resistance depends only on the linear dimensions of the body and on the velocity, density, and kinematical viscosity of the fluid. This relation may be expressed as

[illegible]

where R is the resistance per unit area and F is a function of the one variable rd/ν .

From the foregoing it appears that similarity of motion in fluids at constant values of the variable vd/ν will exist, provided the surfaces relative to which the fluids move are geometrically similar, which similarity; as Lord RAYLEIGH has pointed out, must extend to those irregularities in the surfaces which constitute roughness. In view of the practical value of the ability to apply this principle to the prediction of the resistance of aircraft from experiments on models, experimental investigation of the conditions under which similar motions can be produced under practical conditions becomes of considerable importance, and during the last three years the accuracy of the assumptions made in the derivation of equation (2) has been tested at the National Physical Laboratory under varying conditions. By the use of colouring matter to reveal the eddy systems at the back of similar inclined plates in streams of air and water, photographs of the systems existing in the two fluids when the value of vd/ν was the same for each, have been obtained, and their comparison has revealed a remarkable similarity in the motions.* For the case of surface friction, experiments

* 'Report of Advisory Committee for Aeronautics,' 1911-12, p. 97.

have been made on pipes whose surfaces were smooth or artificially roughened so that the amount of the friction was more than doubled. By making observations on the same pipe first with air flowing through it and then with water at identical values of vd/ν it was found that the corresponding values of $R/\rho v^2$ were also identical both for the smooth pipe and for the rough one.*

Comparisons have also been made between the resistances of models of dirigible balloons made to different scales, in one case when exposed to a current of air and in the other case when towed in an experimental water tank, and results confirmatory of the theory have been obtained.

The experiments described in the present paper form a continuation of the investigation with reference to surface friction, so as to include the highest range in the velocities of flow, the dimensions of the surfaces, and in the nature of the fluids used, as could be conveniently obtained. It will be realised that in order to vary the conditions to the extent desired, and to measure the resistances with the requisite accuracy, the research was practically limited to observations on the resistance to the flow of fluids in pipes. Adopting this method of working, the velocities of flow which have been obtained have ranged from 30 to 6000 cm. per second in pipes varying from 0.3 to 10.0 cm. in diameter. The fluids used in the majority of the experiments have been air and water. The physical properties of these are so widely different that observations on others are hardly necessary, but as a matter of interest some experiments on thick oils are included.

For very accurate comparison the surfaces of the tubes should have been precisely geometrically similar, as regards roughness, but as this condition could not be fulfilled, the experiments were all made on commercially smooth-drawn brass pipes. From the general agreement of the results of different pipes it does not appear that slight irregularities in this respect have a marked effect on the resistance within the range of diameters here used.

Nature of the Observations in the Experiments.

To set up the state of motion required the experimental pipe was, in the majority of cases, connected to a centrifugal fan or pump, driven by an electric motor and provided with speed-regulating devices by means of which the flow of the fluid could be maintained constant throughout the duration of any particular experiment. The length of "leading in" pipe, of the same diameter as the experimental portion, through which the fluid passed before any observations of its velocity or pressure were made, varied from 90 to 140 diameters, as it was considered that this length was sufficient both to enable any irregularities in the distribution of velocity to die away, or any stream-line motion at the inlet to break up, before the measurements were taken.

* 'Roy. Soc. Proc.,' A, vol. 85, p. 366.

To measure the velocity of the current, one of two methods was used according to convenience. By one method the total quantity of fluid passing through the pipe in a given time was either weighed directly, or passed through a water-meter or a gas-holder, which had been designed for the purpose of the experiments and carefully calibrated. By the other method the velocity at the axis of the pipe was estimated by measuring the difference of pressure between that in a small Pitot tube facing the current and placed in the axis of the pipe and that in a small hole in the wall of the pipe. As is now known* this pressure difference is accurately $\frac{1}{2}\rho v^2$, and from this relation the speed at the axis was determined. The mean flow was then calculated from the known ratio of the mean speed to the speed at the axis; an investigation into which forms part of the present paper.

To determine the amount of the surface friction two small holes were made in the walls of the experimental portion of the pipe, one at each extremity, at a known distance apart, and connected to a tilting manometer. The length of this portion varied in different pipes from 20 to 50 diameters, according to the resistance to be measured. In this way the fall of pressure along a given length of the pipe was determined, and from the known diameter of the pipe the surface friction per unit area was calculated. The form of tilting manometer used for the estimation of both the surface friction and the axial velocity, is that devised by Dr. A. P. CHATTOCK and has been previously described.† For the purpose of the present paper it is sufficient to state that in this manometer a pressure difference of the order of 0.003 mm. of water can easily be detected, which is well within the limits of sensitivity required in these experiments. As the fall of pressure in these pipes varied from 0.5 to 150,000 mm. of water, other manometers were required for the higher pressures, and for this purpose water or mercury U-tubes were used for the intermediate pressures, and Bourdon pressure gauges for the highest pressures.

*The Relation between the Mean Velocity of Flow and the Velocity at the Axis in
Tubes of Circular Cross Section.*

In the case of the "stream-line" motion of a viscous fluid through a tube, to which the ordinary equations of motions apply, it appears both from theory and direct experiment that the ratio of the mean velocity over the section to the velocity at the axis is 0.5. For the case of eddying flow the value of this ratio has been investigated by DARCY‡ for water and by THRELFALL§ for air. It might be inferred from the conditions of similarity of flow discussed above, that the ratio in question would be a function of the ratio vd/ν , and in a paper on the Mechanical Viscosity of Fluids read

* 'Report of Advisory Committee for Aeronautics,' 1912-13.

† 'Proc. Inst. Civil Engineers,' December, 1903, and 'Engineering,' September, 1913.

‡ 'Comptes Rendus de l'Académie des Sciences,' vol. 38.

§ 'Proc. Inst. Mechanical Engineers,' 1904, p. 280.

before the Society in 1911* the results of experiments on the radial variation of the velocity of air flowing through smooth brass pipes of 4.9 and 7.4 cm. diameter were given, showing that the velocity distribution curves were only identical when the values of the ratio vd/ν were identical. It was also found that the distribution of velocity was only independent of the values of vd/ν when the surfaces of the pipes were so roughened that the resistance varied as the square of the velocity.

On examining the results of DARCY and THRELFALL no very satisfactory evidence of a variation of the ratio with variations of vd/ν was forthcoming. This may be due to the fact that THRELFALL'S observations are all included in a range of vd/ν of from 66,000 to 300,000, and as the variation in the ratio may be assumed to be small, the difference between the results would not be detected very easily, and also to the possibility that DARCY'S observations were made in a region over which the curve showing the variation of the ratio with vd/ν was nearly horizontal.

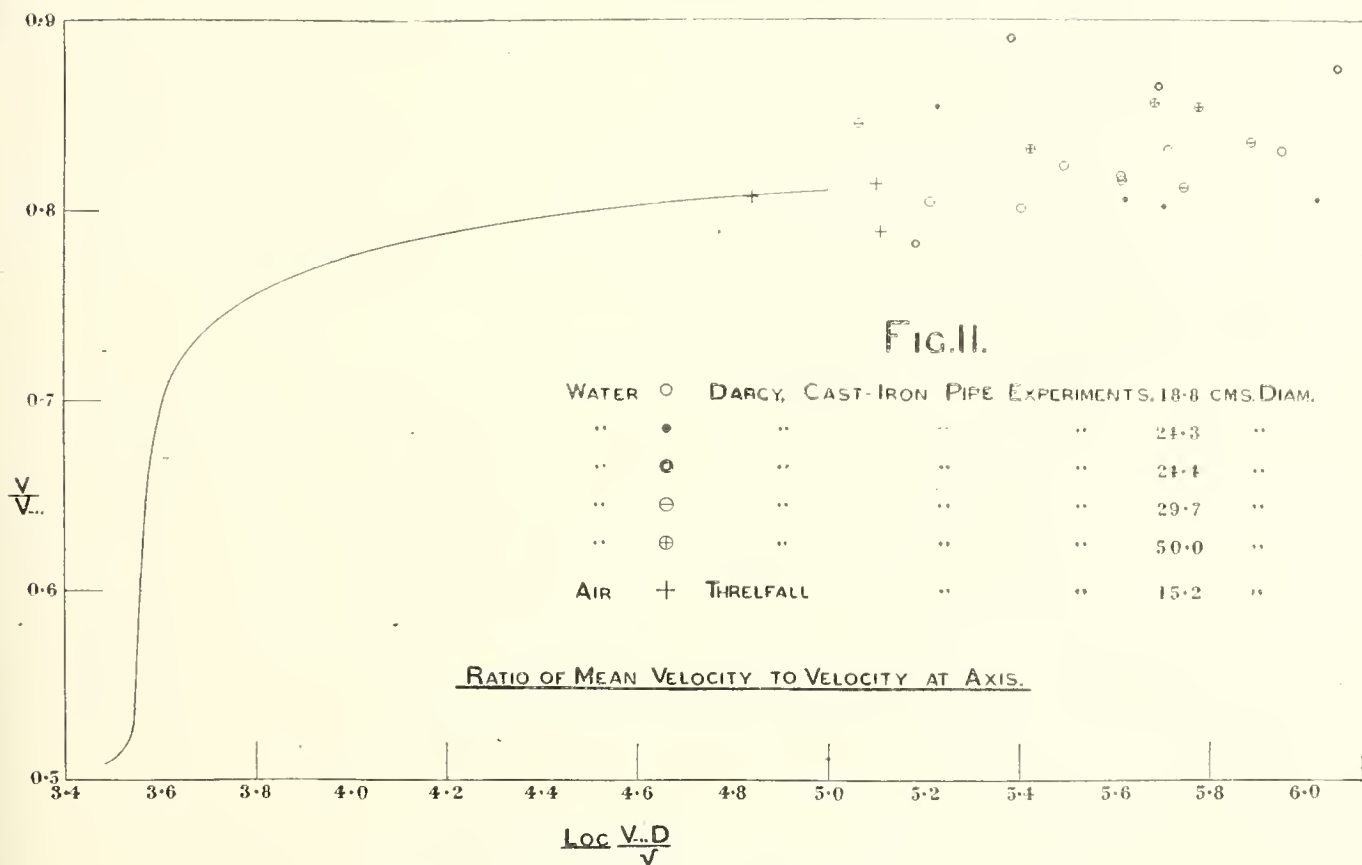
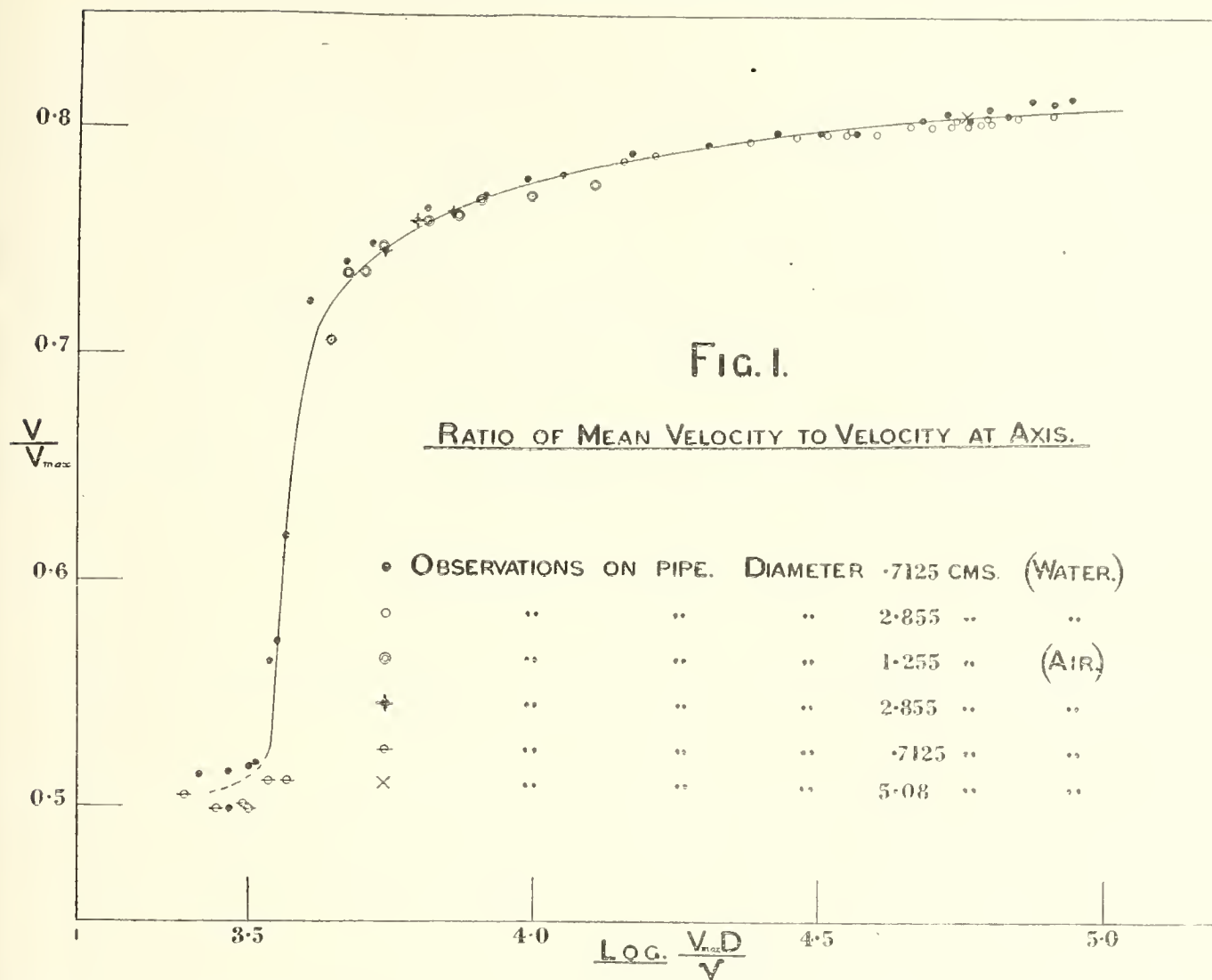
As an accurate determination of the ratio was essential for many of the proposed friction experiments it was decided to make an extensive series of observations with a view to tracing the variation of the ratio from its value 0.5 below the critical value of the ratio vd/ν to as high a value as possible. As the use of an accurately calibrated gas holder was available it was possible to make the determination for the case of air flow as well as for water flow. The pipes used were 0.7125, 1.255 and 2.855 cm. in diameter, and the method of estimating the velocity at the axis was that previously described. It may be mentioned that in this part of the work comparatively small errors in observation would have obscured the effect to be investigated, as its whole amount in the region of eddying flow is of the order of 5 per cent., so that the greatest care and accuracy was necessary.

The calculated values of the ratio of mean velocity to the velocity at the axis are plotted in fig. 1 as ordinates with the corresponding values of $\log v_{\max} \cdot d/\nu$ as abscissæ. The values of the velocity at the axis have been adopted merely for convenience in the use of the curve to determine rates of discharge from a pipe by means of a single observation at the axis, and experience has shown that in this work a better graphical representation of the results is achieved by using a logarithmic scale for plotting the values of vd/ν than by using a simple one.

The observations were continued for smaller and smaller values of vd/ν until stream-line motion was set up, below which it was not considered necessary to go.

The values of the calculated ratios are tabulated in Table I. It will be seen from the mean curve drawn through the plotted points in fig. 1 that the value of the ratio ranges from 0.5 at the critical value of the speed, *i.e.*, at $vd/\nu = 2500$, to 0.81 at the value $vd/\nu = 70,000$. For the comparison of these results with those obtained by other experimenters the mean curve through the National Physical Laboratory results is reproduced in fig. 2, in which the results of DARCY and THRELFALL are plotted. It will be noticed that the curve passes through two of THRELFALL'S points whereas

* 'Roy. Soc. Proc.,' A, vol. 85, p. 366.



The curve represents the author's mean results.

the third is not in agreement. It is possible that this may be due to the third pipe being rougher in surface than the others, as it has been shown by direct observation of distribution of velocity in rough and smooth pipes at the National Physical Laboratory* that the ratio of mean speed to maximum speed in a rough pipe is less than that in a smooth one.

The interpretation of DARCY's results is also difficult for the same reason. According to the determinations of the frictional resistances of these cast-iron pipes given by DARCY, their surfaces are all appreciably rougher than those used in the present experiments, so that the ratio in the case of the former should be less than for the latter, but no direct evidence of this is obtained from the plotted results. In fig. 1 is also plotted the ratio obtained by direct integration from the radial distribution of the velocity in a 5.08 cm. pipe, which agrees well with the other results.

The Surface Friction Experiments.

The ratio of the mean velocity to the velocity at the axis having been determined, it was possible to complete the observations of surface friction for those cases in which no other method of estimating the discharge was available.

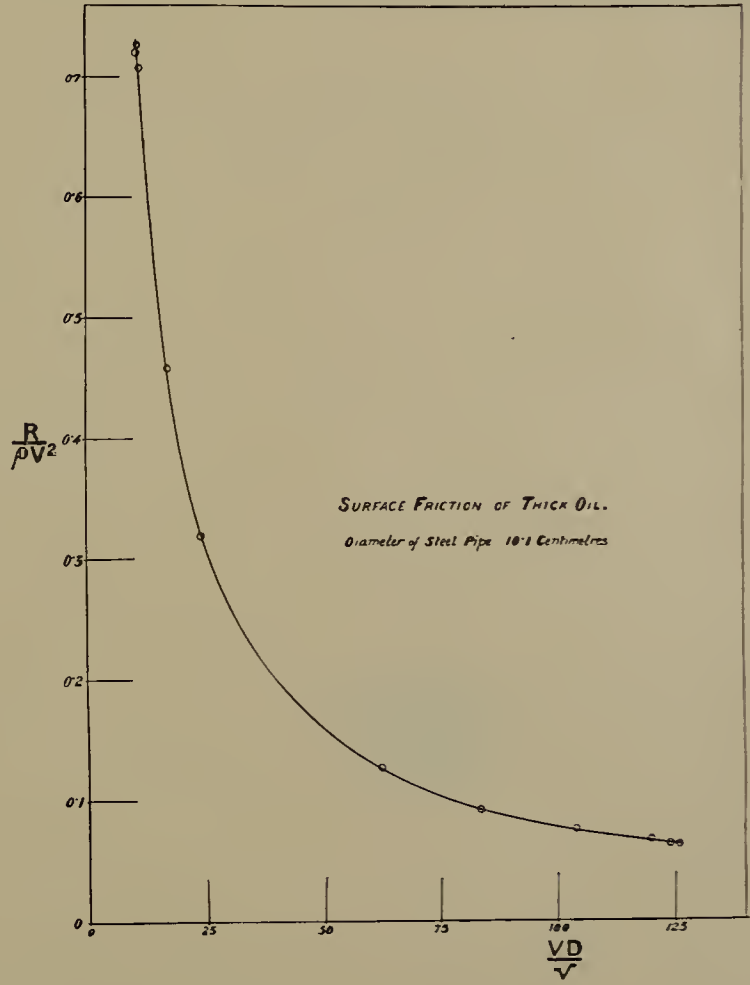
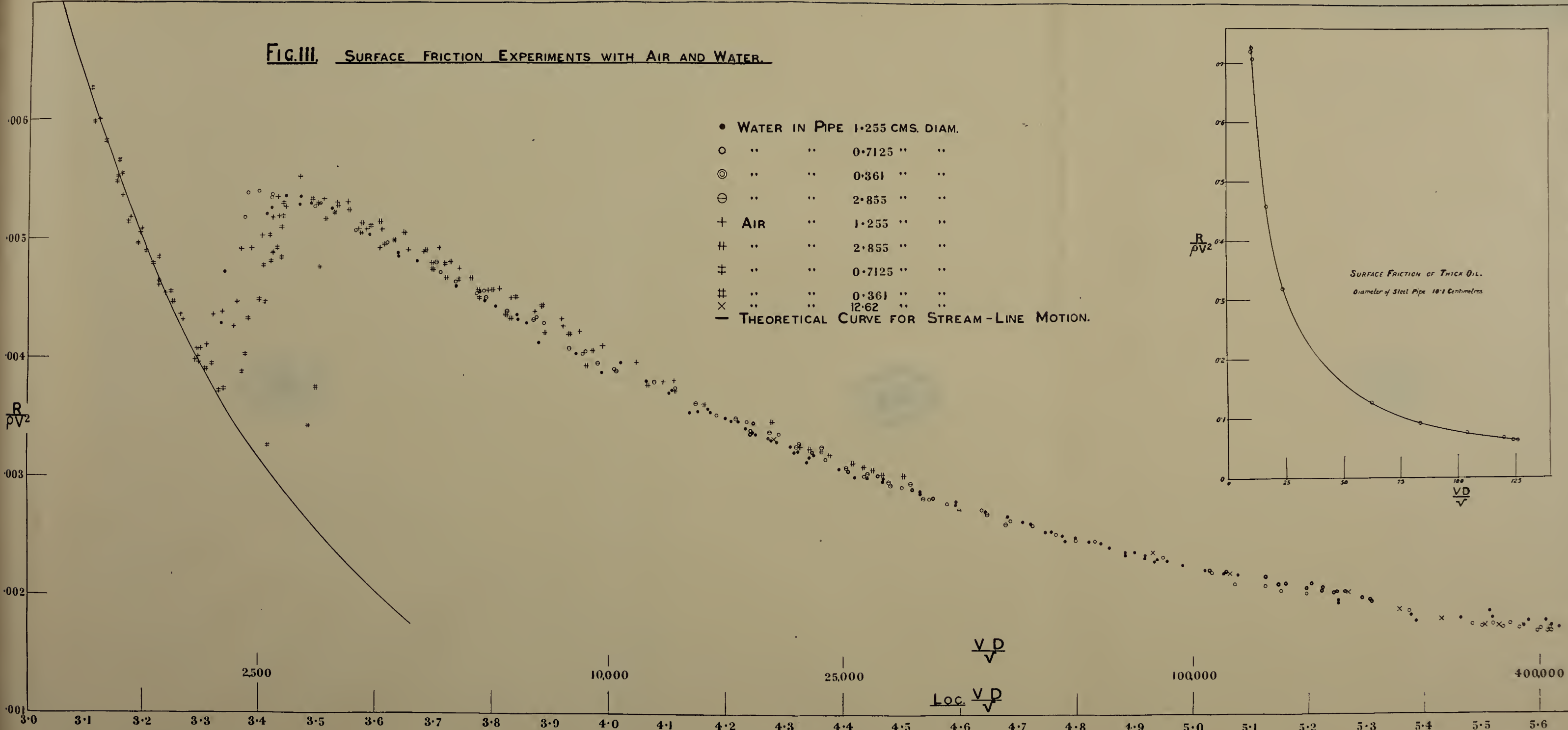
The scope of the experiments will be seen from the table on p. 207 in which the dimensions of the pipes, and the methods of setting up the flow and estimating the velocity and friction per unit area are stated. The actual values of the mean velocity of flow in centimetres per second and the surface friction in dynes per square centimetre of the surface are given in the tables at the end of the paper together with the calculated values of the ratio vd/ν (Tables II. and III.).

The method of representing the results is that, suggested by Lord RAYLEIGH, of plotting points whose ordinates are the values of $R/\rho v^2$ and abscissæ the corresponding values of vd/ν , with the previously stated modification that the logarithms of vd/ν are taken as the abscissæ instead of the actual values. In this way it has been found possible to include all the results on a reasonable length of diagram, and at the same time to show the region of the change from eddying to stream-line motion on a fairly open scale. This has been done in fig. 3.

The extension of the experiments for values of vd/ν greater than 115,000, which was the maximum attainable with the appliances available at the National Physical Laboratory, has been made possible by the kindness of Mr. Edward B. Ellington, Engineer of the London Hydraulic Power Supply Company, who has allowed the authors to connect their experimental pipes to the high-pressure water main at the Grosvenor Road Pumping Station. In this way it has been found possible to reach velocities of 3200 cm. per second in the 1.255 cm. diameter pipe, 5250 cm. per second in the 0.7125 cm. pipe, and 5600 cm. per second in the 0.361 cm. pipe. The maximum value of vd/ν attained was 430,000.

* 'Roy. Soc. Proc.,' A, vol. 85, p. 371.

FIG. III. SURFACE FRICTION EXPERIMENTS WITH AIR AND WATER.





EXPERIMENTS on Surface Friction.

Diameter of pipe in centimetres.	Length on which the friction was measured.	Fluid used.	Method of producing flow.	Method of measuring discharge.	Method of measuring friction.
1.255	52.96	Water	(1, 2, 3) Centrifugal pump (4) Plunger pumps	Meter	(1) Mercury tilting gauge. (2) Water U-tube. (3) Mercury U-tube. (4) Bourdon pressure gauges.
0.7125	30.50	"	(1, 2, 3) Centrifugal pump (4) Plunger pumps	"	(1) Mercury tilting gauge. (2) Water U-tube. (3) Mercury U-tube. (4) Bourdon pressure gauges.
0.361	22.86	"	Centrifugal pump Plunger pumps	"	Mercury U-tube. Bourdon pressure gauges.
2.855	61.2	"	Centrifugal pump	"	Mercury tilting gauge.
1.255	52.96	Air	Suction of gas holder Centrifugal fan	Motion of gas holder Pitot tube in axis	Water tilting gauge.
0.7125	30.50	"	Suction of gas holder	Motion of gas holder	" " "
0.361	22.86	"	Suction of gas holder	" " "	" " " Mercury tilting gauge.
2.855	61.2	"	Suction of gas holder Centrifugal fan	" " " Pitot tube in axis	Water tilting gauge.
10.10	152.5	"	" "	" " "	" " "
10.10	152.5	Thick oil	Centrifugal pump	By direct weighing	Mercury tilting gauge.

Coming to the chief features of interest in the curves of $R/\rho v^2$ it will be noticed that the inclination to the horizontal becomes gradually less as the value of vd/ν increases, indicating that the law of resistance tends to become one in which the friction varies as the square of the velocity and therefore independent of the dimensions and temperature of the fluid. At what value of vd/ν this becomes approximately the case the data available are not sufficient to predict.

As the value of vd/ν diminishes the corresponding values of $R/\rho v^2$ grow at an increasing rate, obtaining a maximum, for pipes of the smoothness used in these experiments, of 0.0054 when vd/ν is equal to 2500 approximately. Throughout the whole of this range in vd/ν of from 2500 to 470,000, with the exception of a few individual determinations due possibly to errors of observation, the variation of $R/\rho v^2$

for either fluid, in any of the four pipes, from its mean value does not exceed 2.0 per cent., so that the similarity of the motions over this range is fully demonstrated. Below $vd/\nu = 2500$ there is a sudden fall in the value of the ordinate showing that a change in the character of the motion is taking place—a change which is revealed to the observer by the rapid fluctuations which take place in the resistance and render accurate determinations practically impossible. It appears that the fluctuations are due to the steepness of the curve in this region, so that small fluctuations of speeds of flow which would produce no appreciable effect on the resistance when the motion was fully eddying, cause such a relatively large change in the resistance that steady readings cannot be obtained.

When this fall in the value of the ordinate occurs, it will be seen that a relatively small reduction in the value of the abscissæ brings all the plotted points on the theoretical curve for stream-line resistance. As the theory of this type of motion is well known and its results have been constantly checked by observations of the resistance, it has not been considered necessary in the case of the air and water experiments to extend the curve further back than to values corresponding to $vd/\nu = 1250$.

It will be noticed from the results plotted in fig. 3 that the values of vd/ν at which the motion changes from sinuous to stream-line in character is practically constant for all the pipes except the smallest (0.361 cm. diameter). In the case of this small pipe, and to a much smaller but perceptible degree in the 0.7125 and 1.255 cm. pipes, it was found that, both for air and water, under the conditions of admission to the pipe, the stream-line motion tended to persist when the critical (2500) value of vd/ν had been exceeded, until at some value of vd/ν depending on the nature of the orifice and the amount of disturbance of the air at the inlet and possibly other factors the value of $R/\rho v^2$ suddenly rose to the value attained in the other pipes under similar conditions.

In explanation of the apparently anomalous behaviour of this small pipe it may be recalled that OSBORNE REYNOLDS defined two critical velocities in pipes. One is the velocity at which a fluid, which enters a pipe in a high state of turbulence, passes from eddying motion to stream-line motion, and which is well defined in all the pipes used in these experiments except the small one under discussion. The other refers to the case in which a perfectly undisturbed fluid enters a pipe in stream-line motion which persists until, with considerable care, a velocity of about seven times that of the former critical velocity can be reached before it breaks down into eddying motion. If slight disturbances are present this breakdown occurs earlier.

It is evident that the behaviour of the small pipe in these experiments is analogous to the second case of critical velocity mentioned by REYNOLDS, and that the turbulence existing outside the orifice to this pipe was not sufficiently violent to correspond to turbulent flow, whereas in the case of the other pipes it was so. Further confirmation of this was found in the fact that by fitting a bell mouth-piece

inlet to the pipe to ensure steadier conditions, and making a series of observations of surface friction, the velocity at which the flow became fully eddying, as shown by the ordinates of the curve reaching the same value as for the other pipes, was almost precisely that found by REYNOLDS for his upper limit, *i.e.*, at a value of $vd/\nu = 16000$.

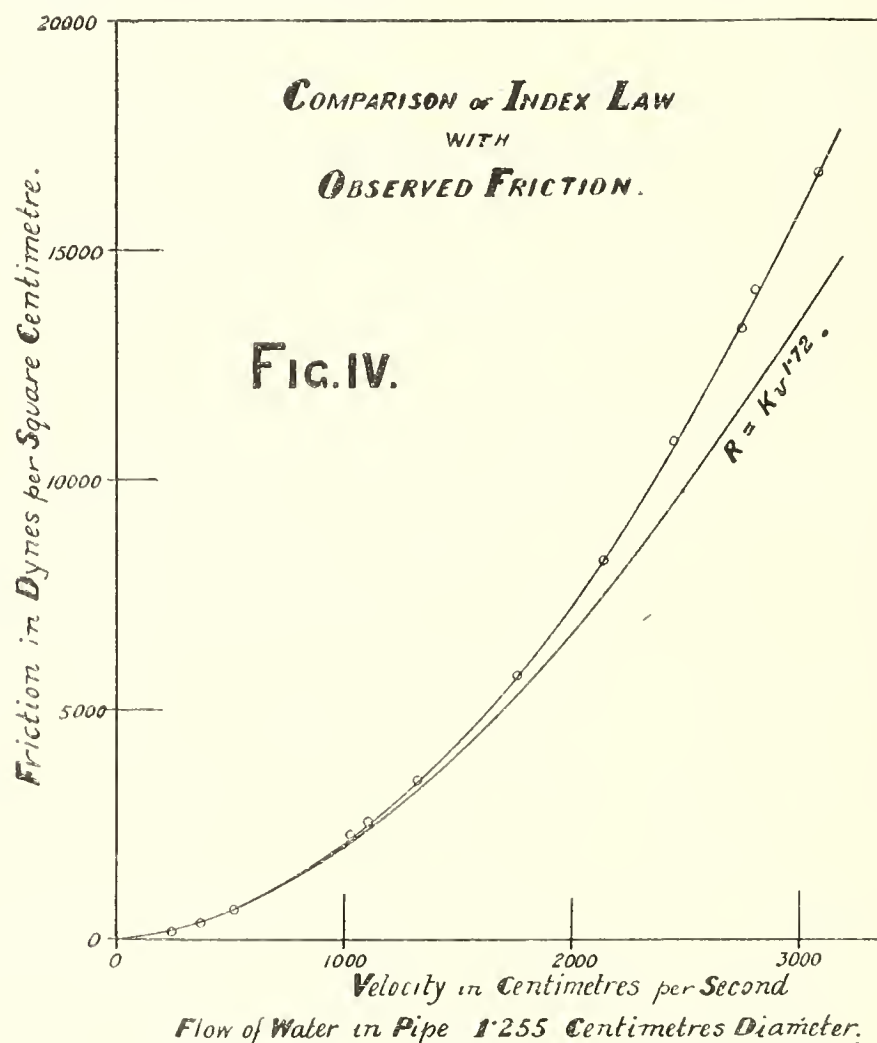
When the mouth-piece was removed and the fluid, which in this case was air, was drawn into the open end of the pipe from the room, by the suction of a gas holder, the observations plotted in fig. 3 were obtained. These show a critical value of vd/ν of about 3000. Beyond this value the results are in complete agreement with the other pipes. Further work in this direction is being made the subject of a separate investigation.

In the experiments on the thick oils it was found impossible, with the pumping appliances available, to reach the critical value of the velocity, so that the experiments were all confined to cases of stream-line motion. The particular oil on which these observations were made had a value of the kinematical viscosity at 15.5°C. of 36.2 , or 3230 times that of water at the same temperature. By heating this to 50°C. the kinematical viscosity could be reduced to 2.1 , but even at this temperature the critical velocity in the 10 cm. pipe used for the experiments would have been $525 \text{ cm. per second.}$ As a matter of interest the results of a series of observations of the surface friction of this oil, when flowing through a steel pipe 10.1 cm. diameter at speeds varying from 5 to $60 \text{ cm. per second,}$ are given in Table IV. and are also plotted in fig. 3. As the value of $R/\rho v^2$ in these observations is in some cases as much as 400 times that of its maximum value in the air and water experiments, a separate diagram has been made in the right-hand corner of fig. 3. In this diagram the curve is drawn from the equation $\frac{R}{\rho v^2} \frac{vd}{\nu} = 8$. A table of viscosities determined by a viscosimeter has been used in the calculation of the values of vd/ν for the abscissæ of the plotted points. Since, in all probability, the apparatus used for the experiments constituted a more refined method of determining the viscosity than the viscosimeter itself, any systematic variation of the plotted points from the curve would not necessarily indicate errors of observation, but it will be seen that the agreement is remarkably good.

The Limits of the Index Law of Resistance.

The determination, in these experiments, of the frictional resistance of a 1.255 cm. pipe, when the velocity has ranged from the first commencement of eddying motion, at a speed of flow of $22 \text{ cm. per second,}$ up to a speed of $3150 \text{ cm. per second,}$ has made possible a check, within these limits, of the accuracy of the well-known index law. This law appears to be due mainly to the observations of WILLIAM FROUDE and OSBORNE REYNOLDS, and there is no doubt that over a moderate range of speed it holds with considerable accuracy. It may be mentioned that the ranges of speeds obtained by OSBORNE REYNOLDS in his experiments on 0.62 and 1.27 cm. pipes were

not large, viz., from 48·0 (the critical velocity) to 469 cm. per second in the 0·62 cm. pipe, and from 23 (the critical velocity) to 705 cm. per second in the 1·27 cm. pipe, and further, that the difficulties in maintaining a uniform speed in these experiments, owing to the fact that the water was drawn direct from the Manchester mains, were such as to render any small variation from the index law, such as would exist over such ranges of speeds, almost impossible to detect. In FROUDE'S experiments* on towing flat boards the range in speed was not greater than from 50 to 500 cm. per second, so that no deviation would be apparent.



In the case of the present experiments the results for the 1·255 cm. pipe were taken and according to REYNOLDS'S method, the logarithms of the friction and velocity from $v = 40$ to $v = 100$ cm. per second were carefully plotted. The points so obtained were found to lie on a straight line whose slope was 1·72 to 1. Assuming a law of resistance $R = kv^n$, where n had this value, k was determined from the low speed observations used for the determination of n and a series of values of R were calculated up to a speed of 3200 cm. per second. Plotting these values and those actually obtained in the experiments the two curves in fig. 4 were obtained, from which it will be seen that by the use of the index law the resistance is underestimated by 5 per cent. at

* 'B.A. Report,' 1872, p. 118.

1000 cm. per second, by 8·5 per cent. at 2000 cm. per second, and by 15 per cent. at 3000 cm. per second.

This, of course, is an extreme case, since if the intermediate speed observations had been taken for the determination of n , a higher value would have been obtained and the error in the use of the index law at the highest velocities would have been much less. In order to show the manner of variation of n throughout the whole range of velocities obtained, the values of n have been determined by the Reynolds method at four different stages and are as follows :—

Velocity in centimetres per second . . .	58.	258.	900.	2250.
Value of n from plotting	1·72	1·77	1·82	1·92.

Similar results showing a gradual increase in the value of n as the velocity increases have been obtained by the reduction of the observations for the 0·7125 and 1·255 cm. pipes, and it may therefore be taken as fully demonstrated that an index law for surface friction cannot be devised which will express the facts with any accuracy, except over a comparatively small range in the value of vd/ν . It will be obvious that this factor must be borne in mind in predicting the skin friction of large bodies moving in a fluid from observations on small-scale models moved in the same fluid.

The Comparison of the Results with those of Previous Experimenters.

As the method of representing the results of the surface friction experiments described in this paper is somewhat novel, it has been considered advisable to reduce the observations of some well-known previous experimenters, in a similar manner, and to plot them all on the same diagram for the purpose of comparison. Taking only experiments on pipes whose roughness was comparable with the ones used by the authors, those chosen for reduction are as follows :—

Experimenter.		Fluid used.	Nature of surface.	Diameter of pipes in centimetres.
DARCY	‘Comptes Rendus de l’Académie des Sciences,’ vol. 38	Water	Drawn lead	2·7 and 4·1
„		„	Bitumen covered	8·26, 19·6, and 28·5
REYNOLDS	‘Phil. Trans. Roy. Soc.,’ 1883	„	Drawn lead	0·62 and 1·27
SAPH and SCHODER	‘Proc. Amer. Soc. Civ. Engs.,’ 1903, vol. 51, p. 253	„	Drawn brass	5·31, 3·81, 3·14, 2·68, 1·60, 0·82, and 0·27
BRIX	‘Phil. Mag.,’ vol. 17, 1909, p. 395	Air	Lead	0·635
„		„	Wrought iron	8·26, 13·2, and 17·1
STOCKALPER	‘Revue Univers. des Mines,’ vol. 7, p. 257	„	Cast and wrought iron	15 and 20

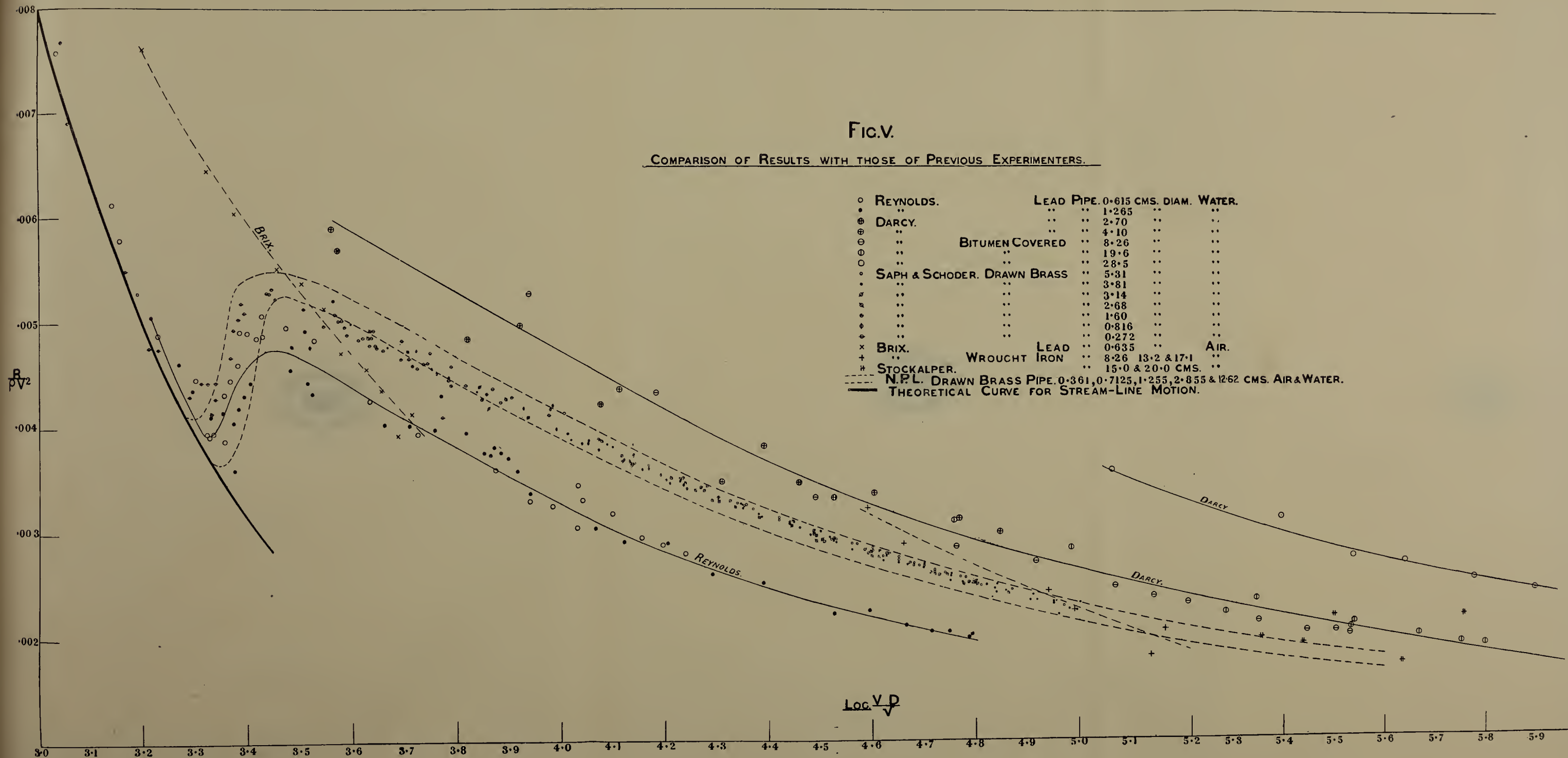
Unfortunately in some of these observations the temperature has not been recorded so that the values of vd/ν calculated are only approximate. The reduced results have been plotted in fig. 5 where, for convenience of comparison, the limits of the results of the present experiments have been indicated by broken lines. In the case of DARCY'S results the points referring to the two lead pipes and the bitumen-covered pipes of 8.6 and 19.6 cm. diameter have been taken as forming one group with approximately the same surface smoothness and a line has been drawn through their mean position. This line shows the same characteristics as the curve of the present experiments, except that the surface is somewhat rougher. The bitumen-covered pipe of 28.5 cm. diameter had evidently a considerably rougher surface, but otherwise the distribution of the points are what would be expected. For REYNOLDS'S experiments on the two lead pipes a mean line has also been drawn which indicates a lower resistance than that found in the pipes used for the present experiments, but the character of the curve and the position of the critical point are identical with those now found. From the point of view of comparison with the present observations neither of the above sets of experiments are so satisfactory as those of SAPH and SCHODER,* since the nature of the surfaces in the two researches was probably identical. The plottings of these, with a very few exceptions, lie within the limits of the present experiments of which they form an excellent check. The whole series of these experiments include observations on pipes of 5.32, 3.81, 3.14, 2.68, 2.08, 1.60, 1.27, 0.95, 0.82, 0.72, 0.66, 0.56, 0.46, 0.38 and 0.272 diameter, of which only the 5.32, 3.81, 3.14, 2.68, 1.60, 0.82 and 0.272 cm. have been reduced for plotting in fig. 5. For the determination of the frictional resistance in the case of the flow of air the amount of reliable data appears to be very small. The six experiments of Dr. BRIX† on iron pipes of 8.26, 13.2, and 17.1 cm. diameter are in fair agreement with the present results, but the interpretation of the series of observations on the 0.635 cm. lead pipe is difficult as the form of the curve appears to indicate that the conditions are those of stream-line flow. The only explanation which can be suggested is that the diameter of the pipe was over-estimated by some 15 per cent., in which case the curve shown through the plotted points would come into fair agreement with the theoretical curve for stream-line flow. The results of STOCKALPER'S experiments, which possess additional interest owing to the fact that the air was under a pressure of about 5 atmospheres, are well in agreement with the present experiments as regards the 20 cm. pipe and less so in the case of the 15 cm. pipe, but unfortunately the range of velocity is not a large one.

In conclusion, a note on the various deductions from theoretical considerations of the criterion for similarity of motion in fluids may be of interest. In a paper read before the Cambridge Philosophical Society in 1850, STOKES‡ considered any number of similar systems composed of similar solids, oscillating in a similar manner in different

* 'Proc. Amer. Soc. Civil Engrs.,' 1903, vol. 51, p. 253.

† 'Phil. Mag.,' vol. 17, 1909, p. 395.

‡ STOKES, 'Mathematical and Physical Papers,' vol. 3, p. 17.





fluids or in the same fluid. Then, if $\alpha, \alpha^1, \dots$, be homologous lines in the different systems, $T, T^1, \dots$, corresponding times, such as the times of oscillation, $c, c^1, \dots$, the maximum excursions of similarly situated points in the fluids and the equations

$$\frac{dp}{dx} = \mu \left\{ \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} \right\} - \rho \frac{du}{dt},$$

$$\&c., \quad \&c.,$$

(which are identical with the general equations of motion of a viscous fluid, in which the terms involving the squares of the velocities are neglected) are satisfied for one system, they will be satisfied for all the systems provided

$$u \propto v \propto w, \quad x \propto y \propto z,$$

and

$$p \propto \frac{\mu u}{x} \propto \frac{\rho u x}{t},$$

i.e., provided

$$u \propto \frac{c}{T}, \quad x \propto \alpha, \quad t \propto T, \quad \text{and} \quad \frac{c^2}{T} \propto \frac{\mu}{\rho}.$$

From this last relation it follows that for dynamical similarity the value of ua/ν must be constant for all the systems.

In 1873, HELMHOLTZ,* in a paper to the Royal Prussian Academy of Sciences, Berlin, gave a somewhat more general treatment of the question. Considering two fluids of densities ρ_1, ρ_2 , and kinematical viscosities ν_1, ν_2 , the conditions under which the motions of the two fluids are similar are determined thus:—

Taking the equation of motion of the first fluid as

$$-\frac{1}{\rho_1} \frac{dp}{dx} = \frac{du}{dt} + u \frac{du}{dx} + v \frac{du}{dy} + w \frac{du}{dz} - \nu_1 \left\{ \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} \right\},$$

and two similar equations, and writing

$$\nu_2 = q\nu_1, \quad \rho_2 = r\rho_1,$$

then in order that these equations may be transformed into the equations for the second fluid

$$-\frac{1}{\rho_2} \frac{dP}{dX} = \frac{dU}{dT} + U \frac{dU}{dX} + V \frac{dU}{dY} + W \frac{dU}{dZ} - \nu_2 \left\{ \frac{d^2U}{dX^2} + \frac{d^2U}{dY^2} + \frac{d^2U}{dZ^2} \right\},$$

&c., under the given conditions of similarity

$$U = nu, \quad V = nv, \quad W = nw,$$

* HELMHOLTZ, 'Wissenschaftliche Abhandlungen,' vol. I., p. 158.

it will be seen that, multiplying (1) by $\frac{n^3}{q}$ the required transformation is effected if the change in the scales of length, time and pressure are

$$X = \frac{q}{n}x, \quad T = \frac{q}{n^2}t, \quad P = n^2rp.$$

It follows, therefore, from the linear scale relation that, if l and L are corresponding linear dimensions of two pipes in which the fluids of densities ρ_1, ρ_2 , and kinematical viscosities ν_1, ν_2 are flowing, in order that the two motions may be similar $L = \frac{\nu_2^n}{\nu_1}l$, or $\frac{VL}{\nu}$ must have the same value for each. Again, from the pressure scale relation it follows that for similar motion the value of $P/\rho V^2$ is the same for each fluid.

The method of Lord RAYLEIGH, which was first applied in considering the size of drops formed under various conditions,* is as follows. Assuming that the resistance depends solely on the velocity, linear dimensions, viscosity and density, and also that the resistance F varies as $\rho^a L^b \nu^c \mu^d$ then if M, L, T are the units of mass, length, and time, the dimensions of F are

$$\left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{T}\right)^c \left(\frac{M}{LT}\right)^d = \frac{ML}{T^2},$$

so that

$$a = 1-d, \quad b = 2-d, \quad c = 2-d,$$

and therefore

$$F \propto \rho v^2 L^2 \left(\frac{\mu}{\rho \nu L}\right)^d,$$

and the resistance per unit area can be written

$$R = \rho v^2 f(vL/\nu).$$

With reference to NEWTON'S Theorem on Similar Motions, in Proposition 32, Book II., of the 'Principia,' the authors are indebted to Dr. R. T. GLAZEBROOK for the following note:—

In this Theorem NEWTON shows that two systems of particles, if started similarly, will continue to move in a similar manner if the acceleration of each system is proportional to V^2/L , V being the velocity and L a linear quantity defining the dimensions of the system—the diameter of a particle. Now denoting by f the acceleration and by ρ the density, we have for fluid friction

$$\frac{L^3 \rho f}{L^2} = \text{force per unit area} = \mu \frac{dV}{dx}.$$

* 'Phil. Mag.,' 1899, vol. 48, p. 321.

Thus the dimensional equation is

$$[f] = \left[\frac{\mu}{\rho} \right] \left[\frac{V}{L^2} \right].$$

But according to NEWTON for similar motion

$$[f] = \left[\frac{V^2}{L} \right].$$

Hence

$$\left[\frac{\mu}{\rho} \right] \left[\frac{V}{L^2} \right] = \left[\frac{V^2}{L} \right],$$

or $\frac{\mu}{\rho} \frac{1}{L \cdot V}$ is independent of dimensions.

The authors also desire to thank Dr. GLAZEBROOK for the facilities given them for carrying out the work and the interest he has shown in the progress of the experiments.

TABLE I.—Observations of Mean and Maximum Velocities.

Mean velocity, centimetres per second.	Observed velocity at axis (maximum).	Rate of mean velocity to maxi- mum velocity.	Nature of flow.
167.5	208.8	0.803	Water in pipe, 2.855 cm. diameter.
142.3	177.5	0.802	
129.9	162.0	0.802	
121.1	150.9	0.802	
105.7	132.4	0.798	
94.6	118.4	0.799	
94.5	118.6	0.798	
85.7	107.4	0.798	
75.7	95.0	0.797	
62.8	79.0	0.795	
42.4	53.7	0.789	
37.0	47.1	0.786	
153.4	191.4	0.802	
145.9	181.2	0.805	
160.6	200.0	0.803	
167.5	207.8	0.806	
217.2	269.4	0.807	
187.5	232.8	0.806	
194.6	246.7	0.789	Water in pipe, 0.7125 cm. diameter.
145.9	187.0	0.780	
125.9	161.9	0.778	
104.4	135.5	0.770	
81.73	107.0	0.764	
65.20	87.07	0.748	

TABLE I.—Observations of Mean and Maximum Velocities (continued).

Mean velocity, centimetres per second.	Observed velocity at axis (maximum.)	Rate of mean velocity to maxi- mum velocity.	Nature of flow.
57·60	77·83	0·740	Water in pipe, 0·7125 cm. diameter.
49·06	67·91	0·722	
39·44	65·70	0·619	
35·12	61·37	0·572	
33·15	58·79	0·564	
28·89	55·60	0·519	
27·28	53·69	0·508	
25·24	49·05	0·515	
24·24	48·63	0·498	
22·25	43·28	0·514	
512·9	642·4	0·799	
450·0	563·3	0·799	
374·8	469·6	0·799	
282·3	356·1	0·793	
668·1	828·6	0·805	
733·0	906·8	0·808	
797·9	991·1	0·805	
872·0	1076	0·810	
940·4	1165	0·807	
1028	1264	0·813	
1117	1376	0·812	
1199	1474	0·814	
573·9	757·2	0·758	Air in pipe, 1·255 cm. diameter.
720·3	937·6	0·768	
115·5	149·0	0·775	
648·8	853·0	0·761	
471·5	631·3	0·747	
363·6	515·4	0·705	
404·2	549·7	0·735	
410·8	558·9	0·735	
893·6	1161	0·770	
240·7	317·4	0·758	Air in pipe, 2·855 cm. diameter.
283·6	372·3	0·762	
207·7	281·8	0·745	
504·5	840·0	0·611	Air in pipe, 0·7125 cm. diameter.
290·5	582·2	0·499	
333·8	669·5	0·498	
364·0	712·2	0·511	
389·0	766·0	0·508	
256·2	505·7	0·505	
324·0	646·2	0·501	

TABLE II.—Experiments with Water.

Mean velocity, centimetres per second. (<i>v</i> .)	Surface friction, dynes per square centimetre. (<i>R</i> .)	Value of $\frac{R}{\rho v^2} 10^2$.	Value of $\frac{vd}{\nu} 10^{-3}$.	Temperature.
				° C.
Pipe No. 1. Diameter 2·855 cm.				
116·3	41·8	0·309	25·32	10·2
125·6	47·8	0·303	27·36	
138·2	56·2	0·294	30·10	
149·7	64·6	0·295	32·62	
136·8	55·2	0·294	30·00	10·5
155·8	68·5	0·282	34·20	
180·0	88·0	0·272	39·5	
201·0	109·2	0·270	44·1	
214·2	120·2	0·261	47·4	10·7
95·0	29·46	0·327	21·0	
104·1	34·20	0·316	23·0	
95·0	29·52	0·328	21·0	
84·8	24·4	0·339	18·8	11·1
73·6	18·95	0·350	16·5	
63·0	14·44	0·363	14·1	
53·7	10·93	0·380	12·0	
46·0	8·26	0·390	10·3	11·3
42·7	7·22	0·396	9·60	
38·2	5·95	0·408	8·58	
33·1	4·74	0·433	7·44	
27·0	3·27	0·449	6·12	11·5
22·3	23·9	0·481	5·06	
29·6	3·86	0·440	6·70	
Pipe No. 16. Diameter 1·255 cm.				
258	199	0·299	29·18	16·0
299	259	0·288	33·80	
343	327	0·278	38·8	
344	331	0·280	38·9	
343	328	0·278	38·8	
258	198	0·298	29·2	
501	640	0·255	56·6	
461	556	0·262	52·2	
423	479	0·268	47·8	
386	404	0·271	43·7	
299	238	0·288	33·8	15·8
447	526	0·263	50·6	
490	612	0·255	55·4	
528	689	0·247	59·7	
552	762	0·250	62·4	
584	842	0·247	66·0	
612	918	0·245	69·2	
632	965	0·242	71·4	
673	1078	0·238	76·2	
699	1162	0·238	79·0	

TABLE II.—Experiments with Water (continued).

Mean velocity, centimetres per second. (<i>v</i> .)	Surface friction, dynes per square centimetre. (<i>R</i> .)	Value of $\frac{R}{\rho v^2} 10^2$.	Value of $\frac{vd}{\nu} 10^{-3}$.	Temperature.
				° C.
Pipe No. 16. Diameter 1.255 cm. (continued).				
730	1247	0.234	82.5	15.8
730	1252	0.235	82.5	
757	1337	0.230	85.6	
796	1460	0.231	90.0	
524	693	0.252	59.2	
232	162	0.301	26.2	
243	177	0.300	27.5	
197	124	0.320	22.3	
151	78	0.342	17.1	14.5
183	108	0.323	20.7	
226	157	0.308	24.6	
199	124.2	0.314	21.7	
201	128.4	0.318	21.9	14.3
193	120.4	0.323	21.0	
187	114.5	0.327	20.4	
173	99.5	0.332	18.9	
163	89.4	0.337	17.8	
152	80.4	0.348	16.6	
136	65.9	0.356	14.9	
170	96.6	0.334	18.7	15.0
169	86.8	0.339	17.6	
147	75.1	0.348	16.2	
134	64.3	0.358	14.8	
116	51.1	0.380	12.8	
116	50.3	0.374	12.8	
129	59.2	0.356	14.2	
124	54.5	0.355	13.7	
115	49.1	0.371	12.7	16.0
105.5	42.4	0.381	11.6	
95.6	36.3	0.397	10.5	
88.4	30.8	0.388	9.75	
79.6	25.6	0.404	8.78	
69.0	19.7	0.413	7.60	
58.1	15.0	0.444	6.40	
54.4	13.5	0.456	6.00	
49.6	11.4	0.461	5.48	16.0
42.7	8.8	0.482	4.70	
36.4	6.4	0.481	4.00	
30.5	4.9	0.526	3.36	
54.0	13.1	0.449	6.12	
61.6	16.6	0.437	6.95	
64.2	17.7	0.430	7.28	
61.7	16.5	0.433	6.98	
33.1	5.6	0.514	3.74	16.0
38.6	7.3	0.488	4.36	
28.3	4.2	0.530	3.20	
26.1	3.6	0.529	2.96	
23.1	2.8	0.521	2.61	16.0
19.6	1.8	0.472	2.22	

TABLE II.—Experiments with Water (continued).

Mean velocity, centimetres per second. (<i>v</i> .)	Surface friction, dynes per square centimetre. (<i>R</i> .)	Value of $\frac{R}{\rho v^2} 10^2$.	Value of $\frac{vd}{\nu} 10^{-3}$.	Temperature.
				° C.
Pipe No. 16. Diameter 1.255 cm. (continued).				
705	1172	0.235	77.0	16.0
798	1480	0.232	87.2	
884	1776	0.227	96.6	
964	2075	0.223	105.2	
1100	2650	0.219	120.2	
1036	2363	0.220	113.2	
38.7	7.28	0.486	4.37	14.5
26.3	3.71	0.536	2.97	
23.5	2.90	0.526	2.66	
19.5	1.63	0.429	2.20	
27.3	3.94	0.530	3.09	
24.8	3.29	0.536	2.80	
140.0	68.8	0.351	15.80	23.0
171.0	96.9	0.331	19.35	
1322	3440	0.197	178	
1767	5755	0.1845	238	
2150	8322	0.1813	289	
2446	10860	0.1816	330	
2762	13360	0.1748	373	23.0
3017	16320	0.1795	407	
1322	3407	0.1950	178	
1800	5796	0.1788	243	
2813	14170	0.1790	380	
3090	16700	0.1750	418	
3188	17660	0.1738	430	
Pipe No. 17. Diameter 0.7125 cm.				
245	225	0.375	13.00	9.3
193	145.9	0.392	10.26	
170.5	117.2	0.403	9.06	
144.4	89.4	0.430	7.74	9.6
114.2	59.6	0.457	6.12	
281	278	0.353	15.27	
316	347	0.348	17.17	10.1
496	742	0.302	27.2	
427	576	0.316	23.4	
356	427	0.337	19.5	10.4
574	962	0.292	31.5	
640	1156	0.282	35.1	
718	1433	0.278	37.6	8.6
822	1847	0.273	43.1	
920	2230	0.264	48.2	
1004	2620	0.260	52.6	

TABLE II.—Experiments with Water (continued).

Mean velocity, centimetres per second. (<i>v</i> .)	Surface friction, dynes per square centimetre. (<i>R</i> .)	Value of $\frac{R}{\rho v^2} 10^2$.	Value of $\frac{vd}{\nu} 10^{-3}$.	Temperature.
				° C.
Pipe No. 17. Diameter 0.7125 cm. (continued).				
1094	3025	0.253	57.7	8.9
1282	4060	0.247	67.6	
1186	3490	0.248	62.5	
169.1	116.0	0.406	9.13	9.0
139.0	84.0	0.435	7.51	
77.5	29.9	0.498	4.18	
68.3	23.8	0.508	3.68	9.7
58.2	17.9	0.528	3.14	
101.3	47.7	0.465	5.47	
49.3	13.0	0.535	2.66	10.6
46.8	11.8	0.540	2.53	
49.4	13.1	0.536	2.66	
44.9	10.8	0.538	2.42	9.8
44.5	10.2	0.518	2.40	
95.5	43.2	0.473	5.15	
110.6	55.8	0.456	5.96	
5254	47500	0.173	415	25.0
5200	45950	0.170	412	
5260	47700	0.1725	416	
2987	16750	0.188	236	
3840	25930	0.176	304	
4180	30700	0.176	331	
1670	58700	0.210	132	
4010	28070	0.175	317	
5070	44350	0.172	402	
4980	42100	0.170	394	
4355	32930	0.174	344	
4470	35350	0.177	354	
4630	36950	0.172	367	
1978	7950	0.203	156	
1117	2970	0.234	88.5	
1353	3970	0.221	107.1	
1485	4665	0.211	117.5	
1776	6490	0.206	140.6	
Pipe No. 18. Diameter 0.361 cm.				
5595	61850	0.1975	202.0	20.5
5410	58330	0.1990	195.3	
5062	52650	0.2045	182.8	
4845	47970	0.2040	175.0	
4425	41420	0.2110	159.7	
3865	31410	0.2100	139.2	
3673	29420	0.2180	132.6	
3140	21880	0.2220	113.3	

TABLE II.—Experiments with Water (continued).

Mean velocity, centimetres per second. (<i>v</i> .)	Surface friction, dynes per square centimetre. (<i>R</i> .)	Value of $\frac{R}{\rho v^2} 10^2$.	Value of $\frac{vd}{\nu} 10^{-3}$.	Temperature.
				° C.
Pipe No. 18. Diameter 0.361 cm. (continued).				
2940	19280	0.2235	106.2	20.5
4640	45000	0.2090	167.6	
4920	49800	0.2050	177.4	
4625	44350	0.2070	167.0	
4325	39000	0.2080	156.2	
3970	33500	0.2120	143.2	
5606	61850	0.1972	202.5	12.0
596	1226	0.346	17.7	
733	1746	0.325	21.2	
873	2353	0.309	25.3	
994	2975	0.302	28.7	
1138	2352	0.290	32.8	
1227	4260	0.283	35.6	12.5
750	1812	0.323	22.0	
602	1233	0.341	17.05	
603	1232	0.339	17.05	

TABLE III.—Experiments with Air.

Mean velocity, centimetres per second. (<i>v</i> .)	Surface friction, dynes per square centimetre. (<i>R</i> .)	Value of $\frac{R}{\rho v^2} 10^2$.	Value of $\frac{vd}{\nu} 10^{-3}$.
Pipe No. 1. Diameter 2.855 cm. Temperature 15° C.			
232.2	0.335	0.506	4.48
252.6	0.384	0.491	4.87
211.2	0.281	0.515	4.07
195.8	0.242	0.514	3.78
186.4	0.224	0.525	3.59
170.1	0.183	0.517	3.28
178.5	0.206	0.528	3.44
278.8	0.456	0.482	5.37
302.0	0.524	0.468	5.82
328.0	0.607	0.458	6.32
346	0.648	0.437	6.67
274	0.430	0.468	5.29
478	1.103	0.394	9.21
404	0.844	0.422	7.78
488	1.190	0.407	9.41
353	0.664	0.434	6.81

TABLE III.—Experiments with Air (continued).

Mean velocity, centimetres per second. (<i>v</i> .)	Surface friction, dynes per square centimetre. (<i>R</i> .)	Value of $\frac{R}{\rho v^2} 10^2$.	Value of $\frac{vd}{\nu} 10^{-3}$.
Pipe No. 1. Diameter 2.855 cm. Temperature 15° C. (continued).			
311	0.544	0.457	6.00
607	1.703	0.378	11.70
675	2.09	0.374	13.00
755	2.53	0.362	14.56
985	4.02	0.348	19.00
1140	5.20	0.325	22.00
1645	10.00	0.302	31.70
1512	8.48	0.303	29.2
1458	8.02	0.307	28.1
1410	7.56	0.310	27.2
1347	6.98	0.313	26.0
1235	5.98	0.320	23.8
1195	5.65	0.323	23.0
1102	4.87	0.327	21.2
Steel Pipe, 10.1 cm. Diameter.			
279	318	0.333	19.1
Pipe No. 16. Diameter 1.255 cm. Temperature 15° C.			
655	2.492	0.476	5.56
383	0.961	0.534	3.25
350	0.830	0.553	2.97
421	1.155	0.532	3.57
482	1.446	0.509	4.09
803	3.570	0.452	6.81
883	4.203	0.440	7.49
454	1.283	0.509	3.85
570	1.952	0.490	4.84
732	3.010	0.458	6.21
537	1.737	0.491	4.55
404	1.062	0.531	3.43
606	2.218	0.493	5.14
323	0.664	0.519	2.74
440	1.206	0.509	3.73
765	3.290	0.459	6.49
978	5.080	0.434	8.30
1152	6.675	0.411	9.78
1317	8.410	0.397	11.18
1465	10.020	0.381	12.43
1051	5.735	0.423	8.91
984	5.060	0.428	8.35
1530	10.930	0.382	12.98

TABLE III.—Experiments with Air (continued).

Mean velocity, centimetres per second. (<i>v</i> .)	Surface friction, dynes per square centimetre. (<i>R</i> .)	Value of $\frac{R}{\rho v^2} 10^2$.	Value of $\frac{vd}{\nu} 10^{-3}$.
Pipe No. 17. Diameter 0.7125 cm. Temperature 15°.5 C.			
460	0.971	0.374	2.215
313	0.617	0.514	1.51
451	0.931	0.373	2.17
579	2.175	0.530	2.78
784	3.800	0.505	3.377
662	2.850	0.530	3.185
579	2.125	0.519	2.780
576	2.075	0.510	2.770
564	1.920	0.493	2.715
557	1.840	0.484	2.768
552	1.790	0.481	2.655
541	1.600	0.447	2.600
504	1.255	0.403	2.425
417	0.870	0.408	2.010
507	1.365	0.433	2.440
536	1.690	0.478	2.580
557	1.850	0.489	2.680
550	1.860	0.503	2.645
441	0.940	0.395	2.120
497	1.175	0.388	2.390
528	1.527	0.449	2.540
387	0.799	0.436	1.860
374	0.779	0.455	1.800
356	0.754	0.484	1.712
305	0.632	0.554	1.468
347	0.706	0.479	1.670
212	0.423	0.768	1.012
285	0.576	0.582	1.370
300	0.602	0.548	1.440
268	0.548	0.627	1.288
273	0.543	0.598	1.310
358	0.727	0.464	1.722
336	0.679	0.490	1.620
327	0.648	0.496	1.570
300	0.610	0.552	1.445
1047	6.440	0.480	5.030
1152	7.750	0.467	5.535
1224	8.360	0.456	5.890
1245	8.560	0.451	5.995
1384	10.300	0.439	6.655
857	4.460	0.497	4.130
Pipe No. 18. Diameter 0.361 cm. Temperature 16°.5 C.			
849	3.46	0.391	2.06
594	2.45	0.566	1.44
344	1.41	0.970	0.838
274	1.11	1.210	0.667
191	0.786	1.756	0.465

TABLE III.—Experiments with Air (continued).

Mean velocity, centimetres per second. (<i>v</i> .)	Surface friction, dynes per square centimetre. (<i>R</i> .)	Value of $\frac{R}{\rho v^2} 10^2$.	Value of $\frac{v l}{\nu} 10^{-3}$.
Pipe No. 18. Diameter 0·361 cm. Temperature 16°·5 C. (continued).			
1080	4·66	0·326	2·63
1270	6·84	0·344	3·10
1310	7·94	0·376	3·19
1310	10·12	0·477	3·20
2390	32·1	0·469	5·95
2710	39·8	0·454	6·80
3040	48·7	0·445	7·67
3370	56·9	0·421	8·55
2020	23·5	0·476	5·01
1580	15·6	0·512	3·91
2050	24·4	0·480	5·25
1370	12·2	0·527	3·39
1740	23·0	0·500	4·30
1260	10·3	0·534	3·10
Pipe No. 12A. Diameter 12·62 cm. Temperature 16°·5 C.			
2183	11·90	0·205	185
2692	16·60	0·188	228
3172	22·15	0·180	269
3790	30·65	0·175	321
4025	34·55	0·175	341
1342	4·84	0·220	114
1004	2·92	0·238	85

TABLE IV.—Experiments with Thick Oil.
Diameter of Pipe 10·13 cm.

Mean velocity, centimetres per second.	Tempera- ture, C.	Density.	Value of ρv^2 .	Value of ν .	Surface friction, dynes per square centimetre. <i>R</i> .	$\frac{R}{\rho v^2} 10^2$.	Value of $\frac{v l}{\nu}$.
45·9	39·5	0·928	1955	3·79	122·6	6·27	123
10·25	37·7	0·929	975	4·29	30·9	31·7	24
35·9	37·2	0·929	1195	4·41	109·8	9·18	82·5
54·9	36·8	0·930	2800	4·58	177·0	6·33	122·0
8·02	36·4	0·930	59·7	4·76	27·2	45·6	17·0
5·12	35·9	0·930	24·4	4·99	17·5	70·0	10·4
57·4	35·9	0·930	3060	4·99	205·0	6·70	117
30·5	35·6	0·930	865	5·11	110·8	12·81	60·6
5·46	35·9	0·930	27·7	4·99	19·5	70·4	11·1
5·80	34·8	0·931	31·2	5·46	22·8	73·1	10·8
55·80	34·5	0·931	2900	5·57	219·3	7·57	101·5

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[PLATE 3.]

BAKERIAN LECTURE.

SERIES LINES IN SPARK SPECTRA.

BY

A. FOWLER, F.R.S.,

ASSISTANT PROFESSOR OF PHYSICS, IMPERIAL COLLEGE, SOUTH KENSINGTON.



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VI. BAKERIAN LECTURE.—*Series Lines in Spark Spectra.*

By A. FOWLER, *F.R.S.*, Assistant Professor of Physics, Imperial College,
South Kensington.

MS. received April 20,—Lecture delivered April 2, 1914.

[PLATE 3.]

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§ 1. *Introductory.*

THE classical work of RYDBERG, and that of KAYSER and RUNGE, dealt mainly with series lines in spectra which are developed in the flame or electric arc, or in vacuum tubes with discharges of moderate intensity. The lines to be discussed in the present communication are some of those which belong to the class of *enhanced* lines as defined by LOCKYER; that is, they are relatively intensified in passing from arc to spark conditions.

In considering these lines it is necessary to take account of the fact that their behaviour in the arc is different for different elements. At least three classes may be recognised:—(I.) Enhanced lines like the H and K lines of calcium, which are well developed in the ordinary arc; (II.) Lines which only appear with small intensities in the arc, such as the enhanced lines of iron; (III.) Lines which do not appear in the ordinary arc (except very locally near the poles), but are strongly developed under spark conditions, as in the case of the well-known magnesium line at λ 4481.

Until very recently the only evidence that enhanced lines may belong to series was that afforded by RUNGE and PASCHEN'S observations of the spectra of magnesium, calcium, strontium, barium, and radium, under the influence of a magnetic field. In the case of each of these elements, three pairs of lines of Class I. were observed, and though no series formulæ could be calculated for them, the magnetic resolutions proved that one pair belonged to the Principal series, another to the Sharp, and the third to the Diffuse series. Further discussion of these series has become possible through the valuable work of LYMAN in the Schumann region, published in 1912.

The present investigation was undertaken in connection with the new series of lines which were produced in 1912 by passing strong condensed discharges through helium tubes containing hydrogen as an impurity.* These lines, of which the strongest is at λ 4686, are of considerable importance in celestial spectroscopy because of their occurrence in the spectra of some of the nebulae, and in stars which are generally regarded as representing the earliest stages of stellar condensation. One of the series, only feebly visible, was in apparent agreement with a series of lines first observed in ζ Puppis by PICKERING, and attributed to hydrogen because of their simple relation to the Balmer hydrogen series. The other was a strong series, which included the lines assigned by RYDBERG to the Principal series of hydrogen from analogy with the spectra of the alkali metals. In addition to the RYDBERG lines, however, the "4686" series included intermediate lines, which the then recognised formulæ suggested was a second Principal series related in a simple manner to the first. As the new lines could not be obtained from hydrogen alone, and in consideration of the occurrence of lines not anticipated by RYDBERG, it was soon felt that further inquiry should be made as to the value of the numerical evidence on which their assignment to hydrogen was chiefly

* 'Monthly Notices, R.A.S.,' vol, 73, p. 62 (December, 1912).

founded. A search for other series of similar character was therefore undertaken, in the hope that some generalisation with regard to such series might be arrived at.

The mode of production of the "4686" lines suggested spark spectra as the most promising source of such series, and, shortly afterwards, experiments on magnesium yielded some new lines of that element which were obviously associated with the spark line 4481 in series of the kind looked for. No satisfactory evidence of relation to other series of magnesium lines, however, was then obtained, and it seemed possible that both the "4686" and the "4481" series were of a new type, having no necessarily simple relation to other known series in the respective spectra.*

The lines of the "4686" and the associated Pickering series have since become of increased importance, in connection with theories of the constitution of the atom, through the theoretical investigations of Dr. BOHR.† Beginning with the "Rutherford" model of the atom, and introducing PLANCK'S quantum, Dr. BOHR has derived a formula for the hydrogen spectrum which excludes these lines, while agreeing closely with the hydrogen series about which there can be no doubt. The "4686" and Pickering series, however, are included in another formula, derived for the emission of helium atoms during the first stage of their reformation when both electrons are supposed to have been removed by the strong discharges employed. This formula is identical, in a first approximation, with that for hydrogen except that the Rydberg constant "N" (= 109,675 for ROWLAND'S scale) has four times its ordinary value. The two "Principal" series previously assigned to hydrogen were thus united in a single formula and attributed to helium, while the Pickering series was made to include intermediate lines coincident with the Balmer series of hydrogen. As the lines in question, whatever their origin, must be regarded as enhanced lines, it was evidently desirable to continue the general investigation of lines of this class.

Further investigation of magnesium, in particular, was also suggested by the discovery made by PASCHEN,‡ and by KING,§ that the line 4481 is a close doublet ($\delta\lambda$ = about 0.2 Å.U.), as it appeared that a valuable indication of the type of series to which it belongs might be obtained if other members could be resolved. Photographs with high dispersion have accordingly been taken for the resolution of the lines and to provide more accurate data for testing the formula for enhanced lines which was suggested by the work of BOHR. Other photographs, taken with smaller dispersion, have resulted in the detection of additional lines which clearly belong to series related to that beginning with 4481. The discussion of the new data for magnesium, and of the data for calcium and strontium given by other observers, has led to some conclusions which are probably of general application to enhanced line series.

The chief results of the investigation are summarised in § 14.

* 'Roy. Soc. Proc.,' A, 1913, vol. 89, p. 133.

† 'Phil. Mag.,' vol. 26, p. 1 and p. 476 (1913).

‡ Communicated privately (1913, August).

§ 'Astrophys. Jour.,' vol. 38, p. 327 (1913).

§ 2. *Wave-lengths of the "4481" Series of Magnesium.*

The members of the "4481" series are essentially spark, or enhanced lines, but they are most conveniently produced in an arc between magnesium poles *in vacuo*, and are best observed in the blue green patch in the region of the negative pole, viewed end-on. Under these conditions they are obtained as very narrow lines, well adapted for accurate measurement. The new determinations of wave-lengths have been based as far as possible on the interferometer values for the lines of iron, or upon grating determinations depending upon such standards, among which those given by Dr. KEIVIN BURNS* have been especially useful.

The Line 4481.—Previous photographs taken in the 1st order of a concave grating of 10 feet radius gave no indication of the resolution of 4481, but photographs recently obtained in the 3rd and 4th orders show it to consist of two clearly separated lines (Plate 3, fig. 6), of which the more refrangible is the stronger, as stated by KING. The lines have no close resemblance to the side components of a reversed line, as the space between them is considerably greater than the thickness of either of them, and, as pointed out by KING, the measured separation is the same in photographs taken with different lengths of exposure, and in different orders of the grating. Iron arc comparisons were exposed both before and after the magnesium, and the plates were measured in the usual way with red right and red left. The mean results from numerous measurements are :—

International scale.	Rowland scale.	$\delta\lambda$.
4481·327	4481·495	
		0·198
4481·129	4481·297	

Dr. KING's values for the two components on ROWLAND's scale are $4481·499 \pm 0·001$ and $4481·284 \pm 0·001$ ($\delta\lambda = 0·215$), but from the reproductions given in his paper, the lines do not appear to have been as narrow as those from which the above results were obtained.

[*Note.*—The new measurements of λ 4481 as a doublet permits of a more satisfactory conclusion as to its presence among the Fraunhofer lines than has hitherto been possible. ROWLAND tabulates two lines, each of intensity "0" at 4481·298 and 4481·515, with an intermediate titanium line (intensity 1) at 4481·438. The agreement for the stronger component, which is a very distinct line in ROWLAND's map, may be considered exact, but the discordance on the less refrangible line exceeds the probable error of measurement, and the intensity is also a little too high in the sun. It may be, however, that both these discordances are caused by a still fainter solar line which is nearly superposed on that of magnesium, since such lines are very numerous. The presence of Mg 4481 as faint lines in the solar spectrum may therefore be considered

* 'Zeitsch. f. Wiss. Photog.,' 1913, XII., 6, p. 219; 'Lick Obs. Bull.,' vol. VIII., No. 247 (1913).

very probable, indicating a condition somewhat more advanced than the ordinary arc, in the direction of the spark.]

The Line 3104.—This line was easily resolved in the 4th order of the grating (Plate 3, fig. 7), and has two components exactly resembling those of 4481, but separated by less than one-tenth of an Ångström unit. The iron arc comparison was photographed through a glycerine solution of para-nitroso-dimethyl-aniline contained in a quartz cell, as recommended by Prof. R. W. WOOD.* This solution absorbs the superposed 3rd order spectrum, but transmits the 4th order lines with reference to which the magnesium lines were measured. The mean results from three excellent plates with five comparison spectra are as follows :—

International scale.	Rowland scale.	$\delta\lambda$.
3104·805	3104·929	
		0·092
3104·713	3104·837	

The Line 2661.—On the assumption of a constant wave-number interval between the components of the resolved lines, the separation at 2661 would be about 0·07 Å.U. which is very near the limit of resolution of the grating. Hence with the exposures of 3 to 5 hours required in the 3rd and 4th orders, a very slight unsteadiness of conditions, such as might arise from small variations of temperature, would tend to mask any resolution which might otherwise be effected. Many 3rd order plates were taken, all of which show the line broadened, but only one suggests resolution, and on none of the 4th order plates was this result improved upon. Measurements of the best 3rd order plate gave the following wave-lengths :—

International scale.	Rowland scale.	$\delta\lambda$.
2660·821	2660·909	
		0·066
2660·755	2660·843	

These values are in good accordance with the mean of several determinations from 1st order plates, namely, 2660·785 (I. Å.), 2660·873 (R).

The Remaining Lines.—It has not been found possible to photograph beyond the first three lines in the higher orders of the grating, but the wave-lengths of the next two, 2449 and 2329, have been checked by a photograph in the first order, giving a dispersion of 5·5 Å.U. per mm. Wave-lengths of iron lines given by BURNS were used for the line 2449, but were not available for 2329. The lines 2253, 2202, 2166 do not appear on any of the grating photographs, but are well shown on the original plates taken with a quartz spectrograph, giving an average dispersion in this region of about 6 Å.U. per mm. Further measurements have been made, but in the absence of better standards it has not been found necessary to modify the values given in the

* 'Phil. Mag.,' Series 6, vol. 6, p. 257 (1903).

previous paper. The reference spectrum for these lines was the copper arc, as given by KAYSER and RUNGE, and the lines are subject to an uncertainty of about 0.05 Å.U. in absolute value.

All the necessary details with regard to the "4481" series are brought together in Table I., the corrections to vacuum having been made from KAYSER's table.*

TABLE I.—The "4481" Series of Magnesium.

λ , wave-length (I.Å.).	Estimated limit of error in λ .	$\delta\lambda$.	n , wave-number (<i>in vacuo</i>).	Estimated limit of error in n .	δn .	Remarks.
4481.327	0.005	0.198	22,308.68	0.025	0.99	From 3rd and 4th order plates.
4481.129	0.005		9.67	0.025		
3104.805	0.005	0.092	32,198.99	0.05	0.94	From 4th order plates.
3104.713	0.005		9.93	0.05		
2660.821	0.01	0.066	37,571.43	0.14	0.94	From 3rd order plate.
2660.755	0.01		2.37	0.14		
2449.573	0.01		40,811.31	0.17		} From 1st order plate.
2329.58	0.02		42,913.30	0.4		
2253.87	0.05		44,354.65	1.0		} From quartz spectrograph.
2202.68	0.05		45,385.36	1.0		
2166.28	0.05		46,147.86	1.1		

From the separations of the three lines which have been resolved, it appears at once that the interval between the components, in wave-numbers, is sensibly constant, and that the series is therefore not of the ordinary Principal type. In a Principal series having a separation of 0.99 for the first member at 4481, the interval at 3104 would be reduced to 0.64, and at 2660 to 0.44, but there is no evidence of such contraction.

§ 3. *Formulae for the Mg "4481" Series.*

The application to these lines of the ordinary formulæ of RYDBERG, RITZ, or HICKS, requires the division of the lines into two series by taking alternate lines, the two

* 'Handbuch der Spectroscopie,' Band II., p. 514.

series thus resembling the Diffuse and Sharp series which occur in the spectra of other elements. Two such series can only be included in a single formula in the special case when the fractional terms associated with the parameter m differ by exactly 0.5, the Rydberg constant N being then replaced by $4N$, if m takes successive integral values. In the general case, an objection to this combination would be the smaller intensities of the lines of the Sharp as compared with those of the Diffuse series. The intensities of the "4481" lines, however, are graded as in an ordinary series, and the chief objection to uniting them in a single formula in the first discussion was the undesirability of introducing a new type of series formula for what seemed to be a special case, so long as the older formulæ could be employed. This objection is no longer valid, because, as will appear later, several other series of the same type, occurring in associated groups, have since been recognised.

From the formulæ previously given for the two divisions of the "4481" lines, it was evident that all the lines could be combined, at least very nearly, in one equation if desired. There were, however, small deviations, apparently systematic, which suggested that the union was not quite exact, but these have been removed by the new measurements, which became necessary when it appeared that the lines were doublets. It can no longer be doubted that the lines form a single series, differing from the more familiar series in that the lines occur twice as frequently; *i.e.*, if the ordinary formulæ involving the Rydberg constant N be employed, we find lines corresponding not only to the integral values m , but also to $m + \frac{1}{2}$. It will, however, be more convenient to employ formulæ in which m takes successive integral values only, N being then replaced by $4N$ or by a number of that order of magnitude.

As the positions of the lines have been determined with considerable precision, and the lines are fairly numerous, it seemed desirable to compare the accuracy of a variety of formulæ, some with N assumed the same as for hydrogen, and others with this term calculated from the lines themselves. The fact that not all the lines have been resolved, and the slight uncertainty as to the absolute positions of the last two or three lines of the series, renders the comparison to some extent wanting in finality, but the results may nevertheless be of interest.

For comparison of observed and calculated wave-numbers, the unresolved lines have been regarded as consisting of two components separated by 0.99, one having a wave-number greater by 0.33 than that of the unresolved line, and the other 0.66 less, since the more refrangible component is about double the intensity of the less refrangible. The wave-numbers adopted in the calculations for the less refrangible components, adjusted in this manner, are given in the first column of Table II. The wave-numbers are on the International scale, and have been corrected to vacuum. On this scale, the value of the Rydberg constant, according to an investigation by W. E. CURTIS* is 109,679.3.

* Not yet published.

The formulæ which have been calculated are as follows :—

I. RYDBERG'S, with N as for hydrogen,

$$n = 49,770.45 - \frac{4 \times 109,679.3}{(m + 0.996943)^2} \quad m = 3, 4, \dots$$

II. RYDBERG'S, with numerator calculated,

$$n = 49,781.21 - \frac{4 \times 109,883.9}{(m + 0.999887)^2} \quad m = 3, 4, \dots$$

III. HICKS', with N as for hydrogen,

$$n = 49,775.20 - \frac{4 \times 109,679.3}{\left(m + 0.994541 + \frac{0.006170}{m}\right)^2} \quad m = 3, 4, \dots$$

IV. RITZ'S, with N as for hydrogen,

$$n = 49,774.76 - \frac{4 \times 109,679.3}{\left(m - 0.004714 + \frac{0.021503}{m^2}\right)^2} \quad m = 4, 5, \dots$$

V. RYDBERG'S, with numerator calculated, and $(m + \mu)$ assumed an integer

$$n = 49,781.63 - \frac{4 \times 109,891.8}{m^2} \quad m = 4, 5, \dots$$

VI. Same as V.; least square solution, using all the lines,

$$n = 49,780.44 - \frac{4 \times 109,885.65}{m^2} \quad m = 4, 5, \dots$$

The observed *minus* computed values resulting from these formulæ are shown in Table II.

TABLE II.—Less Refrangible Components of Mg “4481” Series.

Wave-number (I.Å.), <i>in vacuo.</i>	Esti- mated limit of error.	I.	II.	III.	IV.	V.	VI.	Remarks.
22,308.68	0.02	0.00*	0.00*	0.00*	0.00*	0.00*	-0.35	} Lines unresolved but wave-num- bers adjusted.
32,198.99	0.05	-1.30	0.00*	0.00*	0.00*	+0.05	+0.25	
37,571.43	0.14	0.00*	0.00*	0.00*	0.00*	0.00*	+0.51	
40,810.65	0.17	+1.43	-0.15	+0.21	+0.25	-0.22	+0.47	
42,912.64	0.4	+2.39	-0.63	+0.25	+0.34	-0.75	+0.05	
44,353.99	1.0	+3.48	-0.71	+0.70	+0.85	-0.88	0.00	
45,384.70	1.0	+4.11	-1.05	+0.86	+1.06	-1.26	-0.31	
46,147.24	1.1	+4.57	-1.37	+1.00	+1.23	-1.60	-0.62	
(46,727.2)†				(+1.15)				

* Used in calculation of constants.

† Calculated from equation III. assuming estimated error of formula.

It will be seen that the lines are very closely represented by any of the formulæ. Even the simple Rydberg equation (I.) leaves no error so great as a quarter of an Ångström unit, and it would require wave-lengths of still greater accuracy to test the relative merits of the other formulæ. In each case, however, the residuals, although very small, are sufficiently systematic to indicate that not one of the formulæ can be considered exact, unless some unrecognised source of systematic error in the determination of the wave-numbers may eventually be traced. In choosing between the different forms of equation it is accordingly necessary to be guided by experience of their application to other series of the same character, if they can be found. The result of such trials, as will presently appear, is to indicate that the Hicks form of equation is the one which most closely accords with the observations in general.

In the absence of an exact formula, a consideration of the residuals given in Table II. indicates that the limits of the "4481" series may be taken as 49778.0 and 49779.0 with very small probable error, and these values will be adopted in the subsequent discussion.

§ 4. *The Doublet Series of Calcium, Strontium, and Barium.*

Other series of lines apparently similar in character to the "4481" series of Mg were found in LYMAN'S observations of narrow doublets of Ca, Sr, and Ba in the Schumann region,* combined with SAUNDER'S observations in the ordinary ultra-violet. While the investigation was in progress, however, a discussion of the available observations of these series was published by E. LORENSER,† who also found that the ordinary formulæ could not be applied to them. LORENSER has further established that these series stand in the relation of Fundamental (F), or "Bergmann," series to the systems of series comprising the well-known wider doublets in the spectra of these elements, of which the H and K lines of calcium may be quoted as a familiar example. In each case the separation of the narrow doublets of the F series is identical with that of the first member and its satellite in the Diffuse (D) series of the wider doublets; and the limits of the F are apparently identical with the variable parts of the expressions for these lines.

Omitting Ba, which presents some difficulties, though generally conforming, LORENSER'S formulæ‡ are as follows, the Sharp series being indicated by S:—

* 'Astrophys. Jour.,' vol. 35, p. 341 (1912).

† 'Dissertation,' Tübingen (1913).

‡ Formulæ of this type were first employed for the doublets of Ba by SAUNDERS ('Astrophys. Jour.,' vol. 32, p. 164, 1910). In a later paper SAUNDERS also indicated a connection between the limits of the wide and narrow doublets ('Physical Review,' Series 2, vol. 1, p. 332, 1913).

Calcium—

$$\text{Wide doublets.} \left\{ \begin{array}{l} D(m) = \frac{70,117}{70,340} \\ S(m) = \frac{70,117}{70,340} \end{array} \right\} - \frac{423,416}{(m-0.69117)^2},$$

$$\left\{ \begin{array}{l} D(m) = \frac{70,117}{70,340} \\ S(m) = \frac{70,117}{70,340} \end{array} \right\} - \frac{421,559}{(m+0.617844)^2}.$$

$$\text{Narrow doublets} F(m) = \frac{81,820.0}{81,880.8} \left\{ \begin{array}{l} D(m) = \frac{81,820.0}{81,880.8} \\ S(m) = \frac{81,820.0}{81,880.8} \end{array} \right\} - \frac{430,959}{(m-0.039796)^2}.$$

Strontium—

$$\text{Wide doublets.} \left\{ \begin{array}{l} D(m) = \frac{64,154}{64,954} \\ S(m) = \frac{64,154}{64,954} \end{array} \right\} - \frac{410,836}{(m-0.59254)^2},$$

$$\left\{ \begin{array}{l} D(m) = \frac{64,154}{64,954} \\ S(m) = \frac{64,154}{64,954} \end{array} \right\} - \frac{415,157}{(m+0.684636)^2}.$$

$$\text{Narrow doublets} F(m) = \frac{73,833.5}{74,113.2} \left\{ \begin{array}{l} D(m) = \frac{73,833.5}{74,113.2} \\ S(m) = \frac{73,833.5}{74,113.2} \end{array} \right\} - \frac{430,554}{(m-0.056189)^2}.$$

The numeration of the members is shown in Tables III. and IV. (LOR.=LORENSER; FOW.=FOWLER).

The negative terms corresponding to $m = 3$ in the D series, and to $m = 1.5$ in the S series (the latter giving the first P doublet), are given very erroneously by the formulæ, as is so often the case in the more familiar series. Nevertheless, LORENSER has fully established the connection of the F with the D series, through the usual relations, to which reference has already been made. There is sufficient evidence that the lines in question are to be regarded as enhanced lines, and, though LORENSER does not appear to have recognised this fact, his work proved that enhanced lines may form groups of related series which are generally similar to those occurring in arc spectra, but require modifications of the usual formulæ.

It will be observed that in LORENSER's formulæ the numerator is of the order of $4N$ as in those already found for the Mg "4481" series, and the question arises whether $4N$ may not be the universal constant for enhanced lines, as N is universal for series of lines produced under less violent excitation. The series are not long enough to give a very satisfactory comparison of the relative accuracy of different formulæ, but, so far as it goes, the evidence is distinctly in favour of the Hicks formula with $4N$ for the numerator. For the F series the formula may take the simpler Rydberg form with sufficient accuracy.

The formulæ for the above series are then as follows, the wave-numbers being on ROWLAND's scale to facilitate comparison with LORENSER's figures:—

Calcium—

$$\begin{aligned} \text{Wide doublets} \quad . \quad . \quad . \quad & \left\{ \begin{aligned} D(m) &= \frac{70,289.2}{70,512.2} \end{aligned} \right\} - \frac{4 \times 109,675}{\left(m + 0.396204 - \frac{0.106980}{m}\right)^2}, \\ & \left\{ \begin{aligned} S(m) &= \frac{70,289.2}{70,512.2} \end{aligned} \right\} - \frac{4 \times 109,675}{\left(m + 0.222264 - \frac{0.143832}{m}\right)^2}. \\ \text{Narrow doublets} \quad . \quad . \quad & F(m) = \frac{81,948.6}{82,009.4} \left\} - \frac{4 \times 109,675}{(m - 0.013703)^2}. \end{aligned}$$

Strontium—

$$\begin{aligned} \text{Wide doublets} \quad . \quad . \quad . \quad & \left\{ \begin{aligned} D(m) &= \frac{64,323.3}{65,123.3} \end{aligned} \right\} - \frac{4 \times 109,675}{\left(m + 0.611476 - \frac{0.283578}{m}\right)^2}, \\ & \left\{ \begin{aligned} S(m) &= \frac{64,323.3}{65,123.3} \end{aligned} \right\} - \frac{4 \times 109,675}{\left(m + 0.344681 - \frac{0.233220}{m}\right)^2}. \\ \text{Narrow doublets} \quad . \quad . \quad & F(m) = \frac{73,968.3}{74,248.0} \left\} - \frac{4 \times 109,675}{(m - 0.028718)^2}. \end{aligned}$$

The numeration of the members is shown in Tables III. and IV.

Details as to wave-lengths and wave-numbers of the lines involved, as quoted by LORENSER, are shown in Tables III. and IV., which also indicate the differences between the wave-lengths observed and those calculated from the two sets of equations.

An inspection of the tables will show that the observations are more closely represented in every case, and especially in the case of the negative terms, by the Hicks formula involving $4N$ than by the formula which has been employed by LORENSER. The superiority of the former is further shown in the greater accuracy with which it proves the identity of the limits of the F series with the variable parts of the expressions for the first line of the D series and its satellite. LORENSER has assumed this identity in determining the limits of the F series, thus—

$$\text{Ca: } 70,117 + 11,703.0 = 81,820.0$$

$$70,117 + 11,763.8 = 81,880.8$$

$$\text{Sr: } 64,154 + 9,679.5 = 73,833.5$$

$$64,154 + 9,959.2 = 74,113.2$$

If, however, the formulæ for the F series be calculated entirely from the F lines themselves we get the equations

TABLE III.—Calcium Doublets.

Wide—Diffuse series.					Wide—Sharp series.					Narrow—Fundamental series.				
λ (Row- LAND).	n .	m .		$O - C (\Delta\lambda)$.	λ (Row- LAND).	n .	m .		$O - C (\Delta\lambda)$.	λ (Row- LAND,) [†]	n .	m .	$O - C (\Delta\lambda)$.	
		Lor.	Fow.				Lor.	Fow.					Lor.	Fow.
- 8498.35*	- 11,763.8	3	2	- 2195	- 1825									
- 8542.47	- 11,703.0													
- 8662.50	- 11,540.9													
3181.4*	31,423.8	4	3											
3179.45	31,443.0			0.0	0.0									
3158.98	31,646.8													
2113.01	47,311.2	5	4	0.0	0.0									
2103.47	47,525.8													
1815.0	55,096	6	5	- 0.08	+ 0.24									
1807.8	55,316													
1680.5	59,506	7	6	- 0.76	0.0									
1674.1	59,733													

* Satellites.

† First doublet of Principal series.

‡ Wave-lengths *in vacuo*.

TABLE IV.—Strontium Doublets.

Wide—Diffuse series.					Wide—Sharp series.					Narrow—Fundamental series.				
λ (Row- LAND).	n .	m .		$O - C (\Delta\lambda)$.	λ (Row- LAND).	n .	m .		$O - C (\Delta\lambda)$.	λ (Row- LAND).†	n .	m .	$O - C (\Delta\lambda)$.	
		LOR.	FOW.				LOR.	FOW.					LOR.	FOW.
-10,038.3*	- 9,959.2	3	2	-5945	-4077.88†	-24,515.7	1.5	2	-300.5	2166.11	46,151.6	4	0.0	0.0
-10,328.3	- 9,679.5				-4215.66†	-23,715.7				2152.82	46,436.4			
-10,915.0	- 9,159.2													
3475.01*	28,768.8	4	3	0.0	4305.60	23,219.2	2.5	3	0.0	1778.8	56,217	5	0.0	0.0
3464.58	28,855.4				4161.95	24,020.6				1769.8	56,503			
3380.89	29,569.7													
2324.60*	43,005.2	5	4	0.0	2471.71	40,445.9	3.5	4	0.0	1620.7	61,702	6	-1.4	-0.96
2322.47	43,044.6				2423.67	41,247.4				1613.3	61,985			
2282.14	43,805.2													
1995.7	50,092	6	5	+1.30	2053.3	48,687	4.5	5	+0.93					
1965.2	50,869			+0.5	2020.5	49,477			+0.7					
1847.0	54,142	7	6	+0.06										
1820.0	54,945			0.0										

* Satellites.

† First doublet of Principal series.

‡ Wave-lengths *in vacuo*, except in case of first pair.

$$\text{Ca : F } (m) = \frac{82,044.8}{82,105.6} \left\} - \frac{444,561}{(m+0.00586)^2},$$

$$\text{Sr : F } (m) = \frac{74,236.6}{74,516.3} \left\} - \frac{455,055.1}{(m+0.025265)^2}.$$

Thus, when LORENSER'S formula is used throughout the discrepancies are

$$\text{Ca : } 82,044.8 - 81,820.0 = 224.8$$

$$\text{Sr : } 74,236.6 - 73,833.5 = 403.1$$

The Hicks 4N formula gives the variable parts of the expressions for the first D lines and their satellites as

$$\text{Ca : } 70,289.2 + 11,703.0 = 81,992.2$$

$$70,289.2 + 11,763.8 = 82,053.0$$

$$\text{Sr : } 64,323.3 + 9,679.5 = 74,002.8$$

$$64,323.3 + 9,959.2 = 74,282.5$$

Comparing these values with the limits calculated from the observed F lines by means of the formula with 4N, we have the discrepancies reduced to

$$\text{Ca : } 81,992.2 - 81,949 = 43.2$$

$$\text{Sr : } 74,002.8 - 73,968.3 = 34.5$$

The discussion of the available data for calcium and strontium, therefore, leads to the conclusion that series consisting of enhanced lines may be represented by the formula of HICKS if the ordinary numerator N be replaced by 4N. It is, of course, to be understood that the formula is subject to the same limitations as the one involving N as applied to ordinary series.

There is, however, one important difference between the enhanced line series of Ca, Sr, and Ba, and the ordinary arc series of these and other elements, namely, that in the case of enhanced lines the first observed member of the Diffuse series appears with a negative sign, while in the corresponding ordinary series it has a positive sign. In each case the limit of the F series and the separation of the doublets are nevertheless derived from this first member taken with proper sign. This is the "new relation" discovered by SAUNDERS in connection with the wide and narrow doublets of Ca, Sr, and Ba, namely, "The convergence wave-number of the complex pair series (D) *plus* the wave-number of the first term equals the convergence wave-number of the narrow pair series (F)."

The modified Hicks formula may accordingly be employed with confidence in searching for possible relationships of the "4481" series with other series of enhanced lines of magnesium.

§ 5. *The Doublet System of Magnesium.*

The spectrum of magnesium, as is well known, also contains doublets similar to those occurring in calcium, strontium, and barium. They do not appear in the flame spectrum; they occur in the arc and are certainly intensified in the spark, and are therefore to be regarded as enhanced lines of Class I. LORENSER has discussed these lines, but has not identified any associated Fundamental series such as were found for the other three elements. The lines in question are indicated in Table V.

TABLE V.—Magnesium Wide Doublets.

Diffuse series.				Sharp series.			
λ (ROWLAND).	n .	m .		λ (ROWLAND).	n .	m .	
		LOR.	FOW.			LOR.	FOW.
				— 2795·63* — 2802·80*	— 35,759·9 — 35,668·4	1·5	1
2798·07 2790·88	35,728·7 35,820·7	3	2	2936·61 2928·74	34,043·1 34,134·6	2·5	2
1737·8 1735·0	57,544 57,637	4	3	1753·6 1750·9	57,025 57,113	3·5	3

Unfortunately, the observed lines are so few that the equations of the two series can only be determined completely by utilising all the lines in the calculation of constants, the limit derived from the Sharp series being assumed for the Diffuse. LORENSER'S formulæ with the numeration indicated in Table V. are

$$S(m) = \left. \begin{array}{l} 84,893\cdot0 \\ 84,984\cdot5 \end{array} \right\} - \frac{413,202\cdot5}{(m+0\cdot35060)^2},$$

$$D(m) = \left. \begin{array}{l} 84,893\cdot0 \\ 84,984\cdot5 \end{array} \right\} - \frac{423,376\cdot6}{(m+0\cdot065474)^2}.$$

The Hicks formulæ for these series, with the numeration shown under "Fow." in Table V., are

$$S(m) = \left. \begin{array}{l} 85,479\cdot8 \\ 85,571\cdot3 \end{array} \right\} - \frac{4 \times 109,675}{\left(m + 0\cdot938644 - \frac{0\cdot036421}{m}\right)^2},$$

$$D(m) = \left. \begin{array}{l} 85,479\cdot8 \\ 85,571\cdot3 \end{array} \right\} - \frac{4 \times 109,675}{\left(m + 0\cdot949439 - \frac{0\cdot040110}{m}\right)^2}.$$

* Also the first doublet of the Principal series.

These Mg doublets are analogous to the wider doublets of Ca, Sr, and Ba :—(1) They occur under precisely similar experimental conditions ; (2) They show similar Zeeman effects (RUNGE and PASCHEN) ; (3) The separations of the components—Mg 91·5, Ca 223, Sr 801, and Ba 1691—are roughly proportional to the squares of the atomic weights ; (4) The limits of the series follow a natural sequence, decreasing in the usual way as the atomic weight increases.

The analogy, however, is incomplete. In Ca, Sr, and Ba, the first negative terms of the D series, (given by $m = 2$ in the Hicks formula) correspond, as already mentioned, to observed lines (Tables III. and IV.) having satellite separations identical with those of the doublets in the associated F series ; in these cases the satellite lies on the violet side of the more refrangible component of the D doublet, and not on the red side of the less refrangible component as in the positive terms of the same series. In Mg, on the other hand, the D pair given by $m = 2$ in the Hicks formula has a positive sign, and no lines have been observed near the position of the negative term (25,350 or about λ 3940) given by $m = 1$. An associated F series, by analogy, would be expected to have its limits near 110,830 ($= 85,480 + 25,350$), but there is no evidence of the existence of such a series.

The absence of a negative term in the observed D doublets of Mg suggests, as an alternative view, that the associated F series may be derived from the first line actually observed, namely λ 2798. In that case LORENSER'S formula would give for the limit of the F series : $84,893\cdot0 - 35,728\cdot7 = 49,164\cdot3$, and the Hicks formula $85,479\cdot8 - 35,728\cdot7 = 49,751\cdot1$.

These figures at once suggest the limit 49,778 of the "4481" series ; and the available data for the Wide doublets are, in fact, consistent with the assumption that the agreement is exact. It would seem then that the "4481" series may be a Fundamental series deriving its limits from the first positive line of the D series of Wide doublets. In that case the D line λ 2798 would be expected to have a satellite with the same separation as the lines of the "4481" series, namely, $\delta n = 0\cdot99$. A special search for the satellite has been made in the photographs of the arc *in vacuo* taken in the fourth order of the grating, but it has not been directly observed. Indirect evidence of its possible existence, however, is afforded by careful measurements of the intervals of the Wide pairs. The new wave-lengths are given on the International Scale in Table VI. It will be seen that while the pairs of the P and S series are in close agreement, indicating a normal separation of 91·5, the interval of the D pair is not less, as would usually be the case if a satellite were present, but greater. This is confirmed by the recent measures of these lines by NACHEN.* The measurements thus suggest that there is a satellite on the *more* refrangible side of the chief line, the separation being comparable with that in the "4481" series if we assume that the tabulated λ 2797·989 refers to the chief line itself. Such a reversal of the usual position of the satellite, however, would accord

* 'Zeitsch. f. Wiss. Photog.,' 1913, XII., 2, p. 59.

with the fact that the more refrangible component of the "4481" lines is also the stronger.

While it is thus probable that the "4481" series is numerically related to the D series of Wide doublets, there are objections to regarding it as directly derived from that series—

- (1) The "4481" lines, unlike the doublets, do not occur in the ordinary arc;
- (2) The separation of the components of the "4481" lines is much too small compared with the F doublets of Ca, Sr, and Ba; the latter are roughly proportional to the squares of the atomic weights—Ca 65, Sr 285, Ba 575—and a corresponding F series of Mg would be expected to have a separation of about 20 in place of the 0.99 actually observed in the "4481" lines.

The probable nature of the relation, however, has been revealed by the discovery of a new system of doublets, with which the "4481" series appears to be directly associated. Before referring to these new lines it will be convenient to give further data for the Wide doublets, for comparison with similar data which will appear later for the new series. On the assumption that the variable part of the expression for the first observed pair of the D series is identical with one limit of the "4481" series, the corrected limit of the less refrangible components (I.Å.) becomes $49,779.0 + 35,729.6$, or $49,778.0 + 35,730.6^* = 85,508.6$, and that of the more refrangible components, $85,508.6 + 91.5 = 85,600.1$. These values have been utilised in the construction of Table VI. It is only necessary to explain that the symbols mD , mS ,

TABLE VI.—Mg Wide Doublets. Revised Data.

$$\text{Limits } \begin{cases} 85,508.6. \\ 85,600.1. \end{cases}$$

<i>m.</i>	D series.				S series.			
	λ (I.Å.).	<i>n.</i>	$\delta n.$	$mD.$	λ (I.Å.).	<i>n.</i>	$\delta n.$	$mS.$
1					$-2795.523^\dagger$	$-35,761.15$	91.54	121,269.7
					$-2802.698^\dagger$	$-35,669.61$		121,269.7
2	2797.989	35,729.63 (35,730.6) ‡	92.44	49,779.0 49,778.0	2936.496	34,044.44	91.48	51,464.2
	2790.768	35,822.07		49,778.0	2928.625	34,135.92		51,464.2
3		57,546 57,639		27,962.6 27,961.1		57,027 57,115		28,481.6 28,485.1

* Assumed wave-number of satellite to λ 2797.989.

† First P doublet.

‡ Wave-number of probable satellite.

in the convenient notation adopted by PASCHEN in his extensive researches on series, represent the differences between the wave-numbers of the observed lines and the limits of the series to which they belong. Thus, if the Hicks equation were exact, the symbol 3D would be an abbreviation for $4N / \left(m + D + \frac{\Delta}{m} \right)^2$ when m has the value 3, and so on. Defects of formulæ are eliminated in evaluating the variable parts of the expressions in this manner, except as regards the determination of the limits. In the present case it is probable that the limits are also exact.

The differences between the two values of 3D, and between those of 3S are to be attributed to errors of observation.

§ 6. *A New System of Magnesium Doublets.*

Two conspicuous doublets, a little more refrangible than λ 4481 (Plate 3, fig. 2), were recorded in the magnesium arc *in vacuo* by FOWLER and PAYN in 1903.*

Following E. E. BROOKS,† who also investigated their occurrence in the magnesium spectrum, it will be convenient to designate these the “F and P” or “F.P.” Doublets in order to distinguish them from the “Wide” Doublets which have already been discussed, and to avoid confusion with the term “Narrow” Doublets as applied to the F series of Ca, Sr, and Ba. The wave-lengths of these lines were given more accurately by FOWLER‡ in a later paper as 4434·20, 4428·20, 4390·80, and 4384·86 (ROWLAND), from which the wave-number intervals in the two doublets are 30·5 and 30·8 respectively.

The lines evidently form two related pairs, and the experimental conditions for their production are identical in every respect with those for λ 4481, thus differing from the Wide Doublets in the ultra-violet. It appeared, therefore, that the “F.P.” Doublets might belong to series, the investigation of which would throw further light on the relationships of the “4481” series.

Six other doublets with the same interval were eventually traced, and an additional one in the infra-red has been recorded by LORENSER. The doublets occur in groups of two, the more refrangible being slightly the stronger in each group. They fall readily into two series, and as in the other enhanced line series which have been considered, it is necessary to employ the term 4N for the numerator in the formulæ.

The stronger doublets have been regarded as belonging to the Diffuse, and the weaker ones to the Sharp series. According to HICKS this classification is supported by the signs of the last terms of the denominators when his formula is employed.

Measurements of the lines have been made on photographs taken with the quartz spectrograph (Plate 3, figs. 4 and 5), with the exception of the doublets near λ 4481,

* ‘Roy. Soc. Proc.’ 1903, vol. 72, p. 255.

† ‘Roy. Soc. Proc.’ 1907, vol. 80, p. 220.

‡ ‘Phil. Trans.’ A, 1909, vol. 209, p. 451.

TABLE VII.—New ("F.P.") Doublets of Magnesium.

m .	Diffuse series.					Sharp series.				
	λ (ROWLAND).	λ (I.Å.).	n , <i>in vacuo</i> (I.Å.).	δn .	O - C, Δn .	λ (ROWLAND).	λ (I.Å.).	n , <i>in vacuo</i> (I.Å.).	δn .	O - C, Δn .
2		-10897.8* -10934.2*	-9,172.1* -9,141.6*				-9334.8* -9361.5*	-10,709.8* -10,679.3*		
3	7896.63†	7896.37	12,660.6	30.9	0.0†		8227.5*	12,154.4*		
	7877.39†	7877.13	12,691.5				8206.9*	12,184.9*		
4	4390.751	4390.585	22,769.73	30.86	0.0†	4434.159	4433.991	22,546.84	30.53	0.0†
	4384.809	4384.643	22,800.59			4428.163	4427.995	22,577.37		
5	3539.00	3538.86	28,249.8	30.5	0.0	3553.65	3553.51	28,133.3	30.9	0.0
	3535.18	3535.04	28,280.3			3549.75	3549.61	28,164.2		
6	3169.11	3168.98	31,546.9	30.3	0.0†	3175.97	3175.84	31,478.8	30.3	0.0†
	3166.07	3165.94	31,577.2			3172.92	3172.79	31,509.1		
7	2967.98	2967.87	33,684.5	30.5	+1.8	2971.81	2971.70	33,641.1	30.4	+1.5
	2965.30	2965.19	33,715.0			2969.13	2969.02	33,671.5		

* Calculated only.

† Observed by LORENSER.

‡ Used in calculation of constants.

the wave-lengths of which have been redetermined from plates taken in the 3rd and 4th orders of the grating. International standards were employed in the determinations but the wave-lengths on both scales are given in Table VII.

For these "F.P." doublets the small letters d , s , and p will be used for the Diffuse, Sharp, and Principal series respectively, D , S , and P being reserved for the corresponding series of Wide doublets.

Employing the less refrangible components of the first, second, and fourth doublets for the calculation of constants, the resulting equation for the d series is

$$d(m) = \frac{40,618.45}{40,648.95} \left\} - \frac{4 \times 109,679.3}{\left(m + 0.947218 + \frac{0.042317}{m}\right)^2}$$

and for the s series, assuming the same limits and calculating from the first and third of the less refrangible components

$$s(m) = \frac{40,618.45}{40,648.95} \left\} - \frac{4 \times 109,679.3}{\left(m + 0.930,683 - \frac{0.014220}{m}\right)^2}.$$

These equations represent all the observations very closely as will be seen from the columns "O-C" in Table VII.

§ 7. *Relation between the "4481" and the New Series.*

A connection between the "F.P." doublets and the "4481" series is indicated by the above equation for the d series, since $m = 2$ gives the variable part as 49,790.5 which is very close to the limits of the "4481" series. By analogy with the doublets of Ca, Sr, and Ba, this would be the case if the "4481" series were a Fundamental series forming part of the "F.P." system. Such an association would conform with the rule already indicated by the other three elements that the limit of the Fundamental series in the case of enhanced lines is derived from the first negative term of the d series, and not from the first positive as in arc line series.

For the complete establishment of this relation the lines of the d series should be found to have satellites; and in the case of the first member the separation of the satellite should be equal to that of 4481. The wave-length of the first line is given by the Hicks formula as near λ 10,900, a position which does not fall within the region which could be observed with the instruments available. The satellite separation should be $\delta\lambda = 1.2$. The second d pair (λ 7896), which should have a satellite separation $\delta\lambda = 0.25$, is also outside the region investigated. At the third (λ 4390.6) the satellite separation would be reduced to $\delta\lambda = 0.04$ ($\delta n = 0.21$) which is too small for resolution. There is, however, some evidence of a disturbance of the position of the third line (Table VII.). This part of the spectrum was photographed in the 3rd

and 4th orders of the grating, and careful measurements have shown that the $\lambda 4390$ d doublet is slightly wider (0.3) than the neighbouring doublet of the s series. LORENSER'S interval for the second d pair is also greater than that of the other doublets which have been measured. The intervals of the remaining pairs have been less accurately determined and cannot be used for such a comparison. As in the case of the corresponding line $\lambda 2798$ of the Wide doublet (D) series, the observations of the $\lambda 4390$ doublet suggest a satellite on the *more* refrangible side of the chief line, and this unusual position would accord with the greater strength of the *more* refrangible components of the "4481" series.

The "4481" series may accordingly be considered to be associated with either of the two systems of doublets, but a more direct connection with the "F.P." group is indicated by the fact that these lines occur under precisely the same experimental conditions as 4481. With this relation no difficulty arises with regard to the small interval of the "4481" lines, or to the limits of the series when compared with the corresponding figures for Ca, Sr, and Ba. It may therefore be concluded that the "4481" series is the Fundamental series of the "F.P." system.

§ 8. *The Principal Series of the New Doublet System.*

Associated with the "F.P." doublets a Principal series would also be expected. The approximate formula for the more refrangible components of these doublets, calculated in the usual way from that of the s series, is

$$p(m) = 51,328.2 - \frac{4 \times 109,679.3}{(m + 1.286480)^2}$$

This indicates the neighbourhood of $\lambda 9340$ for the first p doublet (which should have $\delta n = 30.5$) and $\lambda 3643$ for the second. The first is outside the range of the present observations, but the second is probably represented by one of two similar pairs of lines which are developed under the same conditions as the "F.P." lines. In each pair the more refrangible component is the stronger, as it should be in the p series, and the separations are about that which would be expected in the second member. Particulars of these lines are as follows:—

λ (I.Å.).	n .	δn .	Remarks.
3850.40	25,964.1	14.6	Probable "combination" doublet (Plate 3, fig. 8).
3848.24	25,978.7		
3615.64	27,650.0	14.0	Probable p doublet.
3613.80	27,664.0		

It is evident that both pairs cannot belong to the p series, and the near equality of

the intervals therefore suggests that one belongs to the p series while the other is a "combination" doublet. We have, in fact, the following numerical relations with the term $2d$ given by the limit of the more refrangible (stronger) components of the "4481" series:—

$$49,779\cdot0 - 25,978\cdot7 = 23,800\cdot3,$$

$$49,779\cdot0 - 25,964\cdot1 = 23,814\cdot9.$$

$$\left. \begin{array}{l} 23,800\cdot3 + 27,664\cdot0 = 51,464\cdot3 \\ 23,814\cdot9 + 27,650\cdot0 = 51,464\cdot9 \end{array} \right\} \text{mean } 51,464\cdot6.$$

The last number is not far from the approximate limit calculated for the p series, and it is suggested that the lines at λ 3615 form the p doublet while the other pair is a combination derived from this and the limit of the "4481" series ($f \infty - 3p$, or $2d - 3p$).

A striking confirmation of this conclusion is afforded by the fact that the term $2S$ for the Wide doublet series (Table VI.) is $51,464\cdot2$. This is not only in close accordance with the limit for p calculated above, but is an important indication of the intimate connection of the two systems of doublets. It will suffice for the present to note that the limit of the p series of "F.P." doublets appears to be given by the difference between the limit of the subordinate Wide doublets and the wave-number of the first observed S line which has a *positive* value; or, symbolically, $p \infty = 2S = S \infty - S(2)$.

The agreement is too close to be regarded as accidental, and assuming the relation to be true, the limit of the p series can be determined very exactly. The limit of the "4481" series (giving $2D$ and thence $S \infty$) has been found with considerable precision and the wave-number of the line λ 2936·496 giving $S(2)$, has also been accurately determined. Hence, with very small probable error, the limit of the p series may be taken as $51,464\cdot2$. The limits of the d and s series ($40,618\cdot4$ and $40,648\cdot9$) are probably also not far from the truth, and the wave-numbers of the components of the *first* p pair may therefore be deduced by the Rydberg-Schuster law. Hence, although only the second p doublet has been actually observed, there are sufficient data for the calculation of HICKS' formulæ for the series, namely,

$$p_2(m) = 51,464\cdot2 - \frac{4 \times 109,679\cdot3}{\left(m + 1\cdot305940 - \frac{0\cdot041388}{m}\right)^2},$$

$$p_1(m) = 51,464\cdot2 - \frac{4 \times 109,679\cdot3}{\left(m + 1\cdot307258 - \frac{0\cdot041556}{m}\right)^2}.$$

The positions of the first four doublets calculated from these equations are as follows:—

TABLE VIII.—Principal Series of “F.P.” Doublets.

Limit = 51,464·2.

$m.$	λ (I.Å.).	$n.$	$\frac{mp_2}{mp_1}$	$\delta n.$	Remarks.
2	9243·7	10,815·3	40,648·9	30·5	Calculated.
	9217·7	10,845·8	40,618·4		
3	3615·64	27,650·0	23,814·2	14·0	Observed.
	3613·80	27,664·0	23,800·2		
4	2790·84	35,819·9	15,644·3	7·6	} Calculated.
	2790·35	35,827·5	15,636·7		
5	2474·37	40,402·4	11,061·8	4·5	
	2474·09	40,406·9	11,057·3		

The Principal series thus appears to be satisfactorily identified, but it is somewhat remarkable that it does not occur with greater intensity.

§ 9. *Relation between the Two Systems of Doublets.*

The connection of 4481 with the Wide doublets appears to arise from the fact that the two sets of doublets are themselves closely related. An indication of this has already been noted in connection with the limit of the p series. Further evidence is afforded by a comparison of other terms of the two systems. Thus, for the Wide doublets, including the case just mentioned, we have from Table VI.,

$$S_1 \infty = 85,508\cdot6,$$

$$S_2 \infty = 85,600\cdot1,$$

$$2D = 49,778\cdot0,$$

$$2S = 51,464\cdot2,$$

$$3D = 27,961\cdot8,$$

$$3S = 28,483\cdot3,$$

all of which are derived from actual observations, and are, therefore, unaffected by any imperfections of formulæ except as regards the limits of the “4481” series. The equations for the d , s , and p series of “F.P.” doublets (pp. 244 and 246) give corresponding terms,

$$1p_1 = 85,462\cdot9,$$

$$1p_2 = 85,550\cdot2.$$

$$2d = 49,790, \quad 2s = 51,328,$$

$$3d = 27,958, \quad 3s = 28,464.$$

The calculated terms of the "F.P." doublets may thus be considered identical with the corresponding terms of the Wide doublets, within the possible limits of error. In that case the corresponding subordinate series would run parallel to each other in the two systems at distance apart of

$$S_1 \infty - s_1 \infty = 85,508\cdot6 - 40,618\cdot4 = 44,890\cdot2 = P_1(2) - P_1(1)$$

and

$$S_2 \infty - s_2 \infty = 85,600\cdot1 - 40,648\cdot9 = 44,951\cdot2 = P_2(2) - P_2(1).$$

A further important indication of a close connection between the two systems results from a consideration of the P series of Wide doublets, of which only the pair $\lambda\lambda$ 2802, 2795 falls within the region of observation. In view of the relations of the other series to which reference has been made, the limit of the P series is probably given exactly as $85,508\cdot6 + 35,761\cdot1$, or $85,600\cdot1 + 35,669\cdot6 = 121,269\cdot7$ as shown in Table VI. With this limit, the formulæ for the two components of the series, derived from the observed pair are,

$$P_2(m) = 121,269\cdot7 - \frac{4 \times 109,679\cdot3}{(m + 1\cdot263889)^2},$$

$$P_1(m) = 121,269\cdot7 - \frac{4 \times 109,679\cdot3}{(m + 1\cdot265100)^2}.$$

From these equations we find for the P series the results shown in Table IX.

TABLE IX.—Principal Series of Wide Doublets (Preliminary Calculation).

Limit = 121,269·7.

$m.$	λ (I.Å.).	$n.$	$\frac{mP_2}{mP_1}$	$\delta n.$	Remarks.
1	2802·698	35,669·6	85,600·1	91·5	Observed lines (Table VI.).
	2795·523	35,761·1	85,508·6		
2	1248·64	80,087·1	41,182·6	30·5	} Calculated.
	1248·16	80,117·6	41,152·1		
3	1029·45	97,138·8	24,130·9	13·7	
	1029·31	97,152·5	24,117·2		

The first striking result derived from these calculations is that the separation of the second member is 30·5, which is exactly that of the *d* and *s* doublets of the “F.P.” system. Hence it is suggested that the variable parts mP_1 , mP_2 are really identical with corresponding terms of the *p* series (Table VIII.). The differences are in fact not too great to be attributed to defects of the simple formulæ employed. Thus the P and *p* series may also be considered as running parallel to each other at a distance apart of $P \infty - p \infty = 121,269\cdot7 - 51,464\cdot2 = 69,805\cdot5 = S(2) - S(1)$. Since the data for the *p* series depend upon more complete observational material than those for P, the latter may be corrected by the former, with the following results:—

TABLE X.—Corrected Principal Series of Wide Doublets.

Limit = 121,269·7.

$m.$	λ (I.Å.).	$n.$	$\frac{mP_2}{mP_1}$	$\delta n.$	Remarks.
1	2802·698	35,669·6	85,600·1	91·5	Observed lines (Table VI.).
	2795·523	35,761·1	85,508·6		
2	1240·37	80,620·8	40,648·9	30·5	} Calculated.
	1239·90	80,651·3	40,618·4		
3	1026·11	97,455·5	23,814·2	14·0	
	1025·96	97,469·5	23,800·2		

Even the second pair is beyond the region photographed by LYMAN in the case of magnesium, but it is not outside the limit of possible observation.

The foregoing considerations leave no doubt that while the limits and separations of the D and S series of Wide doublets are derived in the usual manner from the first doublet of the P series, those of the *d* and *s* series of the narrower “F.P.” doublets are derived from the *second* P pair (Wide doublets). PASCHEN’S work on Combination series appears to justify the belief that the RYDBERG-SCHUSTER law of limits, and RUNGE’S law as to the derivation of the limit of the Fundamental series from the first line of the Diffuse series, are both exact, and the parallelism of the two systems thus permits the accurate calculation of data for both, since data lacking in one may be adopted from the other. For example, the term 2*s* of the “F.P.” group is given by calculation as 51,328·2, and 1*s* as 119,449·3; both may be obtained with a much higher degree of precision by adopting the values 2S and 1S of the Wide doublet system (Table VI.), which are themselves independent of formulæ except as regards the determination of the limits of the “4481” series. The following table (XI.) may

TABLE XI.—The “F.P.” Doublet System of Magnesium (Wave-numbers and “Variable Parts”).

<i>m.</i>	<i>d</i> series. Limits $\begin{cases} 40,618 \cdot 4. \\ 40,648 \cdot 9. \end{cases}$		<i>s</i> series. Limits $\begin{cases} 40,618 \cdot 4. \\ 40,648 \cdot 9. \end{cases}$		<i>f</i> (“4481”) series. Limits $\begin{cases} 49,778 \cdot 0. \\ 49,779 \cdot 0. \end{cases}$		<i>p</i> series. Limit, $51,464 \cdot 2.$	
	<i>n.</i>	<i>md.</i>	<i>n.</i>	<i>ms.</i>	<i>n.</i>	<i>mf.</i>	<i>n.</i>	$\begin{matrix} mp_2. \\ mp_1. \end{matrix}$
1		(110,836)		121,269·7*		(109,576·1)†	$\begin{matrix} -34,135 \cdot 9\dagger \\ -34,044 \cdot 4\dagger \end{matrix}$	$\begin{matrix} 85,600 \cdot 1 \\ 85,508 \cdot 6 \end{matrix}$
2		49,779·0		51,464·2*		(48,823·6)†		$\begin{matrix} 40,648 \cdot 9 \\ 40,618 \cdot 4 \end{matrix}$
3	12,660·6 12,691·5	27,957·8		28,483·3*	22,308·7 22,309·7	27,469·3	27,650·0 27,664·0	23,814·2 23,800·2
4	22,769·7 22,800·6	17,848·7	22,546·8 22,577·4	18,071·6	32,199·0 32,199·9	17,579·0		$\begin{matrix} 15,644 \cdot 3 \\ 15,636 \cdot 7 \end{matrix}$
5	28,249·8 28,280·3	12,368·6	28,133·3 28,164·2	12,485·1	37,571·4 37,572·4	12,206·6		$\begin{matrix} 11,061 \cdot 8 \\ 11,057 \cdot 3 \end{matrix}$
6	31,546·9 31,577·2	9,071·5	31,478·8 31,509·1	9,139·6	40,811·3	8,967·4		
7	33,684·5 33,715·0	6,933·9	33,641·1 33,671·5	6,977·3	42,913·3	6,865·4		
8		(5,473·9)		(5,502·9)	44,354·7	5,424·0		
9					45,385·4	4,393·3		
10					46,147·9	3,630·8		
11						(3,050·8)†		

* From Table VI. (*mS*); $1s = P \infty$.

† From HICKS' formula for “4481” series (equation III., p. 232).

‡ Observed S doublet (Table VI.), S(2); the variable parts in this case are $1P_2$ and $1P_1 = S_2 \infty$ and $S_1 \infty$.

accordingly be drawn up for the "F.P." system, it being understood that the variable parts of the p , d , and s series are equally applicable to the Wide doublets. In each case the terms md , &c., are given as derived from the observed wave-number of the less refrangible components; they would be the same for the more refrangible components if there were no errors of observation. The wave-numbers included are those which have been actually observed; others may readily be obtained from the limits and variable parts given. Terms depending wholly on calculation from formulæ are enclosed in brackets, and for simplicity the satellite has been disregarded. Further reference to this table will be made in the discussion of Combination series.

§ 10. *Combination Lines and Series of Magnesium.*

In the discussion of series, the wave-number of a line is represented by the difference of two other wave-numbers, the first of which (the limit) is constant for a given series, and the other variable, the limit itself appearing as one of the variable parts in an associated series. Combination lines and series, as is well known from the work of RITZ and PASCHEN, are formed by taking differences between the variable parts from different series. The Fundamental series was regarded by RITZ as a special type of Combination series, in which the variable part, in a doublet series, is given by $m(p_1 - p_2)$, or $m\Delta p$, where p_1 and p_2 are taken from the formulæ for the two components of the Principal series, or derived from the constant separation in the Diffuse and Sharp series. This relation, however, is only approximate, and mf will be used to denote the variable part in the Fundamental series. It should be recalled that if there are no satellites in the D series, the F series consists of single lines, and that when satellites are present in a doublet system, the F lines are also doublets, with a separation equal to that of the satellite and chief line in the first member of the D series.

The whole of the "F.P." system may be considered to consist of Combination series derived from the Wide doublets, or *vice versa*. Retaining the use of capital letters for the Wide, and small ones for the "F.P." doublets, and disregarding the satellites, we have the relations

$$\begin{array}{ll} p_2(m) = 2S - mP_2, & P_2(m) = 1s - mp_2, \\ p_1(m) = 2S - mP_1, & P_1(m) = 1s - mp_1, \\ s_1(m) = 2P_1 - mS, & S_1(m) = 1p_1 - ms, \\ s_2(m) = 2P_2 - mS, & S_2(m) = 1p_2 - ms, \\ d_1(m) = 2P_1 - mD, & D_1(m) = 1p_1 - md, \\ d_2(m) = 2P_2 - mD, & D_2(m) = 1p_2 - md. \end{array}$$

$$f(m) = 2D - mf \quad \text{or} \quad 2D - m(P_1 - P_2).$$

Either of the groups might be approximately determined in this way from the other, but an accurate knowledge of all the data for one of them can only be obtained by utilizing observations of both.

An interesting extension of the investigation of magnesium has been made possible by the production of a number of new lines of moderate intensity, which occur under the same experimental conditions as the "F.P." group (Plate 3, figs. 1, 2, 3). For some time the appearance of these lines seemed rather capricious, and it was thought that they might possibly be due to impurities. It was eventually found, however, that they could always be obtained in a small region near the negative pole of the arc *in vacuo*, provided a sufficiently heavy current (7 or 8 amperes) were employed. The lines were shown clearly in many photographs, where the only lines traceable to impurities were faint ones of Na, Ca, and Ba. No further doubt can be felt as to the magnesium origin of the lines, in view of the manner in which their positions can be calculated from the constants for the doublets, taken from Table XI.

One of the combination terms has already been utilised in the discussion of the *p* series. This is represented by the combination $2d-3p_1$, $2d-3p_2$, giving the lines $\lambda\lambda$ 3850.40, 3848.24, which form a well-marked doublet, shown very clearly in Plate 3, fig. 8.

In addition, two well defined Combination series have been identified. A few of the lines involved appear in a list given by LORENSER, but their connections were not then recognised. One of these series is somewhat stronger than the other and may be distinguished as Series A, the weaker being designated Series B. In each case the intensities degrade in the usual way in passing to the violet (Plate 3, figs. 1, 2, 3). Series A results from the combination $3d-mf$, and B from $3f-mf$. Details of the observations and comparisons with the calculations are given in Tables XII. and XIII.

TABLE XII.—Magnesium Combination Series A ($3d-mf$).

Limit, $3d = 27,957.8$.

<i>m.</i>	λ (I.Å.) (FOWLER).	<i>n</i> (observed).	<i>n</i> (calculated).	O - C, Δn .	Remarks.
5	6346.67	15,752.0	15,751.2	+0.8	LOR. gives λ 6347.27 (Row.).
6	5264.14	18,991.3	18,990.4	+0.9	" " λ 5264.2 "
7	4739.59	21,093.1	21,092.4	+0.7	
8	4436.48	22,534.2	22,533.8	+0.4	
9	4242.47	23,564.7	23,564.5	+0.2	" " λ 4242.60 " Mg?
10	4109.54	24,326.9	24,327.0	-0.1	
11	4013.80	24,907.1	24,907.0	+0.1	

TABLE XIII.—Magnesium Combination Series B ($3f-mf$).Limit, $3f = 27,469\cdot3$.

m .	λ (I.Å.) (FOWLER).	n (observed).	n (calculated).	O - C, Δn .	Remarks.
5	6545·80	15,272·8	15,262·7	+ 10·1	LOR. gives $\begin{cases} \lambda 6546\cdot77 \text{ (Row.)} \\ \lambda 6545\cdot66 \text{ } \\ \lambda 5401\cdot2 \text{ } \end{cases}$,, ,, ,,
6	5401·05	18,509·9	18,501·9	+ 8·0	
7	4851·10	20,608·2	20,603·9	+ 4·3	
8	4534·26	22,048·2	22,045·3	+ 2·9	
9	4331·93	23,078·1	23,076·0	+ 2·1	
10	4193·44	23,840·2	23,838·5	+ 1·7	
11	4093·90	24,419·8	24,418·5	+ 1·3	

It will be observed that in each case there is a systematic difference between observation and calculation. In Series A, however, the differences are very slight, and possibly negligible; but in Series B they are too large to be attributed to faulty observations. The terms mf have been derived from observations of a high order of accuracy, and in the case of Series B, the calculated wave-numbers would not be affected by any error in the adopted value of the limit of the f series. It is scarcely probable that Series B, with its limit so near to $3f$, is independent of the "F.P." system, and it would therefore appear that the combination principle is not exact in every case, except probably as regards the values of the limits.

Calculations of formulæ for the two series from the observed lines themselves confirm the values of the limits given by the combination terms. These formulæ are as follows:—

$$A(m) = 27,956\cdot48 - \frac{4 \times 109679\cdot3}{\left(m + 0\cdot993807 + \frac{0\cdot008967}{m}\right)^2},$$

$$B(m) = 27,468\cdot38 - \frac{4 \times 109679\cdot3}{\left(m + 0\cdot994863 + \frac{0\cdot014621}{m}\right)^2}.$$

The residuals (O - C) left by the formulæ on the observed wave-numbers are—

$m =$		5	6	7	8	9	10	11
O - C {	Series A = 0·0*		+ 0·3	0·0*	+ 0·1	0·0	- 0·3	0·0*
	,, B = 0·0*		+ 1·8	0·0*	+ 0·1	0·0	0·0	0·0*

A general view of the enhanced line series of magnesium now identified is given in

fig. 1, in which the doublet separations have been arbitrarily increased for greater clearness.

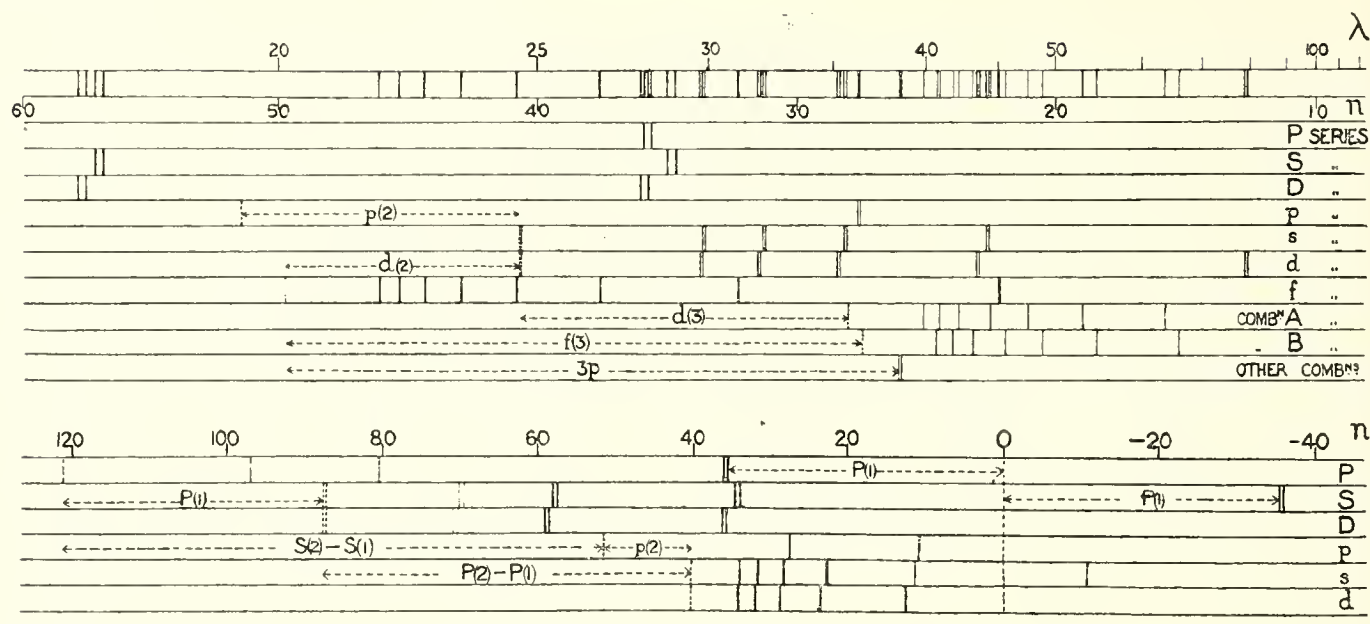


Fig. 1. Enhanced-line series of magnesium.

§ 11. General Comparison of Mg, Ca, Sr, and Ba.

One of the results of the investigation is to indicate that Mg behaves differently from Ca, Sr, and Ba in the spark. All are alike in having series of Wide doublets with separations related to the atomic weights, but the D doublet with negative frequency which occurs in the other three elements is lacking in Mg. While Ca, Sr, and Ba have strong Fundamental series with limits derived from the negative D doublet, the corresponding F series of Mg, like the negative term from which it should be derived, does not appear to exist. In Mg the equivalent of the F series of the other three elements is replaced by the whole system of "F.P." doublets (including the "4481" series as Fundamental), which has no analogue in the other three elements so far as is known at present. The "F.P." system itself is closely related to the family of Wide doublets, but although both consist of enhanced lines they do not necessarily co-exist; the wider system occurring in the ordinary arc, while the closer requires the conditions of the spark or its equivalent, the negative pole of an arc *in vacuo*.

The "4481" series is therefore established as a Fundamental series, but the system of which it forms a part is in some respects exceptional.

It is important to observe that no numerical relations between the enhanced and arc line series occurring in the spectra of any of the four elements have been recognised.

§ 12. The Spectra of Hydrogen and Helium.

The now demonstrated existence of series having $4N$ for the "universal" constant has an interesting bearing on the question of the origin of the special lines produced

by strong discharges in helium tubes, to which reference has already been made. Some of these were at first regarded as being identical with the lines of the supposed P series of hydrogen calculated by RYDBERG on the assumption that the Pickering (ζ Puppis) lines formed the S series. The lines in question are compared with RYDBERG'S values (Rowland scale) in the appended table:—

TABLE XIV.—The “4686” Series.

Observed λ (FOWLER).	Calculated λ (RYDBERG).	Difference $\Delta\lambda$.
4685·98	4687·88	1·90
3203·30		
2733·34	2734·55	1·21
2511·31		
2385·47	2386·50	1·03
2306·20		
2252·88	2253·74	0·86

In order to conform with RYDBERG'S theory, and to adapt the formulæ then ordinarily employed, it was necessary, as in the first discussion of the Mg “4481” series, to divide the series into two parts by taking alternate lines. One series, as will be seen from the table, then agreed very closely with the calculated values for the P series of hydrogen, and the other was regarded as a second P series having a simple relation to the first.

The conditions of appearance of these lines in helium tubes indicates that they are enhanced lines, and calculation shows that they can be united in a single series of the 4N type, as in the case of the other series of lines of this class which have been discussed. Thus, the first line, which has been most accurately measured, gives the equation

$$n = 438,879 \cdot 1 \left(\frac{1}{3^2} - \frac{1}{m^2} \right).$$

The differences O—C (Δn) for the seven observed lines are 0·0, −0·3, +1·4, +0·6, +1·0, +2·3, −1·4. All are within the estimated limits of error of observation, or very nearly so, except the third, and even in this case a correction of 0·1 Å.U. to the observed wave-length would reduce the difference to zero.

The “4686” series is accordingly a series of the enhanced line, or 4N, type, and can no longer be considered to belong to the same group as the Balmer series of hydrogen which is of the arc, or N, type. The numerical relations indicated by RYDBERG'S calculations are therefore not significant, and, in view of the experimental evidence, it must be concluded that the “4686” series is not due to hydrogen, but to helium, as first indicated by Dr. BOHR from theoretical considerations. This conclusion is further supported by EVANS'S observation of the line 4686 in a helium tube in

which the ordinary lines of hydrogen could not be detected.* In accordance with the convenient nomenclature of LOCKYER it is suggested that the lines in question should be designated "proto-helium" lines.

Analogy with the magnesium "4481" series suggests that the "4686" series should be regarded as one of the F type, and the question arises as to how the associated P, S, and D series are represented in the proto-helium spectrum. A knowledge of the F series alone does not permit the complete calculation of any of the three associated series, but only indicates the "variable part" of the first line of the D series. In the case of proto-helium, the limit of the "4686" series, $N'/3^2$, shows that the variable part of the D formula is identical with that of the F, but the limit of D remains undetermined. A consistent arrangement is possible, however, if the D be regarded as having the same limit as the F series, in which case the four series would coalesce in the formula

$$n = N' \left(\frac{1}{3^2} - \frac{1}{m^2} \right)$$

where $N' = 438,879.1$. This only requires that the zero values of n given by $m = 3$ be regarded as negative terms, so that if the formula be taken to represent the coalesced D and S series, the limit of P derived from S, and that of F derived from D would also be $N'/3^2$. The lines of the four series would thus be coincident. The great strength of the "4481" series of magnesium as compared with that of the other series of the "F.P." group, however, suggests that the "4686" series is primarily of the F type.

As pointed out by BOHR, the substitution of 4 for 3 in the above formula for the "4686" series yields a series of lines of which the Pickering (ζ Puppis) lines, also formerly attributed to hydrogen, are alternate members, the remainder being nearly coincident with the Balmer lines of hydrogen. It is probable, therefore, that the Pickering lines are also due to proto-helium, as they have only been observed in association with 4686. The calculated wave-lengths of the earlier members of the extended series are compared in Table XV. with the Balmer series, and with the hypothetical "sharp" series of hydrogen calculated by RYDBERG.

TABLE XV.—The "Pickering" Series.

Proto-helium, "Pickering" series, λ (calculated).	Hydrogen, Balmer series, λ .	"Hydrogen," RYDBERG'S S series, λ (calculated).
6560.37	6563.07	
5411.74		5412.8
4859.53	4861.52	
4541.79		4543.3
4338.86	4340.64	
4200.02		4201.5

* 'Nature,' 1913, September 4, p. 5.

The astrophysical and experimental data have as yet given no proof of the existence of the lines adjacent to those of hydrogen, but there is no evidence against the supposition that they exist.

The assignment of these lines to proto-helium is supported by the observations of magnesium, which have shown analogous series associated with the "4481" series. The series A of magnesium, given by the combination $3d-mf$, would become $4D-mF$ in proto-helium—the numeration being slightly different in the two cases*—and this would be identical with the formula for the extended Pickering series. Series B of magnesium, given by $3f-mf$, would similarly become $4F-mF$ and would also coalesce into the Pickering series. Alternate lines of series B are in fact not far removed from the Balmer lines, while intermediate lines are near the observed Pickering lines, as will be seen by comparing Tables XIII. and XV.; lines near these positions might also be derived from the F series of other elements in which mF approximates to $4N/m^2$. The relative weakness of the Pickering series of proto-helium, as compared with 4686, is matched in magnesium by the relatively low intensities of the analogous combination series A and B.

The close relations between the lines of hydrogen (real and hypothetical) and some of those of proto-helium are simply accounted for by BOHR's theory of the origin of these spectra. The general formulæ, in which p and m can only take integral values, may be written

$$\text{Hydrogen: } n = N \left(\frac{1}{p^2} - \frac{1}{m^2} \right) \quad p = 2 \text{ for Balmer series.}$$

$$\text{Proto-helium: } n = N' \left(\frac{1}{p^2} - \frac{1}{m^2} \right) \quad \begin{cases} p = 3 \text{ for "4686" series,} \\ p = 4 \text{ for Pickering series.} \end{cases}$$

If N' were exactly equal to $4N$, alternate lines of the Pickering series would evidently coincide with the Balmer series; and the hypothetical P series of hydrogen, given by $p = 1.5$ in the first formula, would fall on alternate lines of the "4686" series.

The hydrogen lines, according to the theory, are produced during the binding of an electron by an atomic nucleus having a single positive charge, and those of proto-helium during the binding of an electron by a nucleus having four times the mass and a double positive charge. If the mass of the electron were negligible in comparison with that of the nucleus, N' would be equal to $4N$, but BOHR has shown that when correction is made for the mass of the electron, the theoretical ratio for these constants is in nearly perfect agreement with that of the values deduced from the spectroscopic observations.† His expression for the constant of the hydrogen series may be written

$$N_H = 109,675 \equiv \frac{10^8 2\pi^2 E^2 e^2 M m}{c h^3 (M + m)}$$

* The numeration would be the same if m in the proto-helium formula were written $(m+1)$ to correspond with $(m+0.99)$ of magnesium.

† 'Nature,' 1913, October 23, p. 231.

where c is the velocity of light, h is PLANCK'S constant, e and m the charge and mass of the electron, and E and M the charge and mass of the nucleus of the hydrogen atom. In the case of proto-helium, the value of E is doubled and M is increased four times. The theoretical ratio between N' and N is therefore

$$\frac{N_{\text{pHe}}}{N_{\text{H}}} = \frac{16(M+m)}{4(M+m)}.$$

Introducing the recognised value $M/m = 1835$, the resulting ratio is 4.001635, as compared with 4.001632 derived from the observations.

A spectroscopic method of determining the mass of the electron, from the constants of the hydrogen and proto-helium series, is thus suggested. The value of N' obtained from $\lambda 4685.98 \pm 0.01$ (ROWLAND'S scale) is $438,879.1 \pm 1.0$, and combining this with hydrogen 109,675, the resulting value for the mass of the hydrogen atom in terms of that of the electron is 1836 ± 12 . A similar calculation from wave-lengths on the International scale, using Mr. CURTIS'S new value of the hydrogen constant (109,679.3) gives the ratio $M/m = 1855 \pm 12$. This result, however, is only to be regarded as provisional; more exact measures of the proto-helium lines may be possible, and the formula employed may require a small correction for the alteration in the mass of the electron due to its velocity.

The assignment of the "4686" series to proto-helium may nevertheless be considered to be independent of BOHR'S theory. It really depends upon the recognition of the new class of series associated with enhanced lines, and the better accordance with experimental results. Apart from the theory it might still be considered possible that the hydrogen spectrum, under appropriate conditions, would include the Rydberg lines. The well-known line about $\lambda 4686$ which occurs in the solar chromosphere and, in some of the nebulae, however, is undoubtedly the proto-helium line, and there are no indications in these spectra of another line near 4688 which would correspond with the hypothetical line of hydrogen. Until other evidence is forthcoming, it may therefore be considered that the line spectrum of hydrogen, as required by BOHR'S theory, consists only of the Balmer series and parallel series in the infra-red and Schumann regions. These are all included in the simple formula previously given.

The proto-helium spectrum is apparently of the same simple character as that of hydrogen, but the series in the infra-red and Schumann regions have not yet been investigated.

§ 13. *Enhanced Lines in Relation to BOHR'S Theory.*

The apparently successful application of BOHR'S theory to the lines of proto-helium renders it desirable to consider briefly the implications of this theory as regards other systems of enhanced lines.

Since enhanced lines have been found to occur in families of series similar to those previously recognised in arc spectra, the only new theoretical problem presented by them is to account for the appearance of the series constant with four times the value associated with arc lines. Otherwise, the extension of the simple formulæ for proto-helium to enhanced lines in general is identical with the extension of BALMER's formulæ for hydrogen to the ordinary systems of series of arc lines. Hydrogen and proto-helium may thus be regarded as limiting cases of the two kinds of series.

Independently of the series evidence there are numerous indications that enhanced lines originate in vibrating systems differing from those which give rise to arc lines. The mode of occurrence of the two sets of lines in the spectra of celestial bodies furnishes important evidence in favour of this view, and experimental evidence in the same direction is abundant. The extensive researches of HEMSALECH on the phenomena of the spark discharge, for example, have revealed striking differences in the behaviour of the two classes of lines.*

It has been shown that in a single discharge, the durations of the enhanced lines of Ca, Sr, and Mg, are much shorter with respect to their intensities than those of the arc lines of the same elements; and in another investigation it was shown that the velocities of the particles emitting the enhanced lines of Ca were nearly twice as great as those of the particles producing the arc lines.†

These experiments suggested that centres of emission of different constitution were concerned in the production of the two classes of lines, but HEMSALECH hesitated to adopt this view until other experiments had been made.

The series investigation not only points to the existence of different emitting systems, but, in the light of BOHR's theory, may possibly give a clue to the nature of the difference. The appearance of $4N$ in the formulæ for series of enhanced lines suggests that, as in the case of proto-helium, such series are produced during the binding of an electron by each of the atoms from which two electrons have been removed by the exciting source; while arc series lines are emitted during the re-formation of atoms from each of which only one electron has been removed. Although the neutral atoms of different elements contain different numbers of electrons, the approximate constancy of N for arc series, according to BOHR, is explained by the fact that the force on an electron entering a system consisting of a positive nucleus and a number of electrons one less than that required to render it neutral would not be very different from the force in the case of the binding of an electron by a hydrogen atom. A similar explanation would evidently account for the constancy of $4N$ in the formulæ for series of enhanced lines. Variations from the simple series conditions presented by hydrogen and proto-helium would be expected in consequence of the presence of additional electrons, but these have not yet been worked out theoretically by Dr. BOHR.

* 'Comptes Rendus,' vol. 151, pp. 220 and 668 (1910).

† 'Comptes Rendus,' vol. 154, p. 873 (1912).

The varying facility with which enhanced lines can be produced in the spectra of different elements would seem to depend upon the intensities of the forces which bind the electrons to the nuclei. In some elements, such as lithium, no enhanced lines at all have been observed. The lithium atom is credited with three electrons, and BOHR's calculations indicate that while the outer electron is lightly bound, the inner two are very strongly bound as compared with the electron in an atom of hydrogen, and even more rigidly than the two electrons in a helium atom. Theory and experiment are thus in agreement, and the production of enhanced lines of lithium would appear to require more powerful spark discharges than those hitherto employed.

In the case of elements like calcium, which give enhanced lines of Class I., it may be supposed that one electron is lightly bound, so that even a flame may effect its removal, while a second is removable by the moderate increase in the energy of excitation when passing from the flame to the arc. The further increase in the intensities of the enhanced lines on passing to the spark may be attributed to the relatively greater number of atoms from which two electrons are simultaneously detached. In elements like iron, giving enhanced lines of Class II., the conditions may be supposed similar, except that the second electron is removed with somewhat greater difficulty.

Enhanced lines like Mg 4481 require a modified explanation. Although the series to which this line belongs is produced only under spark conditions (or their equivalents in modified arcs) it is important to bear in mind that the related system of Wide doublets, consisting of enhanced lines of Class I., is developed in the ordinary arc. Since the term $4N$ appears in the formulæ for both systems, BOHR's theory would lead to the supposition that in this case the greater energy of the spark does not result in the separation of a third electron from any of the atoms but merely produces some change in the configuration of the atomic systems. The experimental data are not sufficiently complete to justify the conclusion that such an explanation would be applicable to all enhanced lines of Class III.

If, by any means, three electrons could be separated from the nucleus, the appearance of lines for which the series constant was raised to $9N$ would be expected, if BOHR's theory is a safe guide. Great energy would apparently be needed to bring about this result, and no such series have yet been recognised. A sufficient increase in the energy of excitation would presumably yield series requiring still greater multiples of the constant N in the formulæ.

It will be observed that BOHR's theory gives a more definite conception to LOCKYER's earlier view that the varying spectra given by the same substance represent the vibrations of different "molecular groupings," such groupings being simplified by successive dissociations brought about by increase of temperature or electrical excitation.

It should be noted that some of the conclusions drawn by STARK from his experiments on canal-ray spectra are inconsistent with the views of BOHR. In the case of helium, STARK has found that the Döppler displacements are smaller for the doublet

series (helium group) than for the single line series ("parhelium" group), and he infers that the atoms which act as carriers in the former case have a single positive charge, while in the second case there is a double positive charge.* Both sets of series, however, are of the "arc" type, with N for the series constant, whereas on BOHR's theory, one should be of the N , and the other of the $4N$ type if STARK's deduction is correct.

In the case of aluminium, STARK has similarly found evidence of atoms with one, two, and three positive charges.† The first is associated with the doublets of the arc spectrum, the second with a spark line at 4663·5, and the third with three spark lines at 4529·7, 4513·0, and 4480·0. Attention is drawn to the fact that the higher charges are connected with the enhanced lines, and differences in the behaviour of the two sets of spark lines when the spark is passed in hydrogen are indicated. The series to which the spark lines of aluminium belong have not yet been recognised, but STARK's results for this element are in general agreement with the conclusion indicated by BOHR's theory that enhanced lines are associated with atoms of greater positive charges than those giving rise to arc lines. In the case of helium it does not seem possible to reconcile STARK's conclusions with those of BOHR, since the doubly-charged atoms, on BOHR's theory, can only give rise to the system of series of which 4686 is the leading member.

STARK's conclusions are equally at variance with those of LENARD,‡ who found reason to believe that while principal series are produced by neutral atoms, the subordinate series originate in atoms which have become positively charged through the loss of one or more electrons. BOHR's theory differs from both in leading to the conclusion that all series having the same series constant should be produced by atoms having the same charge.

A knowledge of the charges of the atoms giving the arc and enhanced lines respectively would possibly aid in the interpretation of the peculiarities of the spectrum of the solar chromosphere, and further investigations bearing on this subject are very desirable.

Whatever may be its significance in connection with the atom, the change in the character of the series in passing from arc to spark lines suggests the spectra of "early-type" stars as a possible source of series requiring still greater multiples of the ordinary series constant in the formulæ representing them. In passing from the later to the earlier type stars, as shown by the work of LOCKYER and others, the arc lines of various metallic elements are gradually replaced by enhanced lines, while lines of hydrogen, helium, and other gases become more prominent. For such metallic elements as are known to belong to series, the change is definitely from series of the arc (N) type to series of the spark ($4N$) type. A considerable number of lines remain

* 'Ann. der Physik,' vol. 40, p. 540 (1913).

† 'Ann. der Physik,' vol. 42, p. 254 (1913).

‡ 'Ann. der Physik,' vol. 17, p. 197 (1905).

without identification, but a preliminary examination of these has given no certain evidence of series for which the series constant would be $9N$.

The Wolf-Rayet ("bright-line") stars are of special interest in this connection, as they are generally considered to represent the first stage in the condensation of nebulous matter into stars, and in many of them the proto-helium lines are a prominent feature. NICHOLSON has suggested that some of the lines may belong to series which can be represented, in wave-lengths, by the formula

$$\lambda = 3646 \frac{(m \pm \frac{1}{3})^2}{(m \pm \frac{1}{3})^2 - 4},$$

where 3646 is the limit of the hydrogen series.* The lines were regarded as a possible extension of the hydrogen spectrum, but on the supposition that other lines associated with them may coincide with the Balmer series, Dr. BOHR has pointed out that all the lines might be united in a single series and might be attributed to lithium. The binding of an electron by a lithium atom from which the three electrons have been removed would, on the theory, give rise to such a series, namely,

$$\lambda = 3646 \frac{m^2}{m^2 - 36},$$

or

$$n = 27418 - \frac{9 \times 109675}{m^2}.$$

This hypothetical series would thus be of the " $9N$ " type, but its existence in this form is not confirmed by the observational data. Omitting the lines which would coincide with the ordinary lines of hydrogen, the earlier members of the series would be 5697, 5193, 4633, 4466, 4243, &c. Of these only the first, third, and fourth are possibly represented in the Wolf-Rayet stars, and since the intensities should degrade in regular order in passing from red to violet, the absence of a line at 5193 is conclusive proof that the observed lines do not form a series of the " $9N$ " type.

In a further discussion of the spectra of the Wolf-Rayet stars, NICHOLSON has arranged most of the lines in a number of series of a different character, in which the Rydberg constant is replaced by various *fractional* parts of its value for hydrogen.† The numerical relations traced in this way are very striking, but the individual series indicated are very fragmentary, and such lines do not always occur together in the same star. The suggested series are also remarkable as involving different fractional values of the Rydberg constant for series proceeding to the same limit.

The general progression from series of the N type to those of the $4N$ type in passing through the stellar sequence would suggest that further change, if any, would be in the direction of multiples, rather than fractions, of N in the series formulæ. It is

* 'Monthly Notices R.A.S.,' vol. 73, p. 382 (1913).

† 'Monthly Notices R.A.S.,' vol. 74, p. 118 (1914).

doubtful, however, whether any change beyond that represented by enhanced lines should be expected. It is now easy to produce a spectrum in which the proto-helium line 4686 exceeds any of the ordinary helium lines in brightness, and from the persistence of the ordinary helium lines in many of the Wolf-Rayet stars it seems reasonable to infer that even in these stars the conditions are not indefinitely in advance of those obtainable in laboratory experiments.

§ 14. *Summary.*

(1) The enhanced (spark) lines of magnesium, calcium, strontium, and barium form series of doublets which occur in groups similar to those previously recognised in arc spectra. The formulæ representing these series, however, differ from those employed for arc lines in that RYDBERG'S constant "N" ($= 109675$ for ROWLAND'S scale) has a value equal to $4N$.

(2) The Fundamental series, in the Doublet systems of calcium, strontium, and barium, as shown by SAUNDERS and LORENSER, derives its limit and the separation of its components from an observed negative term of the Diffuse series. In the case of magnesium, the corresponding negative term of the Diffuse series, and the Fundamental series which might have been expected, do not exist.

(3) A new group of series of narrower doublets has been found in magnesium, in which the separation of the Sharp and Diffuse pairs is identical with the (calculated) separation of the *second* pair of the Principal series of wider doublets. The new group is designated the "F.P." system; in addition to the ordinary series it includes two well-marked combination series, each consisting of seven observed lines.

(4) The "4481" series of magnesium consists of very close doublets with constant separation, and forms the Fundamental series of the "F.P." system. Its limits are derived from the first negative term of the Diffuse series.

(5) Corresponding series of the "F.P." and Wide doublet systems run parallel to each other, but notwithstanding this numerical relation, the two systems do not necessarily co-exist, the wider doublets occurring in the ordinary arc as well as in the spark, while the "F.P." group occurs only under spark conditions or their equivalent in the region of the negative pole of an arc *in vacuo*.

(6) No numerical relations have been traced between any of the enhanced line series and the series of arc lines of the same element.

(7) The "4686" series produced in helium tubes is of the enhanced line ($4N$) type, and can no longer be considered to belong to the same group as the Balmer series of hydrogen, which is of the arc (N) type. It is concluded that these lines are due to helium, as indicated by Dr. BOHR from theoretical considerations. It is suggested that they should be designated "proto-helium" lines.

(8) Analogy with the "4481" series of magnesium indicates that the "4686" series

of proto-helium is primarily of the Fundamental type; the three associated series may be regarded as coincident with it.

(9) The "Pickering" lines associated with the "4686" series are also probably due to proto-helium, in which case the series would include lines nearly coincident with the Balmer series. The observational evidence on this point is incomplete, but the assignment of these lines to proto-helium is supported by the fact that one of the new combination series of magnesium is related to the "4481" series exactly as the extended Pickering series would be related to the "4686" series.

(10) The slight differences between the observed positions of alternate lines of the "4686" series and those calculated by RYDBERG for the "Principal" series of hydrogen are in very close agreement with BOHR's theoretical formulæ for hydrogen and proto-helium. Adopting these formulæ, the spectroscopic data give a provisional value for the mass of the hydrogen atom, in terms of that of the electron, as 1836 ± 12 ; or 1855 ± 12 when the data are corrected to the International scale of wave-lengths.

(11) The appearance of the Rydberg constant in enhanced lines series with four times its usual value may be explained on BOHR's theory by supposing that series of the arc type are produced when only one electron is removed from each of the atoms involved, while in the case of enhanced lines two electrons are removed.

(12) A preliminary examination of terrestrial and celestial spectra has given no indications of series requiring still greater multiples of the Rydberg constant in the formulæ representing them.

The author has pleasure in acknowledging his obligations to Mr. W. JEVONS, A.R.C.S., D.I.C., B.Sc., who has taken the new photographs of the magnesium spectrum, and has given much valuable assistance in other ways.

APPENDIX.

The wave-lengths (on the International scale) of the enhanced lines of magnesium, as observed in the arc *in vacuo* in the course of the foregoing investigation, are brought together in Table XVI. The letters indicating the series to which the lines belong have the significance given in the description of Plate 3. For completeness additional lines observed by LORENSER and by LYMAN are also included.

DESCRIPTION OF PLATE.

The photographs are of the spectrum of the magnesium arc *in vacuo*. The letters indicate the series to which the lines belong, and have the following significance:—

Enhanced lines . . .	{	P,	Principal series	}	Wide doublet system.
		D,	Diffuse ,,		
		S,	Sharp ,,		
		<i>p</i> ,	Principal ,,	}	“ F.P.” doublet system.
		<i>d</i> ,	Diffuse ,,		
		<i>s</i> ,	Sharp ,,		
		<i>f</i> ,	Fundamental series		
		A,	Combination ,, A		
		B,	,, B		
		C,	Other combination lines		
Arc lines	{	R ₁ ,	Rydberg series of single lines (Diffuse).		
		R ₂ ,	,, ,, ,, ,,	(Sharp).	
		T ₁ ,	Diffuse series of triplets.		
		T ₂ ,	Sharp ,, ,,		

1, 2, 3 show chiefly the two Combination series of enhanced lines and the “F.P.” doublets near the Fundamental line λ 4481.

4, 5 show further members of the “F.P.” system and the Wide doublets in the ultra-violet.

6, 7—Grating, 4th order—enlarged about eleven times. λ 4481 ($\delta\lambda = 0.198$) and λ 3105 ($\delta\lambda = 0.092$) respectively, each with a longer and a shorter exposure.

8, the “F.P.” Combination doublet (λ 3850, λ 3848) near the Diffuse triplet.

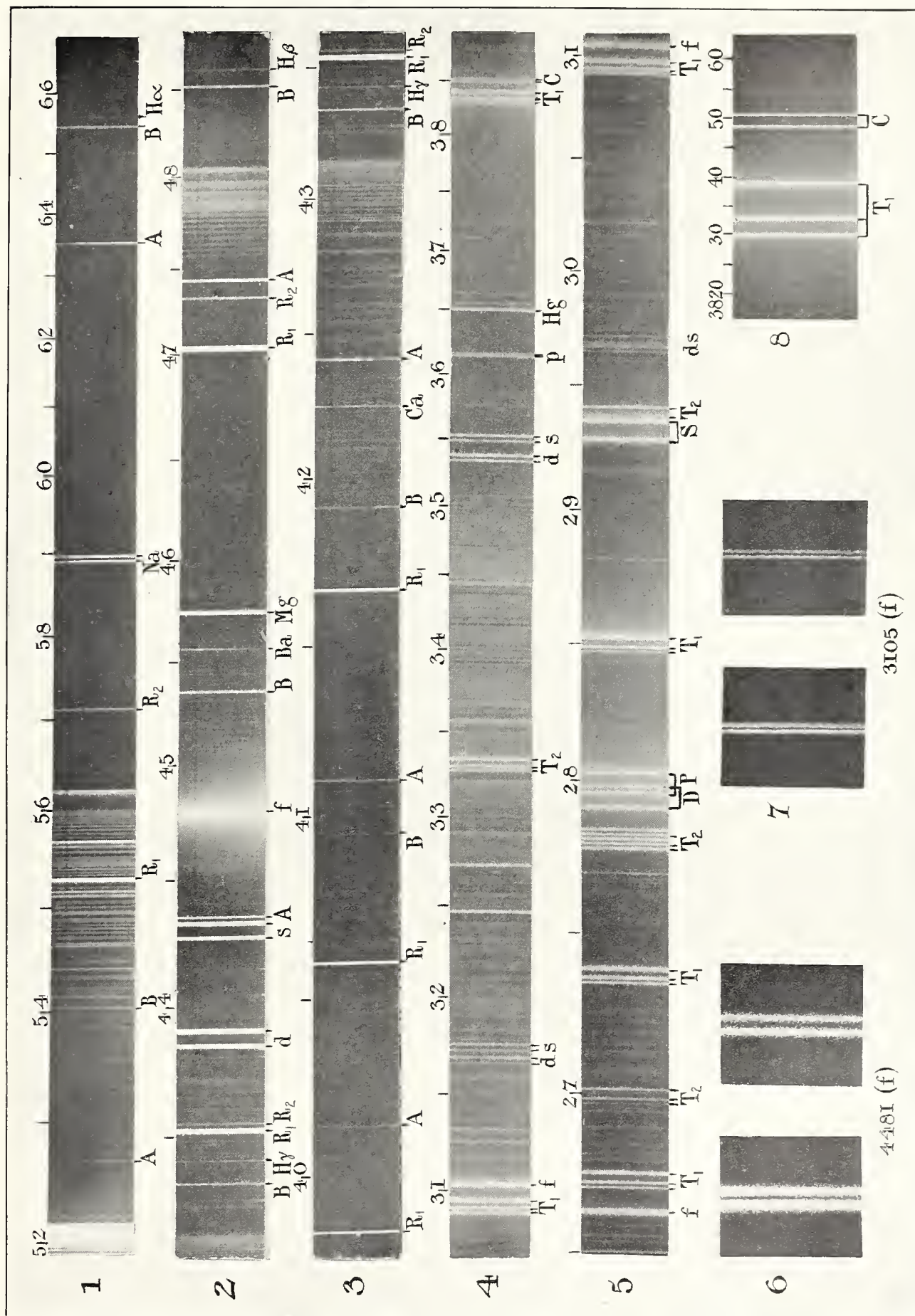
Lines of impurities — Na, Hg, Ca, Ba, H.

Bands of magnesium hydride (5211, 4802).



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[PLATES 4-5.]

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(FROM THE GOLDSMITHS METALLURGICAL LABORATORY, UNIVERSITY OF CAMBRIDGE.)



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VII. *Dilute Solutions of Aluminium in Gold.*By C. T. HEYCOCK, *F.R.S.*, and F. H. NEVILLE, *F.R.S.**(From the Goldsmiths Metallurgical Laboratory, University of Cambridge.)*

Received June 16,—Read June 25, 1914.

[PLATES 4–5.]

THE following is an account of an attempt we have been making to determine the complete equilibrium diagram of aluminium and gold. We have been occupied with the problem during the last two years, and, as the experiments consume a great deal of labour and time, we are not likely to complete the work for at least another year; we therefore venture to ask the Royal Society to accept a *résumé* of the portion which is practically complete.

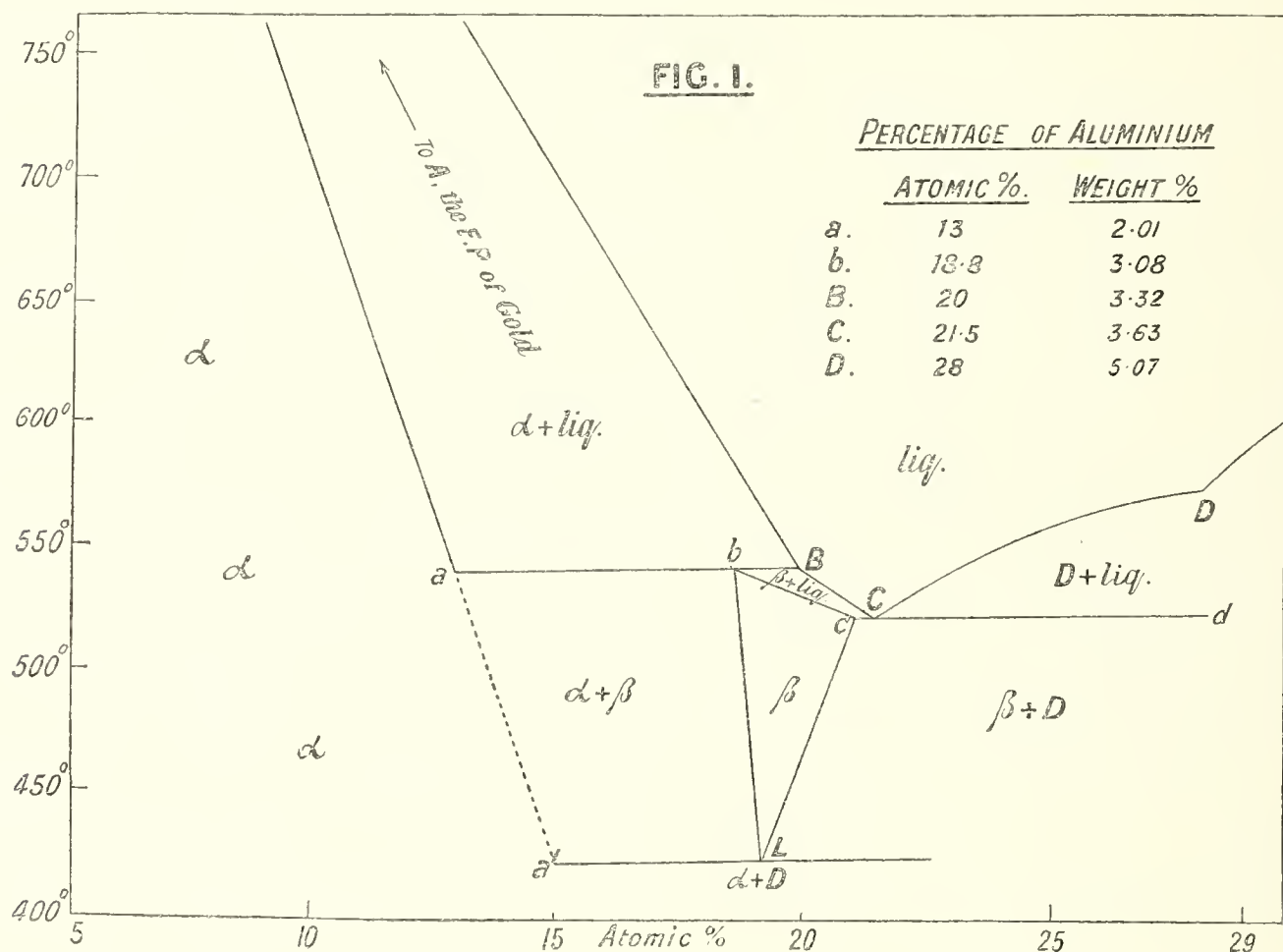
The results, in part embodied in the accompanying diagram, are based on two kinds of experiment—the tracing of cooling curves and the microscopic examination of ingots that have been annealed at certain temperatures, specially treated, and finally chilled by sudden immersion in water, the chill preserving to a great extent the structure existing a moment before the chill.

The diagram, fig. 1, is a temperature-concentration diagram, the temperature being expressed in degrees Centigrade and the concentration in atomic per cents. of aluminium. We also give a table stating the percentage by *weight* of aluminium at important points. It will be seen that none of the alloys contains more than 5 per cent. by weight of aluminium. It may very well be thought that the percentage compositions at points, such as *b* and B, *c* and C, respectively, differ by amounts comparable with possible experimental errors. This is to a certain extent true, but, for reasons which we give towards the end of the paper, we think that the only probable error of any importance is that we may have slightly overestimated the amount of aluminium in the alloys; this error will be of the same sign and almost of the same amount for all the alloys, and its only effect will be to shift a little to the left the points *b*, L, and perhaps *c*, a matter of no importance for our conclusions.

The points B and, to a less degree, C were determined very exactly in a previous research; *b* and *c* were fixed partly from theoretical considerations and partly from a minute study of the alloys.

The upper line, or liquidus, ABCD, was taken from our previous memoir ('Phil. Trans.,' A, 1899). The solidus begins at A, the freezing-point of gold, and the first branch ends at *a*, with 13 atomic per cents. of gold, or, in our notation, at Al 13; it continues horizontally to *b*, then probably follows the line *bc*, and is continued horizontally through C to a point *d*, which lies nearly under the summit D of the liquidus. It is, perhaps, unnecessary to say that at points below the solidus all alloys are wholly solid.

The diagram deals with three substances—the solid solutions alpha and beta, which are of variable composition, and with a third substance, D, which is the sole



constituent of the alloy Al 28 at the summit D of the liquidus. D may very well be the compound Al_3Au_8 ; it is very constant in appearance in all the alloys from Al 18 to Al 28, in which we have studied it, and from the fact that second freezing-points occur along the line *Cd* to a point almost under the summit D of the liquidus it does not appear probable that D is a solid solution.

The diagram ends at 400° C., and all alloys considered in it are supposed to have been melted, cooled to a temperature not below 400° C., and chilled. This limitation is important, as alloys cooled below 400° C., even if afterwards heated, generally require another diagram.

Microscopic Structure.

All alloys whose percentage composition places them to the left of the line Aaa' show the same patterns of uniform polygons or counties; these are closely packed, there being no interstitial matter, and they often meet in straight lines (Plate 4, fig. 1). In other words, the alloys contain only one phase—the solid solution that we call alpha. The alpha of the point a contains 2 per cent. by weight of aluminium, but at lower temperatures alpha can apparently contain a little more aluminium, as indicated by the slope of the line aa' ; we are, however, not quite satisfied as to the exact position of the point a' .

In the region $aa'Lb$ the ingots consist of a complex of alpha and beta, the alpha decreasing in amount as we go to the right and vanishing as we cross the line bL (figs. 2 and 3). The alpha vanishes at or about Al 19, and for about 2 atomic per cents. we have in the solids at a high temperature beta only (figs. 4 and 18).

Polished and etched surfaces of beta that have been chilled at a high temperature always present the appearance of angular rocks seen in relief, a high power showing groups of fine parallel lines, the direction of the lines changing from grain to grain so as to give the effect of shading. This is due to a laminated structure in the beta, perhaps to an incipient decomposition. In some cases the ruled structure is extraordinarily regular and resists etching, remaining bright when the rest of the beta is darkened (fig. 5).

The triangle bLc contains all the alloys that can exist in equilibrium as pure beta; it will be seen that this substance becomes unstable at 424°C . To the right of the point C , which appears to be an eutectic point, we get primaries of the substance D embedded in beta, the D generally etching bright; indeed everywhere to the right of the line Lc the alloys are a complex of D and beta (figs. 6 and 16).

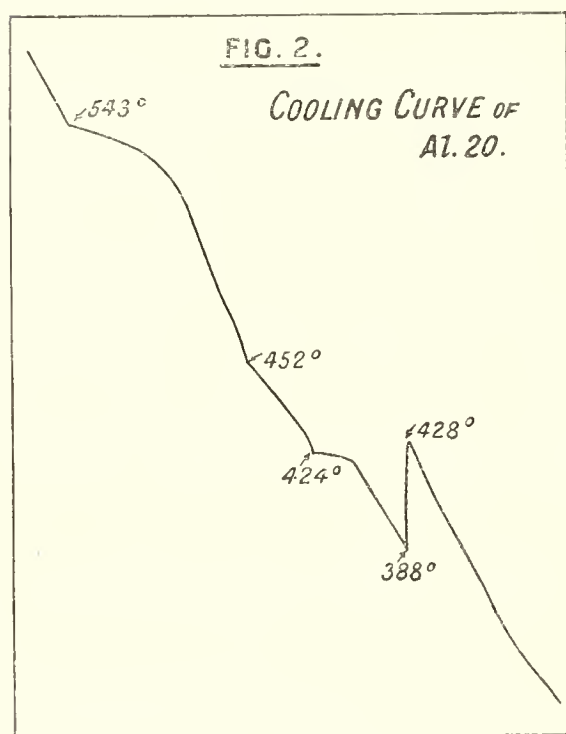
We are hardly entitled, as far as experimental evidence goes, to separate the points C and c , for we have no certain evidence of eutectic structure in any alloys of the BC branch, but we think it probable that the solidus of these alloys ends a little to the left of C as we have drawn it. It must be remembered that the C eutectic can only contain a very small proportion of D , and would be hard to detect; alloys such as Al 22, Al 23, and Al 25, when appropriately treated, do show a minute network of D in the matrix, this is perhaps the eutectic (fig. 7).

The relations of the three bodies alpha, beta, and D are, so far as this diagram extends, very similar to other cases already studied, for example, in the copper-tins. The lines bL and cL are the lines of incipient crystallisation of alpha and of D out of the solid beta; they correspond to the liquidus in crystallisation out of a liquid. The point L is an eutectoid point; when any alloy containing beta cools slowly through the L temperature the residual beta in it breaks up into a complex of alpha and of D . We have verified this in alloys ranging from Al 18 to Al 23 (figs. 8, 9, and 14). If the composition of the alloy as a whole lies to the left of L , the primary, or pseudo-

primary, crystals are alpha, if to the right of L they are D. In this way the microscope shows that L lies between Al 19 and Al 19.5.

Cooling Curves.

We have determined the cooling curves of a number of alloys; we will discuss that of Al 20, of which we give a reproduction. This curve, fig. 2, shows a period of heat evolution beginning at the B temperature and continuing for some time with a slightly falling temperature; this is due to the solidification of the solid solution beta, and is very characteristic of the solidification of solid solutions. The next heat



evolution, near 450° C., is probably due to the crystallisation of D as the solid solution cools across and below the cL line. The formation of the L eutectoid is indicated by a well marked flat at 424° C., the invariable temperature at which this phenomenon occurs. But the most striking feature of the curve is the recalescence that began at 388° C. and, almost instantly, raised the temperature to 428° C.

A similar recalescence can be detected in all alloys from Al 10 to Al 27, though in those remote from Al 20 it is slight, while in those near Al 20 the effect is almost as marked as in that alloy. Both the initial temperature and the rise in temperature are very variable, the latter being sometimes as much as 70° C. This is essentially an explosion phenomenon, that is,

a change commencing below the temperature at which it ought to take place, and the temperature to which the recalescence carries the ingot may not reach that at which the new system would be in equilibrium with the old one. We have also traced the *heating* curves of Al 20 and neighbouring alloys. In these heating curves the first marked absorption of heat, causing a flat, is seen to begin at 520° C., and we think that it indicates the equilibrium temperature of the recalescence reaction. A strong reason for this opinion is that if we take several pieces of an ingot that has undergone recalescence and heat them, annealing and chilling at successively higher temperatures, we find, on examination with the microscope, that the pattern produced by the recalescence persists up to 520° C., but that if the alloy be ever so little further heated this pattern disappears and is replaced by that characteristic of the pre-recalescence state.

By introducing a thermocouple into a molten ingot, tracing the cooling curve, remelting, cooling to selected points on the curve and chilling at these points, we can by microscopic examination correlate the change in structure with the thermal

changes. The change in structure occurring at the recalescence is especially remarkable. For example, Al 14 chilled at any temperature above 420° C. is a uniform solid solution of closely fitting polygons showing no trace of a second material, but chilled at 400° C. it becomes filled with polygons, bars, and sprays of a second material, and from the fact that the new substance shows a tendency to margin the old polygonal divisions, it is probable that it is richer in aluminium than the original alpha (Plate 5, fig. 10).

The same change occurs spontaneously in all alloys from Al 14 to Al 20 when they are cooled below 400° C. and either chilled or not. Ingots of Al 15 and of Al 18, chilled the first at 380° C. and the second at 410° C., show the striation in the alpha and the specks and crinkles in the beta, both alpha and beta having been uniform before the recalescence (figs. 11, 12, and 13). The change in the beta seems to be the most important and to contribute most of the heat evolved.

The recalescence may occur in the beta itself before the temperature has fallen low enough to allow of the formation of the L eutectoid, but, as a rule, the eutectoid forms before the recalescence, and in such cases we can analyse the phenomenon more minutely. In the case of an ingot in which the eutectoid already exists, the chemical change causing the recalescence is, essentially, a reaction between the two phases alpha and D which make up the eutectoid complex; it results in the complete disappearance of all the D and of most of the alpha of the complex, a new body, which for the moment we will call Y, being formed. Hence, if an ingot, which has been successively through the eutectoid and the recalescence changes, be chilled immediately after the latter, we find, instead of the L complex, a very delicate reticulation of threads which etch bright, while in the cells of this network there is the Y body, etching a uniform brown. The primary, or pseudo-primary, whether it be of alpha or of D, is unaffected at this stage. Ingots of Al 23, chilled one at 420° C. the other at 400° C., are excellent examples, the first of the eutectoid, the second of what it becomes after the recalescence (figs. 14 and 15. Compare also figs. 8 and 8A, 9 and 9A).

As the recalescence of the eutectoid always leaves a slight excess of alpha, it follows that the substance Y must contain rather less gold than the pure eutectoid of the L point; this points to the hypothesis that Y is the compound AlAu_4 .

Such an alloy as Al 23, chilled at 400° C. immediately after recalescence, contains three phases, alpha, Y, and D, while the Phase Rule only allows of two; this anomaly is removed by annealing, when the D primaries slowly react with the residual alpha to form more Y, the process resulting in the disappearance of the D or of the alpha, according to whether the one or the other was in excess; in Al 23 the alpha disappears, while in Al 20 a long anneal after spontaneous recalescence always leaves us with a trace of alpha (fig. 17). Two ingots of Al 20, the first (fig. 16) chilled at 440° C., before recalescence, and the second (fig. 17) chilled at 410° C., after spontaneous recalescence and a long anneal at that temperature, show the enormous

change caused by recalescence followed by annealing. The white pattern of the 440° C. chill is of D, and is characteristic of the D that crystallises out of uniform beta when the temperature of the ingot reaches that of the *cL* line. In the second ingot, at one time like the first, the L complex has been formed and destroyed, and the reaction between the alpha, remaining from the complex, and the D bars has removed the latter completely. A comparison of these two ingots only might lead one to think that the residue was D, but a study of ingots intermediate in temperature of chilling proves that the white residue is alpha.

Forced Recalescence.

The fact that recalesced ingots lose all trace of the recalescence pattern when heated above 520° C., at which temperature the beta is solid, makes us doubt if it would be possible to trace the liquidus of Y by seeding liquid alloys with fragments of a recalesced alloy. But we have been able to show that beta, which, spontaneously, does not often recalesce above 410° C., can be stimulated to do so at all temperatures up to 515° C.

The experiment was conducted as follows:—An ingot was melted, a thermocouple placed in it and the ingot allowed to cool slowly while its cooling curve was traced on the recorder. At a selected temperature the ingot was touched with a cold steel wire. The contact generally caused a slight and momentary drop in temperature followed by a sudden jump which sometimes rose as high as, but never above, 524° C. We have in this way forced the recalescence of alloys varying in composition from Al 15 to Al 23.

The recalescence of Al 15 could not be effected above 408° C. The change of structure is well seen by comparing the pattern of a recalesced ingot with that of a control ingot that was not recalesced but otherwise had precisely the same thermal treatment. In the case of Al 15 the control proved to be uniform solid solution with hair-line boundaries between the polygons and a few small triangular pools of matrix. In the recalesced ingot there are broad jagged edges of a new material separating the polygons, as well as a certain amount of this material inside the polygons.

The change in the case of Al 15 took place in the alpha and is not altogether comparable with the cases of forced recalescence of the beta that we are about to describe. In all the cases in which a uniform beta was forced to recalesce two new substances appear, a network or diaper of a somewhat golden material that resists etching and has all the character of alpha, and which we shall call alpha, and another material the crystallisation of which determines the pattern; this second substance etches dark and is eaten away by the etching reagent, it fills the cells of the alpha network; we have already called this substance Y. These two substances preserve their character in all the cases of forced recalescence from Al 18 to Al 23.

Al 18 was forced to recalesce at 500° C. and the temperature rose to 504° C. The

recalescence produces a fine diaper of alpha, but the alpha primaries previously existing remain uniform, as apparently the alpha cannot change at this temperature.

An ingot of Al 19 was, as the etched surface shows, forced to recalesce at $507^{\circ}\text{C}.$; although no thermal effect was noticed it shows a network of alpha with no primaries. Another ingot of Al 19 was recalesced at $504^{\circ}\text{C}.$, the temperature rising to $515^{\circ}\text{C}.$, and was chilled a few minutes later at $510^{\circ}\text{C}.$ It shows a magnificent pattern of bars, many of them large, and of feathery masses of Y, this substance being everywhere outlined by bright alpha. It is very evident in this case that a complete recrystallisation has taken place throughout the ingot, the new pattern being determined by the shape of the crystals of Y. The control ingot was fine-grained, rocky, very uniform beta.

We have forced the recalescence of Al 20 at $397^{\circ}\text{C}.$ and $445^{\circ}\text{C}.$, these two experiments only confirming our previous results. A forced recalescence at $498^{\circ}\text{C}.$, rising to $520^{\circ}\text{C}.$, is a remarkable example of the recrystallisation of a solid being brought about by a stimulus. The picture is one of broad bands, bars, and sprays of Y, margined as usual by alpha; at the highest temperature reached the Y must have been very near its melting point. Another ingot, recalesced at $514^{\circ}\text{C}.$, rose to $524^{\circ}\text{C}.$ and remained at that temperature for some minutes. The pattern shows that a good deal of the Y melted in consequence of the high temperature attained. The pools of matrix are not pure alpha but a complex resembling the feathery pattern of a high chill of beta. Here we have a case of a solid that, when touched by a cold wire, rises ten degrees, completely recrystallises and partly melts. Another ingot was recalesced at $508^{\circ}\text{C}.$ and rose to $523^{\circ}\text{C}.$ The figures 18 to 21, inclusive, illustrate forced recalescence.

These results confirm the view that Y cannot exist much above $525^{\circ}\text{C}.$ Another experiment supports this conclusion. A portion of the ingot recalesced at $498^{\circ}\text{C}.$, and described above, was reheated to $525^{\circ}\text{C}.$, kept at this temperature for 12 hours, and then chilled. The pattern shows that quite half the Y melted, or disappeared, the solid part appearing to have remained in equilibrium with the liquid; but we do not know whether the matrix continued liquid until the chill or solidified previously as beta. The solidified matrix has a coarse striation and might be a complex of alpha and Y. The equilibrium between Y and liquid, that we perhaps see here, could only be arrived at in an indirect way; if we had attempted to reach it by cooling molten Al 20 the alloy would, at $525^{\circ}\text{C}.$, have become a mass of solid beta.

Al 22 was recalesced at $478^{\circ}\text{C}.$, the temperature rising to $520^{\circ}\text{C}.$, the ingot cooling in a few minutes to $486^{\circ}\text{C}.$, when it was chilled. A control was treated in the same way but not recalesced. Both ingots have large crystals of D approximating in shape to those that separate along the cL line, but the recalesced ingot, alone, has a golden network of alpha in the matrix identical with that of the recalesced Al 20. We have also forced a recalescence in Al 23, but the pattern shows very little change.

The recalesced ingots retain the new pattern when annealed at any temperature below 520°C. , the only changes produced by the anneal being to segregate any fine pattern into larger particles and, if there is a redundant phase, to remove it by the reaction $\alpha + \text{D} = \text{Y}$.

It would now be possible to determine the complete equilibrium diagram of Y, by giving a prolonged anneal, at successive temperatures, to a number of recalesced alloys.

Experimental Details.

The alloys used in determining the liquidus, and also for many of the cooling curves, were in large quantities of several hundred grammes and were carefully analysed at the end of each experiment; but such a large number of alloys had to be made that, as a rule, we dispensed with analysis, each alloy being prepared by weighing out appropriate quantities of pure gold and pure aluminium and melting them together in a vacuous silica tube. This method of making the alloy seems to exclude the probability of any error except perhaps that of assuming the percentage of aluminium to be slightly higher than the true value.

Etching.

The etching reagents were bromine water or *aqua regia*; the two give practically the same pattern, *aqua regia* being more rapid and clean in removing the smear of metal produced in polishing. Alpha always etches bright whatever other phase is present. D etches bright in the presence of beta or of Y, but generally darkens if alpha is present. Beta and Y both etch dark, and are rapidly eaten away, in the presence of either of the other two phases. The result is a curious one in the case of alloys containing primaries of D; if the ingot is chilled above the temperature of the L eutectoid the D is bright and the ground is dark, but if the eutectoid is present, then the alpha in it generally causes the D to darken; if the alpha is removed, by recalescence and annealing, the D again etches bright, so that even without the microscope one could pretty safely infer what phases were present. Perhaps the only case where it is difficult to distinguish between alpha and D is in such an alloy as Al 20, after spontaneous recalescence and a long anneal (fig. 17). In this case there is so little of the bright substance, and the particles are so minute, that the microscope, alone, would leave us uncertain.

We have to thank Mr. F. E. E. LAMPLOUGH for very valuable help in the thermometric work and for devising an excellent method for the automatic regulation of high temperatures.

We have also to thank Prof. POPE, F.R.S., for his kindness in placing the resources of the University Laboratory at our disposal, and, in especial, for lending us valuable apparatus and lenses for microscopic photography.

DESCRIPTION OF THE MICROPHOTOGRAPHS (Plate 4).

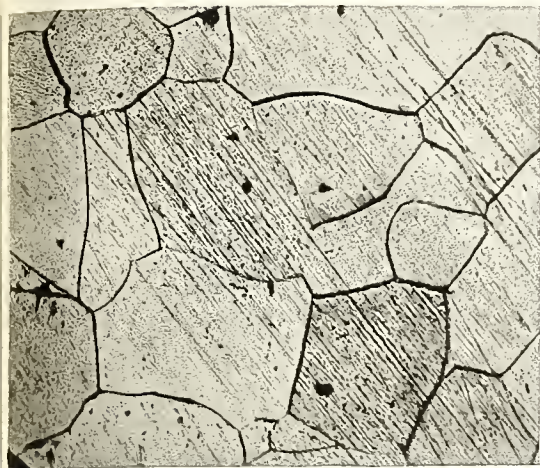
1. Al 15. 50 diameters.
Plunged molten into bath at 470° C., annealed for 20 hours and chilled.
A uniform solid solution of alpha.
2. Al 15. 50 diameters.
Annealed at 545° C. for 23 hours and chilled.
The white is alpha, the dark matrix is beta.
3. Al 18. 50 diameters.
Plunged molten into bath at 520° C., annealed for 8 hours, chilled.
Shows the decrease in the alpha as the content of Al grows.
4. Al 20. 18 diameters, oblique illumination.
Plunged molten into bath at 535° C., annealed for 17 hours, chilled.
Beta only. The relief effect is due to minute striations.
5. Al 18. 350 diameters.
Plunged molten into bath at 470° C., annealed 20 hours, chilled.
The greater part of the beta of this ingot etches the usual uniform dark, but some patches give the bright ruling of the photo. This is, no doubt, due to a very perfect laminar crystallisation of the beta, or of a body derived from the beta. The smooth parts are alpha primaries. This bright ruling can be found in a good many alloys from Al 18 to Al 23 when chilled above 440° C. Almost equally perfect ruling can be found in the dark beta.
6. Al 23. 20 diameters.
Plunged molten into bath at 506° C., annealed for 3 hours, cooled to 410° C., annealed for 18 hours, chilled.
The white is D, the ground is now, presumably, Y. The photo shows three crystal skeletons of D. It is typical of all the primaries of D.
7. Al 25. 50 diameters.
Plunged molten into bath at 535° C., annealed 20 hours, chilled.
The white is D, the dark is beta. The small pattern, which must have formed rapidly during the chill, but a good deal of it at the eutectic temperature of the point C, seems to be made up hexagonal rods of beta surrounded by a margin of D. This is the nearest approach to the C eutectic that we have.
8. Al 20. 160 diameters.
Plunged molten into bath at 510° C., annealed 1 hour, cooled to 440° C.; annealed 3 hours, cooled to 418° C.; annealed 9 hours and chilled.
The large dark bars are pseudo-primary of D that separated when temperature fell to the cL line. The small pattern is the L eutectoid of alpha and D.
- 8A. Al 20. 160 diameters.
The same ingot as that of fig. 8, but after the further treatment of dropping the ingot cold into bath at 400° C. and so provoking the recalescence. The ingot was annealed for 10 minutes at 400° C.
The small white is the alpha left after the decomposition of the eutectoid. The dark ground is Y. The large white is the residue of the pseudo-primary of D; this would be removed by a longer anneal and would leave a pattern like that of fig. 17.
9. Al 18. 300 diameters.
Plunged molten into bath at 418° C., annealed 22 hours and chilled.
A pre-recalescence pattern. The large white is primary alpha. The smaller complex is the L eutectoid of alpha and D.
- 9A. Al 18. 300 diameters.
The same ingot as that of fig. 9, but after recalescence brought about by heating for 10 minutes to 400° C.
The large primaries of alpha are no longer uniform, but are full of the recalescence pattern (compare with fig. 10). The small white is the residue of alpha left over after the decomposition of the L eutectoid.
- 9B. Al 23. 350 diameters.
Plunged molten into bath at 560° C., cooled in 2 hours to 420° C., chilled.
A pre-recalescence pattern. The large dark masses are primary D, the complex is the L eutectoid.

Plate 5.

10. Al 14. 80 diameters.
Molten and annealed at 535° C. for 4 hours, cooled to 440° C.; annealed for 17 hours, cooled to 418° C.; annealed 4 hours, cooled to 400° C.; annealed 1·5 hours, chilled.
Shows the effect of the recalescence on the alpha; a control ingot similarly treated, but chilled at the termination of the anneal at 418° C., has no pattern except traces of hair-line polygonal boundaries.
11. Al 15. 200 diameters.
Annealed at 545° C. for 23 hours, chilled.
Shows the white, uniform, alpha, and the dark, almost uniform, beta. A pre-recalcescence pattern.
12. Al 15. 200 diameters.
Plunged molten into bath at 535° C., annealed for 6 hours, cooled to 520° C.; annealed 2 hours, cooled to 380° C.; annealed 12 hours and chilled.
A post-recalcescence pattern, showing new detail in alpha and in beta.
13. Al 18. 350 diameters.
Plunged molten into bath at 506° C., annealed 3 hours, cooled to 410° C.; annealed for 18 hours and chilled (compare with fig. 3).
Shows the recalcescence detail in both alpha and beta.
14. Al 23. 320 diameters.
Melted and plunged into bath at 560° C., cooled in 1·5 hours to 420° C. and chilled. Same ingot as that of fig. 9B.
The large dark is D primary. The small pattern is the L eutectoid complex of alpha and D.
15. Al 23. 320 diameters.
Similarly treated to the preceding, but cooled to 400° C. before chilling.
Shows the effect of the recalcescence on the eutectoid.
16. Al 20. 20 diameters.
Plunged molten into bath at 530° C., annealed 1 hour, cooled to 440° C.; annealed 17 hours and chilled.
The white polygons of D crystallised out of the uniform solid beta when the temperature fell to the ϵ L line.
17. Al 20. 50 diameters.
Plunged molten into bath at 506° C., annealed 3 hours, cooled to 410° C.; annealed 18 hours and chilled.
This represents the final condition of Al 20 after recalcescence and a long anneal; it consists of Y with a trace of residual alpha in small particles.
18. Al 20. 50 diameters.
A control ingot to the forced recalcescences of the alloys described below. The ingot was placed molten in bath at 560° C., cooled to 508° C., and chilled.
It consists of uniform beta; under oblique lighting it resembles fig. 4.
19. Al 20. 50 diameters.
Similar treatment to the preceding, but, in addition, the ingot was forced to recalcesce at 508° C. by touching with a cold wire, the reaction causing the temperature to rise to 523° C. A few minutes later, when the temperature had again fallen to 508° C., the ingot was chilled.
There are large crystal skeletons of the new body Y bordered by a white substance which is, presumably, alpha.
20. Al 20. 160 diameters.
The same ingot and treatment as that of fig. 19. An enlarged view.
21. Al 20. 50 diameters.
A forced recalcescence at 514° C. Temperature rose to 524° C. and remained at that temperature for some minutes through the heat evolved in the reaction. There was consequently some fusion and the dark matrix is beta reformed on solidification. The crystal skeletons are of Y.



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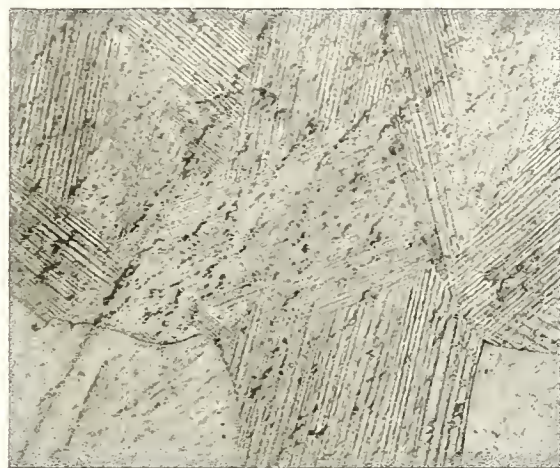
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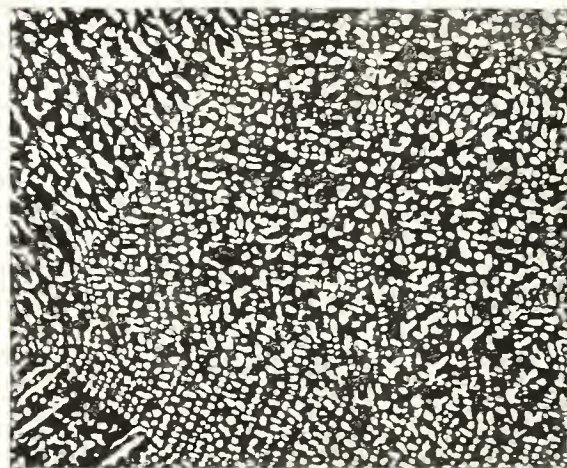
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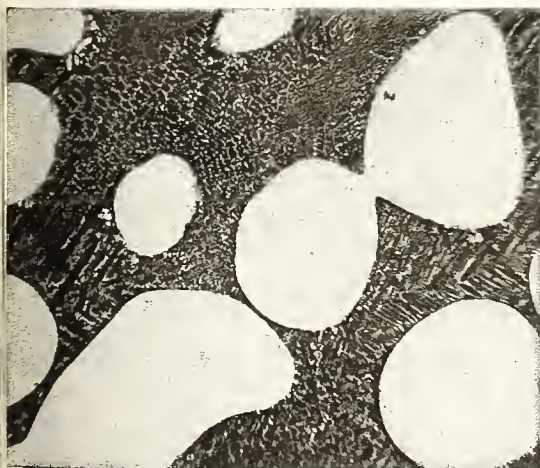
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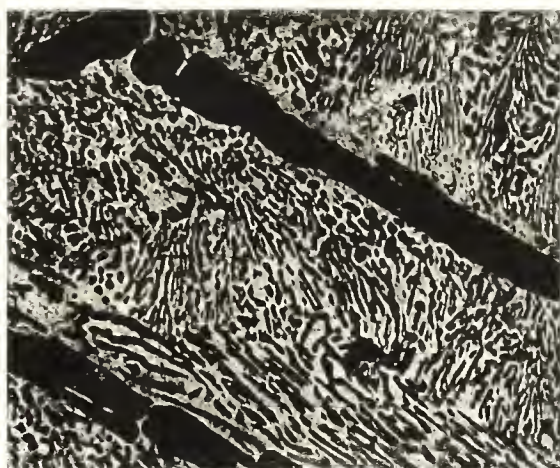
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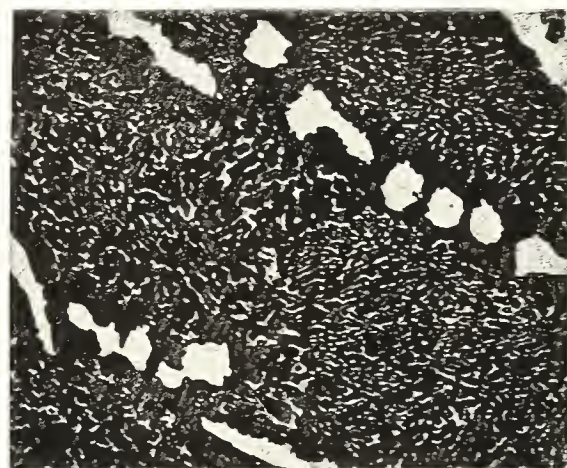
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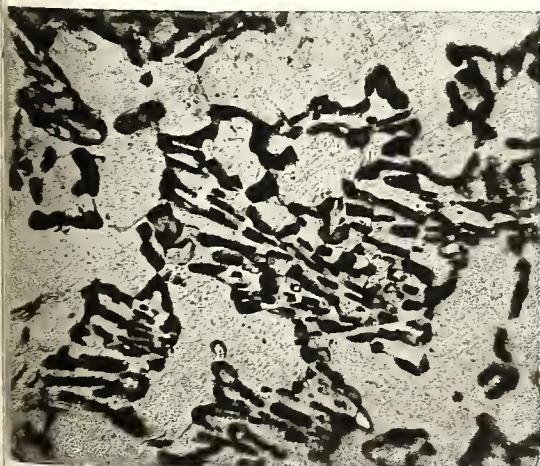
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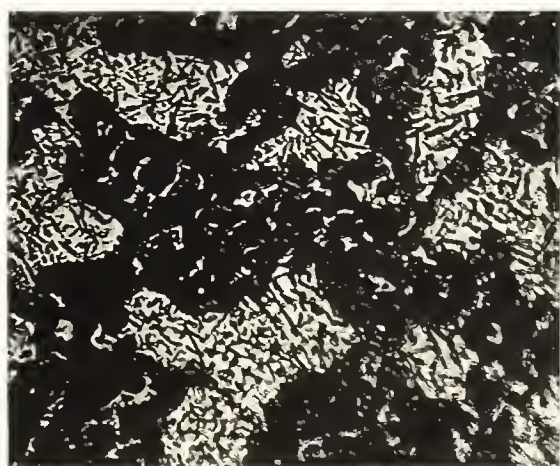
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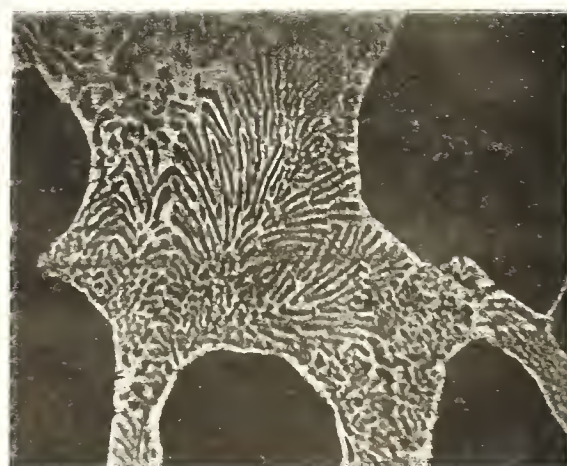
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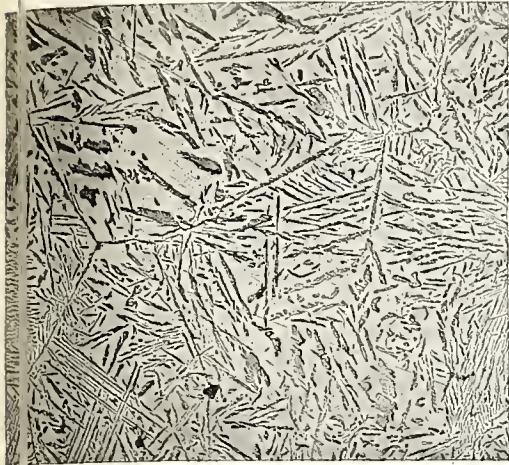


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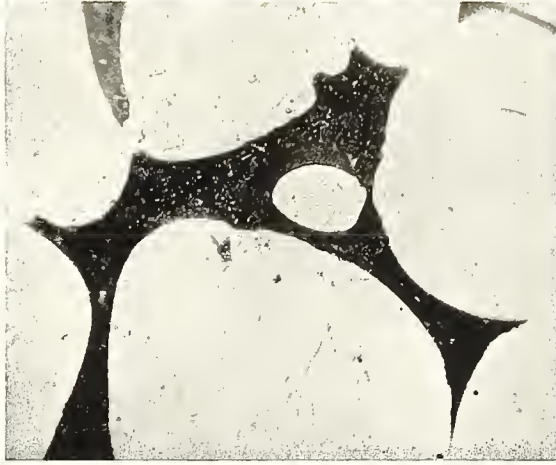


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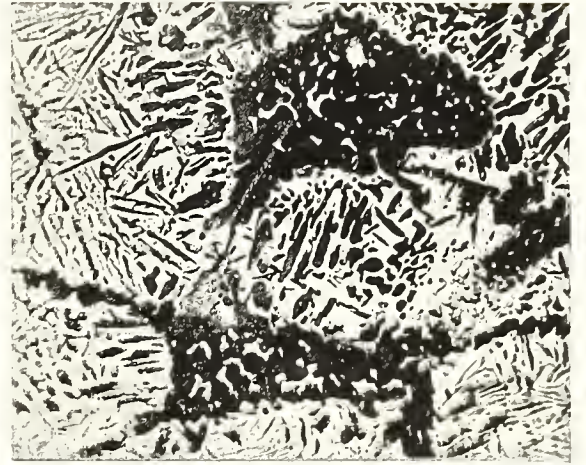




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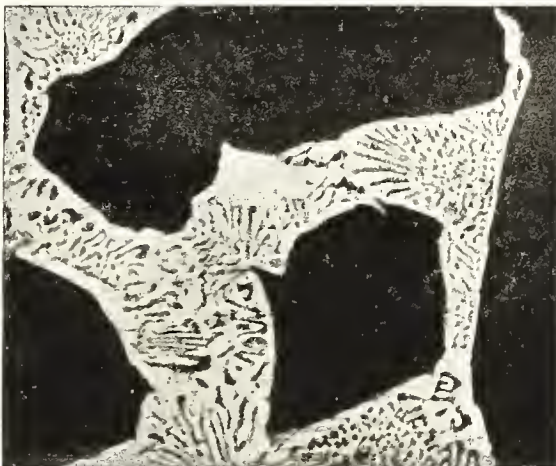
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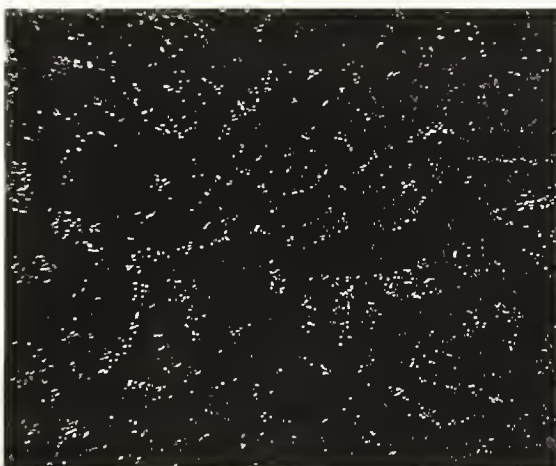
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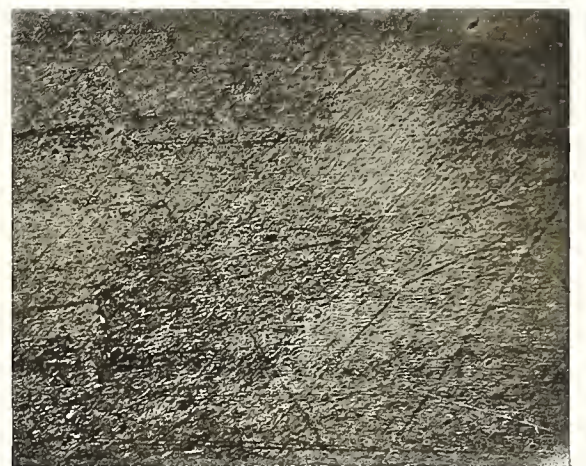
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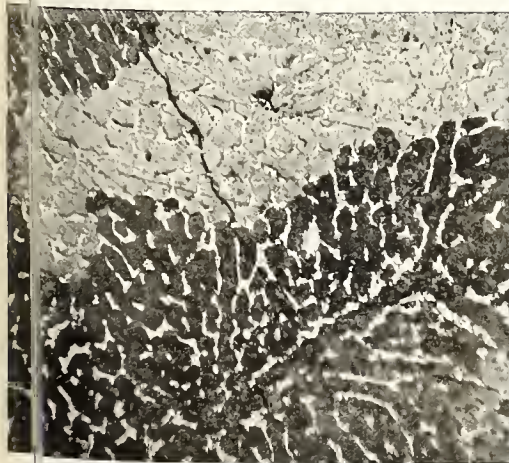
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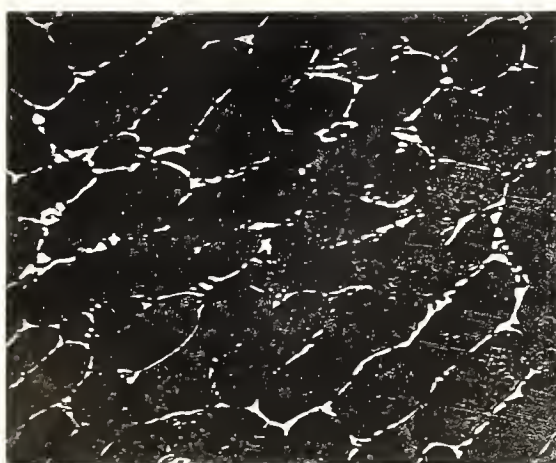
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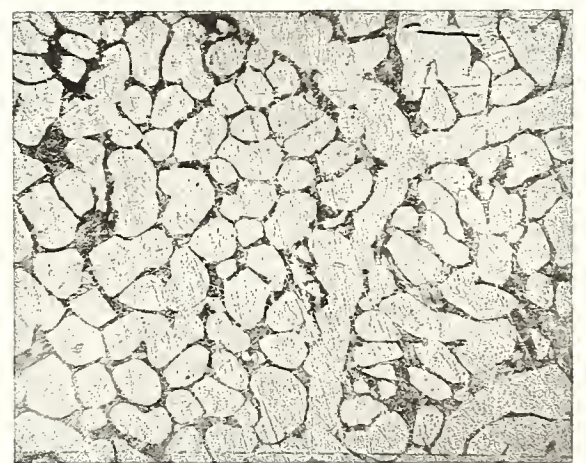
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VIII. *On the Origin of the Electron Emission from Glowing Solids.**By* FRANK HORTON, *Sc.D.**Communicated by Prof. Sir J. J. THOMSON, O.M., F.R.S.*

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RECENT researches on the negative ionisation from glowing solids have thrown doubt upon the accuracy of the view of the origin of this ionisation which has been developed by O. W. RICHARDSON.* This view is based upon the electron theory of metallic conduction, and supposes that an electron entering the surface layer with a normal velocity component greater than a certain amount is able to escape into the surrounding space and to take part in carrying the thermionic current. On this view, as RICHARDSON has shown, the electron emission increases rapidly with the temperature, the connection between these quantities being given by an exponential formula, $i = A\theta^{\frac{3}{2}}e^{-\frac{Q}{2\theta}}$, where i is the thermionic current, θ the absolute temperature, and A and Q are constants. That this formula represents, with fair accuracy, the observed results in the case of platinum and of certain other substances for temperatures up to about 1500° C. has been shown by the experiments of RICHARDSON,* by H. A. WILSON,† by the author,‡ and by other experimenters. Experiments made at higher temperatures, however, have generally shown that the current increases less rapidly with the temperature than is required by this formula. Even if in all cases the temperature variation of the thermionic current were in agreement with the above formula, this could not be taken as a proof of the theory of the origin of the effect, for it has been shown by H. A. WILSON that the current measured at any temperature depends very largely upon the nature of the residual gas present in the apparatus.§ This has led to the view that the ionisation is entirely due to chemical action between these gases and the heated solid—a view which is also in agreement with the exponential relation between the current and the temperature, for such chemical changes as might occur would probably increase with the temperature in this manner.

* O. W. RICHARDSON, 'Phil. Trans.,' A, vol. 201, p. 297, 1903.

† H. A. WILSON, 'Phil. Trans.,' A, vol. 202, p. 243, 1903.

‡ F. HORTON, 'Phil. Trans.,' A, vol. 207, p. 149, 1907.

§ *Loc. cit.*

This chemical action theory has received considerable support from the experiments of HABER and JUST,* and of FREDENHAGEN† on the alkali metals, and of PRING and PARKER‡ on carbon. The experiments with the alkali metals have shown that by repeated purification (re-distillation) and continued improvement of the vacuum the electron emission from these substances can be continually reduced. If, however, a small amount of air or of any gas having chemical action upon the metal under test is allowed to enter the apparatus a large increase in the thermionic current at once occurs.

Similarly with carbon, the negative thermionic current decreases continuously as the gas pressure is reduced and with progressive purification of the carbon, and, moreover, by admitting small amounts of different gases to the carbon an increased current is obtained, the increase being proportional to the known chemical activity of these gases.

RICHARDSON§ has recently described some experiments with a tungsten filament heated to a high temperature in a good vacuum, and has considered the following different hypotheses as to the possible mode of origin of the electronic emission :—

- (1) The emission is due to the evolution of gas by the filament ;
- (2) The emission is caused by chemical action or by some other cause depending on impacts between gas molecules and the filament ;
- (3) The emission is the result of some process involving consumption of tungsten.

Experiments were devised for the purpose of testing each of these theories, and in each case it was found that the observed emission was very much greater than could be accounted for by the hypothesis. It was, therefore, concluded “that the emission of electrons does not arise from any interaction between the filament and the surrounding gases or vapours, nor from any process involving consumption of the material of the filament. It thus follows that the emission of electrons from hot tungsten, which there is no reason for not regarding as exhibiting this phenomenon in a typical form, is not a chemical but a physical process.” These experiments cannot be regarded as conclusive, for there is a possible cause of the electron emission which has not been considered, namely, that it arises from an action between the metal filament and gases contained in it. In testing the second hypothesis mentioned above, RICHARDSON considers only the possibility of chemical action between the metal filament and the gas surrounding it. The fact that the thermionic emission from a hot wire in a good vacuum decreases on driving out the gas by long continued

* HABER and JUST, ‘Ann. der Phys.,’ vol. XXX., p. 411, 1909, and vol. XXXVI., p. 308. 1911.

† FREDENHAGEN, ‘Verh. d. Deutsch. Phys. Ges.,’ vol. 14, p. 384, 1912.

‡ PRING and PARKER, ‘Phil. Mag.,’ vol. 23, p. 192, 1912, and PRING, ‘Roy. Soc. Proc.,’ A, vol. 89, p. 344, 1913.

§ RICHARDSON, ‘Phil. Mag.,’ vol. 26, p. 345, 1913.

heating is evidence that the emission depends either upon the evolution of gas, or upon the gas remaining in the hot wire. RICHARDSON's experiments to test the former hypothesis may be taken as showing that the emission does not depend upon the evolution of gas, but there remains the possibility that it depends upon the gas which is still in the wire. This latter view was put forward by H. A. WILSON* to explain the results of some experiments with a platinum wire heated in hydrogen gas. A series of observations were made at different pressures and it was found that on changing the gas pressure the resulting change in the thermionic current took some time to become established. A similar effect was obtained when the temperature of the wire was altered, keeping the gas pressure constant, in which case the current variation lagged behind the alteration of temperature. These experiments indicate that the increase in the thermionic emission produced by hydrogen is due to the gas inside the platinum, and that it takes time for equilibrium between the gas in the wire and that surrounding it to become established after any change in the conditions has been made.

It appeared probable that information as to the origin of the electron emission from glowing solids could be obtained by experimenting with those substances which give rise to exceptionally large emissions, namely, the oxides of the alkaline earth metals. These oxides are all stable substances and ones which would not be expected readily to enter into reaction with the residual gases left in the apparatus. The possibility of chemical action in these cases would appear to depend on the oxide being electrolysed by the passage of the discharge, in which case there might be some re-combination of the constituents. The view that the electron emission from glowing lime occurs as a result of the re-combination of calcium and oxygen which have been liberated electrolytically has been put forward by FREDENHAGEN.† The author does not think that there is very strong experimental evidence that such chemical changes do give rise to the emission of electrons, but this point will be referred to later. If, for the present, we admit the possibility of the chemical union of calcium and oxygen giving rise to an emission such as is observed in the case of the Wehnelt cathode, the plausibility of FREDENHAGEN's theory depends upon whether the conductivity of these oxides can be shown to be accompanied by electrolysis.

In 1905 the writer investigated the electric conductivity of lime at high temperatures and came to the conclusion that the conductivity is mainly carried on by electrons set free in the interior of the oxide, but that some electrolysis also occurs. Thus there is the necessary condition for the hypothesis as to the origin of the activity of a lime cathode put forward by FREDENHAGEN. The following experimental results are given by FREDENHAGEN as supporting this hypothesis:—

* H. A. WILSON, 'Phil. Trans.,' A, vol. 202, p. 243, 1903.

† K. FREDENHAGEN, 'Ber. K. Sächs. Ges. Wiss.,' Leipzig, vol. 65, p. 42, 1913.

- (1) The emission from calcium is increased when traces of oxygen are present ;
- (2) After much use a Wehnelt cathode becomes "exhausted." The oxide gradually disappears and the metallic surface of the supporting strip is perceptibly altered, probably owing to the action of the liberated calcium or oxygen ;
- (3) When lime is heated so that no current goes through it there is no large emission of electrons.

The only one of these which appears to the writer to be conclusive is the last. The first result follows from the fact that the emission from lime is much greater than that from calcium,* and it is not necessary to attribute the emission to the process of oxidation. The second result supports the view that electrolysis takes place, but does not support the theory further than that ; but from the third result it appears that electrolysis is essential in order to obtain a large electronic emission from lime.

The experiments described in the present paper were performed to test the "chemical action" theory of the working of an oxide cathode. FREDENHAGEN experimented with a mixture of lime and magnesia which is a better conductor than lime alone. The experiments about to be described were made with Nernst filaments which consist of a mixture of oxides having a much larger conductivity than the mixture used by FREDENHAGEN. It has been found by OWEN† that these filaments give a large electron emission at high temperatures. The following points have been investigated :—

(1) Does the electron emission from a Nernst filament depend upon whether the heating current is alternating or continuous? One would expect that, if the conduction through these filaments is electrolytic, the re-combination of the constituents would be more complete with an alternating current than when a continuous current is employed ; so that, if the electron emission depends upon this re-combination, the thermionic current at a given temperature should be greater in the former case.

(2) Is the emission from a Nernst filament the same when it is heated by conducting an electric current and when it is heated by a metal wire passing through it, the wire being raised to incandescence by an electric current? In the latter case practically the whole of the heating current is carried by the metal wire, and any electrolysis which occurs is due to the thermionic current ; so that re-combination must be very much less under these circumstances than when the filament itself conducts the heating current.

* F. HORTON, 'Phil. Trans.,' A, vol. 207, p. 149, 1907.

† G. OWEN, 'Phil. Mag.,' vol. 8, p. 230, 1904.

(3) The emission from a Nernst filament heated in the ordinary manner has been compared with that given by the same material when powdered and heated upon platinum. This experiment is similar to that just mentioned.

The discharge tube used in these experiments is similar to that described in a former paper,* the only difference being that in the present apparatus the two parallel platinum plates which form the anode are 1.5 cm. apart. The cathode, when in position, is parallel to these plates and mid-way between them. It can easily be removed from, or replaced in, the discharge tube. Experiments were made with twenty different Nernst filaments of the B type and meant for use on a 100-volt alternating supply. They were all of about the same dimensions, the length of the glowing portion being about 9.5 mm. and the diameter about 0.78 mm. When a filament was to be heated by passing an electric current through it, the platinum wires attached to it were welded on to the platinum leads of the discharge tube. When an alternating current was used, this was obtained from the secondary of a transformer, the primary coil of which was connected to the alternating town supply, and the current through the filament could be varied by changing the resistance in both the primary and the secondary circuits. One of the fine iron wire resistances supplied with Nernst lamps was always kept in series with the filament. As this resistance has a large positive temperature coefficient, it tends to steady the current through the circuit. In order to start the filament glowing it had to be taken from the discharge tube and heated by holding it above, and near to, a glowing "heater" of the kind supplied with an ordinary Nernst lamp. It was then replaced in the apparatus which could be rapidly exhausted, when required, by means of a water-pump, a mercury pump, and a charcoal tube cooled in liquid air. In this way the gas pressure in the apparatus could be reduced to .0001 mm. within twenty minutes from the time that the filament was started glowing.

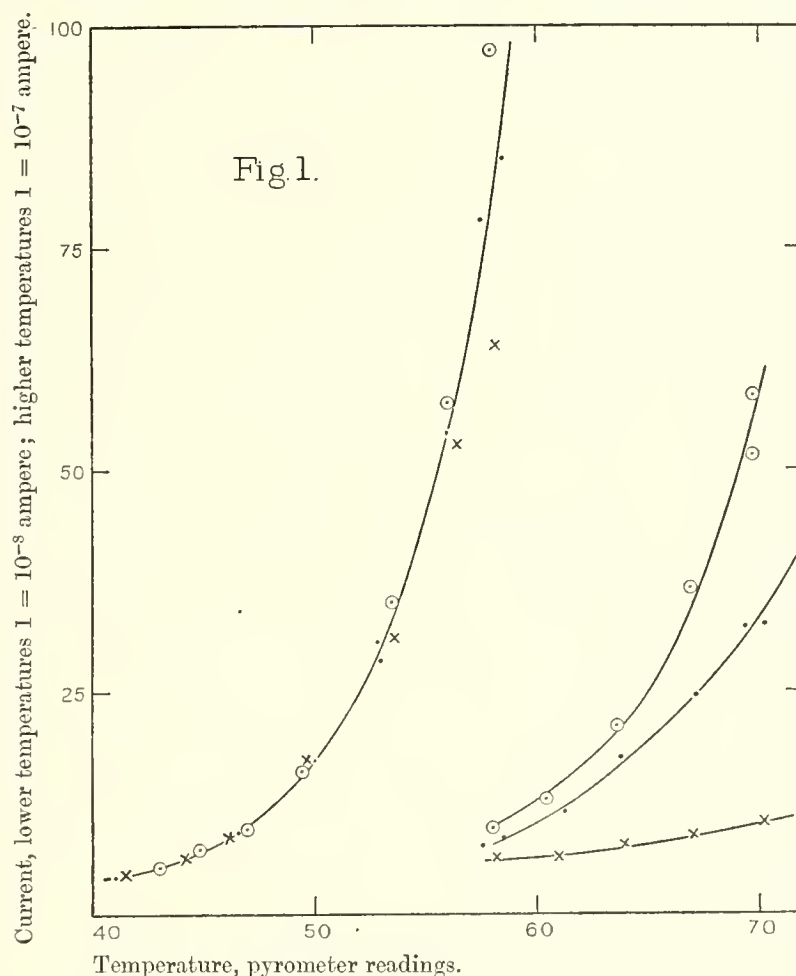
The temperature of the glowing filament was determined by means of a Féry optical pyrometer for the use of which I am indebted to Prof. T. MATHER, of the City and Guilds College, London. This instrument was standardised by observations of the readings corresponding to different temperatures of a platinum tube of about the same diameter as a Nernst filament and having a standardised thermocouple of fine wires of platinum and platinum-rhodium welded to it. In order that the surface of the platinum tube should be exactly similar to that of the filament, a filament was finely powdered and mixed with water, and the platinum tube was then covered with a thin layer of Nernst filament material by evaporating this mixture upon it. This platinum tube was fitted up in the place of the filament in the discharge tube, and observations of the thermo-electromotive force and readings of the optical pyrometer were taken at several temperatures between 900° C. and 1600° C. From these observations, the temperature of a Nernst filament corresponding to any reading of the pyrometer between these limits can be ascertained with fair accuracy.

* 'Roy. Soc. Proc.,' A, vol. 88, p. 117, 1913.

I. *A Comparison of the Electronic Emissions from a Nernst Filament when Heated by Continuous and by Alternating Currents.*

It was originally intended to perform this experiment with the filament in a vacuum, but it was soon found that, although with an alternating current the filament continued to glow steadily in a vacuum, when a continuous current was used one end of the filament became much hotter than the other end. At the same time the gas pressure in the apparatus increased, so that evidently some electrolysis of the material of the filament was proceeding. It was always the positive end of the filament which became the hotter, probably because some metallic constituent of the filament is set free by electrolysis at the negative end and lessens the resistance in that part of the circuit. In order to maintain the whole of the glowing portion of the filament at a uniform temperature when a continuous heating current was used, it was found to be necessary to have oxygen present in the discharge tube. This no doubt oxidises the metal liberated by electrolysis and so maintains the uniformity of the filament. The comparison was therefore performed with air in the apparatus at atmospheric pressure, and the observations were taken in the following manner:—The leads to the filament were connected to a switch so that either an alternating current from the transformer or a continuous current from a battery of storage cells could be used. In addition to the adjustable resistances in the transformer circuit there was another set of adjustable resistances in the battery circuit, so that the E.M.F. applied from the storage cells could be made equal to that being supplied at any instant by the transformer before switching the filament on to the continuous current circuit. In this way observations at about the same temperature could be made alternately, with the filament connected to the transformer or to the storage cells. A filament was first of all heated on the alternating circuit and a series of observations of the negative thermionic current at different temperatures was taken, the potential difference applied across the discharge tube, in addition to that due to the heating circuit, being 210 volts. The negative emission at atmospheric pressure from a new Nernst filament generally increased during the first day's heating, after which it remained fairly constant. When the steady state had been reached a comparison of the emissions from the filament heated on the "alternating" circuit and on the "continuous" circuit was made. The temperature of the filament was adjusted to be as low as practicable, and readings of the thermionic current and of the pyrometer were made; the filament leads were then switched on to the continuous current circuit, and the pyrometer and galvanometer readings were again taken. The filament was immediately switched back on to the alternating circuit, so that it never remained for more than a minute or two with a continuous current flowing through it. The direction in which the continuous current was to flow through the filament was then reversed by means of a reversing key connected with the storage cells, and the observations of the emission and temperature, with alterna-

ting and with continuous current heating, were repeated. The temperature of the filament was then raised by lessening the resistance in the transformer circuit. A corresponding alteration was made in the battery circuit, and measurements of the thermionic emission and the temperature of the filament were made on both circuits. The observations were repeated at gradually increasing temperatures up to about 2050°C ., after which the temperature was gradually lowered again, the measurements being repeated at frequent intervals. In this way three series of measurements of the thermionic current at different temperatures were obtained, with the filament heated (1) by an alternating current; (2) by a continuous current in one direction;



Points marked ● are for heating by alternating current.

Points marked ○ are for one direction of continuous current.

Points marked × are for reverse direction of continuous current.

(3) by a continuous current flowing in the opposite direction through the filament. The values of these thermionic currents were plotted against the corresponding temperatures, and it was found that for temperatures up to about 1750°C . the points all fell fairly well upon one curve, but that at temperatures much higher than this they separated out into three curves; the curve corresponding to the alternating heating current falling between the two curves corresponding to the different directions of the continuous current. A set of curves obtained with one of the filaments experimented on is given in fig. 1. In this the thermionic currents for the

higher temperatures are plotted to one-tenth the scale used for the lower temperatures. The temperatures are represented by the pyrometer readings, to which they are roughly proportional. I was unable to calibrate the pyrometer accurately in degrees centigrade up to the highest temperatures used in these experiments. The lowest temperature represented in the figure (pyrometer reading = 40) corresponds to 1637°C. ; the highest temperature represented (pyrometer reading = 72) corresponds to about 2050°C.

The curious difference between the values of the thermionic current at a high temperature for the two directions of the continuous heating current arises from the potential difference of the heating circuit. There was a fall of potential of some 80 volts along the glowing portion of the filament, and a steady potential difference of 210 volts was maintained between one end of the filament and the anodes. The potential difference between the anodes and a point near the other end of the filament therefore changes considerably when the direction of the heating current is reversed. At the lower temperatures the current does not vary very rapidly with the potential difference when this is in the neighbourhood of 200 volts, but at the higher temperatures used in these experiments the current is far from being saturated with 200 volts, so that the total thermionic current will change considerably when the direction of the current in the heating circuit is reversed. If this explanation is correct we should expect that the difference between the emissions in the two cases would be much less if a considerably larger potential difference were maintained between the electrodes of the discharge tube. To test this, the potential difference was increased to 600 volts and the experiment was repeated. It was then found that even up to 2000°C. there was no marked difference in the value of the thermionic current when the direction of the continuous heating current was reversed. The experimental points for both alternating and direct current heating fell equally nearly to a single curve, showing that the electron emission from the filament under these conditions is the same whether an alternating or a direct heating current be used.

On the theory that the electron emission depends upon the re-combination of the electrolytically separated constituents of the oxides of which a Nernst filament is made, it would be expected that a larger emission would be obtained when an alternating heating current is employed, for then the re-combination is complete. The experiment just described would be more conclusive in disproving this theory if it could have been performed in the absence of oxygen, but, as already stated, the filament could not be made to continue uniformly hot with a direct heating current for long enough to enable accurate observations to be made. When the experiment was tried in a vacuum there was the further difficulty that the liberation of oxygen by the continuous current altered the gas pressure in the apparatus, and ionisation by collisions greatly increased the measured thermionic current. Experiments were also made at as low temperatures as possible in pure nitrogen gas at atmospheric

pressure so that this increase in pressure should be negligible, but the results only justify the statement that there was no great difference between the emissions when the filament was switched momentarily from the alternating circuit on to the continuous current circuit, the resistances in the two circuits having been adjusted so as to maintain the temperature of the filament as nearly constant as possible. Since the filament could not be left on the direct current circuit for more than a second or two at a time, the comparison was very difficult to make. For this reason the following experiments were devised.

II. *A Comparison of the Electronic Emissions from a Nernst Filament when heated by conducting an Electric Current and when heated without conducting the Heating Current.*

The ideal method of making this comparison would, at first sight, appear to be to heat the filament in a vacuum, firstly in the usual manner by means of an alternating current, and secondly by placing the discharge apparatus in some form of furnace. Such an experiment is, however, not practically possible, for a good vacuum cannot be maintained in a vessel at a high temperature; not only do the walls of the vessel evolve gases, but the material which would otherwise be most suitable for the experiment—quartz—becomes permeable to gases at about 1000°C .

A Nernst filament is in the form of a fine tube and the method of thermal heating adopted was by placing the filament upon a tantalum wire which just fitted it, and heating the wire by an alternating current from the transformer. Tantalum wires of the required diameter were prepared for me by Messrs. Siemens, and supplied in straight pieces about 5 cm. long. In performing the experiment the negative emission from a filament was investigated when the filament was heated by an alternating current in a vacuum obtained by the use of charcoal cooled in liquid air. The thermionic current as a rule decreased slightly during the first few hours heating of a new filament; after this it was fairly steady even when tested on different days. The filament was then disconnected from the platinum leads and its two ends were cut off, leaving the middle glowing portion only. This was mounted on a tantalum wire cut to such a length that about 3 mm. of wire protruded from each end of the filament. The ends of the tantalum wire were fitted inside two short pieces of platinum tube which had already been welded on to the platinum leads of the discharge tube. The platinum tubes were of about the same dimensions as the filament, and were pinched up tightly on to the tantalum, thus making good electrical contact. In this way none of the tantalum wire was exposed; the central part was covered by the filament and the ends were encased in the platinum tubes through which the heating current was conducted. These leads never became sufficiently hot to give a thermionic emission comparable with that given by the filament. After exhausting the apparatus, the tantalum wire was heated by an

alternating current and in this way the temperature of the filament was raised, and its electron emission was investigated at different temperatures. The thermionic currents were plotted against the corresponding temperatures and a curve was drawn through the points so obtained. Some readings from this curve are given in the following table which also contains, for comparison, the values of the thermionic current from the filament at the same temperatures when heated by an alternating current passing through it. In both cases the potential difference applied between the terminals of the discharge tube was 208 volts, and the pressure of the residual gas in the apparatus was 0.0002 mm.

Temperature, degrees Centigrade.	Thermionic currents : 1 = 10^{-7} ampere.	
	Filament heated by alternating current.	Filament heated on tantalum wire.
1754	585	480
1690	319	256
1637	163	146
1590	78	78.5
1546	35.2	42.8
1503	22.6	23.4
1466	8.6	9.7

An inspection of the above table shows that the electron emission is about the same whichever method of heating the filament be employed. The greatest difference is about 20 per cent., which is not large considering the experimental difficulties. From the table it appears that the emission from the filament when heated by conducting a current increases more rapidly with the temperature than it does when heated upon a tantalum wire. This was not always the case. Several filaments were tested and it was found that the emission was sometimes greater with one method of heating and sometimes with the other. The rate of increase with temperature of the thermionic emission from a Nernst filament heated by an alternating current in a vacuum depends very largely upon the previous heating of the filament. With filaments which had been heated for many hours at a high temperature it was generally found that the thermionic current above about 1500° C. increased with the temperature much less rapidly than with a new filament.

It is clear that in these experiments there must be very much more electrolytic decomposition of the Nernst filament, followed by re-combination of the constituents, when the filament is itself conducting the heating current, than when it is heated on the tantalum wire. In the latter case only a very small portion of the heating current flows through the filament, for the resistance of this, even at the highest temperature, is very large compared with that of the tantalum wire.

In comparing the electron emission in the two cases it is necessary to consider the

possible effect of the electron emission from the tantalum wire itself. If the electrons emitted by the wire escape through the Nernst filament they would take part in carrying the measured thermionic current. In order to see whether the current so carried is likely to be a considerable fraction of the whole, a piece of tantalum wire of the same dimensions as that used inside the filament was fitted in the apparatus, and the emission from this was measured at different temperatures. At the highest temperature recorded in the table the emission was only 9.7×10^{-7} amperes. It is, however, probable that in the experiment proper the wire was considerably hotter than the filament surrounding it, and there is therefore some doubt as to what part of the measured current may be due to the tantalum; but I think it can safely be stated that the emissions from the filament alone with the two methods of heating are of the same order of magnitude, and there is certainly no such difference as would be expected if the emission were caused by the re-combination of the electrolytically separated elements of which the filament is composed. In order to eliminate the uncertainty as to the effect of the heating wire the following experiment was made.

III. *A Comparison of the Electronic Emission from a Nernst Filament Heated in the Ordinary Manner with that given by the same Material when Heated upon Platinum.*

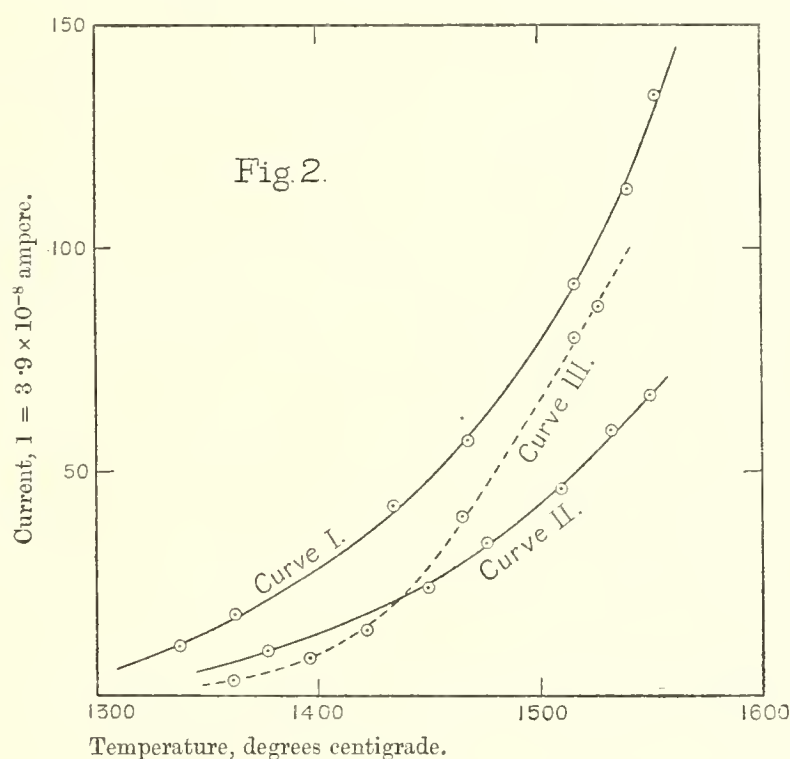
In making this comparison the emission from a filament at different temperatures when heated by an alternating current in a good vacuum was investigated as before. When the thermionic current tested under similar conditions on different days had become fairly constant, the filament was detached from the apparatus and a piece of fine platinum tube was substituted for it. The external diameter of the platinum tube was a little less than that of the filament, and the internal diameters of the two were the same. The length of the tube used was rather greater than that of the glowing part of the Nernst filament so as to make the surface areas about the same in the two cases. A thermocouple of fine wires of platinum and platinum-rhodium was carefully welded on to middle of the platinum tube. This junction was standardised in the usual manner and served to determine the temperature of the tube during the experiments. Observations of the negative emission from this platinum tube were then made under similar conditions to those employed with the Nernst filament, except that the temperature was determined by the thermocouple and not with the pyrometer. The emission from this platinum tube was rather larger than I had expected it would be, and much larger than that measured from the tantalum wire. It was therefore taken down and boiled in strong nitric acid and afterwards heated for some time in air at atmospheric pressure. This treatment reduced the emission considerably, but the thermionic current at 1516° C. was found to be 9×10^{-6} amperes per sq. cm. of heated surface, which is about 20 times the value to which the emission from a platinum wire can be reduced. However, this value of the current was fairly constant, and the values now obtained at different

temperatures were therefore taken as measuring the steady emission from the platinum.

The glowing portion of the filament, the emission from which had already been tested, was finely powdered in an agate mortar; a little water was added, and the particles in suspension were transferred to the platinum tube by evaporating this mixture upon it, a drop at a time. In this way a thin uniform coating of Nernst filament material was obtained upon the platinum tube, which was heated to a bright red heat in the air for a few minutes before being fitted into the apparatus. After reducing the gas pressure to 0.0001 mm. the thermionic emission from the heated tube was again investigated, with, as usual, a potential difference of about 210 volts between the terminals of the discharge tube. A series of observations at gradually increasing temperatures was made, and it was found that the negative emission was now very much smaller than the emission from the uncoated platinum tube had been. The observations showed, however, that the emission was tending to increase, and the tube was therefore left heating for some three or four hours, after which another series of measurements at different temperatures was made. The emission was found to have increased to about 30 times the original values and was now remarkably constant. It was thought desirable to treat the tube exactly as the Nernst filament had been treated when its emission was measured, so before testing on the following day, air was let into the apparatus to atmospheric pressure, and the tube was raised to incandescence for a few minutes, after which the apparatus was evacuated with the platinum tube still glowing. The emissions at various temperatures were at once measured and were found to be about 30 per cent. smaller than those finally obtained on the previous day. On continuing the heating, however, the emission soon rose to about the same value as before. These operations were repeated on four days, and in every case the final steady values of the thermionic current were within 5 per cent. of the steady values first obtained after the initial increase had taken place. It was assumed that these currents represented the sum of the emission due to the platinum tube and that due to the material upon it, and to obtain the effect of the material alone the previously determined values of the emission from the clean platinum tube at corresponding temperatures were subtracted. In making this correction the current from the thermocouple welded on to the platinum tube was used as an indicator of its temperature, and the part of the thermionic current taken as being due to the platinum at each temperature was the value which had been obtained from the tube alone at a temperature giving the same deflection of the galvanometer attached to the thermocouple.

In comparing the corrected value of the emission from the filament material heated on the platinum tube with the emission from the whole filament at the commencement of this series of experiments, the optical pyrometer readings were used for temperature measurements in both cases. The comparison is best made by means of emission-temperature curves, and in fig. 2 three such curves are given. Of these, Curves I.

and II. are for the Nernst filament when heated in the usual manner by conducting an alternating current. Two curves are given so as to show the magnitude of the variation of the negative emission with this method of heating. The curves represent the last two series of observations made before the filament was broken up. Curve II. was obtained immediately after heating at a high temperature in air at atmospheric pressure, and Curve I. shows the values to which the emission had risen after heating for some hours in a vacuum. The increase is rather greater than that obtained when the material on the platinum tube was similarly treated. The corrected "steady values" of the emission from the material of this filament when heated on the platinum tube are shown in Curve III. The differences between these values and those given by the filament itself are not greater than the differences often obtained in observations



Curves I. and II. are for a Nernst filament heated by an alternating current.
Curve III. is for the material of a Nernst filament heated upon platinum.

of the thermionic current from a glowing cathode under apparently identical conditions at different times, and we may safely conclude that there is no marked difference between the emission from the material heated upon platinum—when practically none of the heating current flows through it—and that obtained when the material is heated by conducting an electric current. Such a result could not be obtained if the electrons are liberated only as a result of the chemical action between the products of electrolysis of the material of the filament.

It has been mentioned that the emission from the material heated upon platinum was, at first, considerably less than that from the platinum alone, and that this emission gradually increased up to a steady value in the course of three or four hours' heating.

The fact that the negative emission from a Wehnelt cathode often takes some time before it reaches its full strength is considered by FREDENHAGEN to support his view of the action of these cathodes. An investigation of this effect was therefore made in the case of the Nernst filament material heated upon platinum. For this purpose a strip of platinum foil was welded on to the platinum leads of the discharge tube and was covered with the material of a Nernst filament in the manner which has already been described. The temperature of this strip was determined by means of a thermojunction welded on to it, and the thermionic emission was tested under different potential differences and with different gas pressures in the apparatus. At a low pressure and with 40 volts applied potential difference the negative emission was, at first, very much smaller than that from platinum alone; the measured current at 1440°C. being 10^{-8} amperes cm.^2 , and at 1500°C. 3.2×10^{-8} amperes cm.^2 —values which remained fairly constant during two hours' heating.

The positive emission was also tested, and this was very much larger. At 1260°C. it began by being 2.3×10^{-6} amperes cm.^2 and decreased to 1.7×10^{-6} amperes cm.^2 after heating for 40 minutes, the platinum strip being charged to +40 volts throughout this period. It was found that leaving the strip charged positively for some time did not cause any increase in the negative emission when tested immediately afterwards, but that after the strip had been charged negatively for a few minutes, the positive emission was abnormally large. On FREDENHAGEN'S view of the activity of the oxide cathode, the passage of the feeble negative thermionic current from the platinum strip is supposed to start the electrolytic decomposition of the oxide, and the re-combination of the elements separated by electrolysis is supposed to give rise to a further electron emission, thus increasing the current through the oxide layer and causing the gradually increasing effect sometimes observed with these cathodes. The material of a Nernst filament certainly appears to suffer electrolysis when a current flows through it, so that electrolytic decomposition may be produced by a thermionic current flowing through the material on the platinum strip; but although a strong positive thermionic current was measured at 1265°C. , no negative emission could be detected at this temperature on reversing the direction of the electric field. This experiment shows that the activity of the oxide cathode does not arise in the manner suggested by FREDENHAGEN.

On the day following that upon which the observations just recorded were made, the effect of a stronger electric field was investigated. A potential difference of 209 volts was used between the electrodes and the temperature of the platinum strip was gradually raised to 1500°C. The negative thermionic current at this temperature was 1.9×10^{-7} amperes cm.^2 . It began to increase, very slowly at first, but after about 10 minutes a sudden rapid increase to 1.3×10^{-6} amperes cm.^2 took place, after which the slow increase again continued during a further 10 minutes' heating which was given. The apparatus was then left until the following day when the temperature was at once raised to 1500°C. The negative emission was 2.0×10^{-6} amperes

cm.² and gradually increased to 7.7×10^{-6} amperes cm.² during two hours' heating; but the increase did not stop here, and further heating on the two following days increased the emission to 1.1×10^{-5} amperes cm.² at 1500° C., at which value it remained constant for some time. This gradual increase in the thermionic current was not due to an increasing gas pressure, for this was kept constant by continual pumping; nor did it depend upon the passage of the thermionic current, for it took place when the potential difference was removed from the electrodes during part of the heating, and, as will be seen from the figures already given, the increase continued on one occasion when the cathode was left cold over night with no electric field on. Experiments showed that the current at 1500° C. and 0.005 mm. pressure was approximately saturated with 20 volts, but that complete saturation was never obtained, the current increasing very slightly with each increase of the applied E.M.F. up to 400 volts.

It seems possible that the smallness of the negative emission from the freshly coated platinum may be due to an electrical double layer at its surface which tends to prevent the escape of electrons, so that a large electric field has to be applied to counterbalance this action. Continued heating may have the effect of removing this double layer and hence causing the gradual increase of the thermionic current to a steady maximum value. The existence of such double layers is not easily proved, and further experiments in this direction are necessary.

The Chemical Action Theory of the Electron Emission.

The connection between ionisation and chemical action has been investigated by many observers and very inconsistent results have been obtained. The inconsistency is perhaps most marked in the case of the alkali metals. By recent experiments HABER and JUST* and FREDENHAGEN† claim to have shown that when the alkali metals enter into chemical action there is a liberation of electrons. On the other hand BROGLIE and BRIZARD‡ have concluded that the ionisation which occurs when sodium oxidises is due to the formation on the surface of a host of microscopic bubbles, and that the effect is merely an example of "ionisation by bubbling." A similar result has been announced by REBOUL,§ who also states that the presence of water vapour is essential. A large number of chemical reactions which are not accompanied by ionisation have been investigated by L. BLOCH||; these even include certain cases where the action is accompanied by flame as in the combustion of sulphur or arsenic. But the literature of this subject is so extensive that it is impossible to give an adequate summary of it in the present paper.

* HABER and JUST, 'Ann. der Phys.,' vol. xxx., p. 411, 1909, and vol. xxxvi., p. 308, 1911.

† FREDENHAGEN, 'Verh. d. Deut. Phys. Ges.,' 14, p. 384, 1912.

‡ BROGLIE and BRIZARD, 'Comptes Rendus,' vol. 149, p. 923, 1909.

§ REBOUL, 'Comptes Rendus,' vol. 151, p. 311, 1910.

|| BLOCH, 'Ann. de Chim. et de Phys.,' vols. 22 and 23, 1911.

The arguments in support of the chemical action theory of the origin of the electron emission from glowing solids are based upon two experimental results :—

- (1) The reduction which takes place in the electron emission on purifying the element under test and on improving the vacuum in which the test is conducted.
- (2) The increased activity which is produced when substances known to have chemical action upon the cathode are introduced into the discharge chamber.

These results point to a connection between electron emission and the possibility of chemical action, and it is claimed that the small value of the emission which is still obtained after continued purification must not be regarded as a true thermal effect, for it can be accounted for as being due to remaining traces of gaseous or other impurities. On the other hand, it can be urged with equal force that this remaining effect should not be regarded as due to chemical action until it has been shown to occur under conditions which preclude the possibility of an emission of electrons as a result of an increase in their thermal energy by rise of temperature. Experiments which have been made with these precautions have almost invariably indicated that chemical action is not by itself the cause of ionisation. Evidence to this effect was obtained by the author some time ago when comparing the electron emission from calcium with that from lime. In these experiments a platinum strip was covered with calcium by sublimation in a vacuum; an excess of oxygen was let into the discharge tube and the calcium was oxidised to lime. No detectable ionisation occurred during this process of oxidation in the cold, nor even at a temperature of 500°C . or 600°C ., under which conditions the oxidation must have been very rapid. It was only when the lime formed had been raised to 700°C . or 800°C . that a measurable electron emission was obtained. In the case of the strongly electro-positive alkali metals it is not impossible that an electron emission (as a result of thermal energy) occurs at the ordinary temperature of the laboratory, and if an increased emission does occur in the presence of a reacting gas, this may be due, not directly to the chemical action, but to the rise of temperature which accompanies it. This momentary increase of temperature may be very considerable in the case of the surface layer, although the rise in temperature of the bulk of the metal may be quite inappreciable. The fact that chemical action and rise of temperature are usually closely associated in the type of reaction with which we are concerned makes it very difficult to prove that an observed thermionic emission is a result of either chemical action or temperature alone.

A process which causes the liberation of electrons from matter must act either by increasing the kinetic energy of the electrons or by diminishing the work which an electron must do in order to escape. An increase of temperature acts in the former manner, but a disturbance of the motions of some of the electrons would also

appear to be an inevitable result of an atom taking part in a chemical reaction, and if this does occur, it would be not improbable that such disturbance should lead, in some cases, to an electron emission. Experimental evidence shows that when atoms enter into chemical reactions at ordinary temperatures, it is rarely (if ever) that the accompanying disturbance increases the kinetic energy of any of the electrons sufficiently to enable them to escape; but if the conditions are arranged so that the electrons already possess considerable kinetic energy in virtue of their thermal agitation, it would seem possible that the stimulus of chemical action should have the effect of increasing this energy sufficiently to cause some of the electrons to overcome the forces tending to retain them in the substance. It seems, therefore, that chemical action at a high temperature might be expected to result in an electron emission in cases where the temperature itself is not sufficient to produce this effect, or where it produces it only to a much smaller extent. This conclusion is supported by the fact that an increased emission occurs when certain chemically active gases are allowed to enter the discharge tube, but it must be remembered that there is another manner in which these gases may act in aiding the escape of electrons from the glowing cathode, namely, by causing an alteration in the surface conditions whereby the minimum amount of energy an electron must possess in order to escape from the cathode is reduced. Such a reduction might be caused by the formation of an electric double layer at the surface of the cathode which would create a force tending to drag out the electrons. There is already a considerable amount of evidence of the existence of these double layers in certain cases, and this view has been put forward to explain the action of hydrogen in increasing the electron emission from platinum.*

The author is of opinion that at present there is not sufficient experimental evidence to enable us to explain the manner in which the presence of a gas in the discharge tube acts in increasing the negative thermionic emission, but it is not improbable that the method by which this increase is brought about is different in different cases; for instance, when oxygen comes in contact with a calcium cathode at a high temperature, the increased emission is doubtless due to the fact that the emission from the product of the chemical action—lime—is greater than the emission from calcium at the same temperature. The possibilities of ionisation by collisions need also to be carefully investigated. Recent experiments have shown that the electric intensity required to produce this effect is very much smaller than has been supposed, and the process is probably always going on near a glowing cathode at low pressures. It must be remembered also that the gas pressure in the neighbourhood of a cathode which is continually evolving gas may be considerably greater than that measured in more remote parts of the apparatus, so that it is difficult to estimate the magnitude of the effect of ionisation by collisions.

* 'Conduction of Electricity through Gases,' Camb. Univ. Press, 1906, p. 203.

Summary and Conclusion.

The experiments with Nernst filaments have shown that the oxides of which these are made suffer electrolysis when conducting an electric current, but that the electron emission which occurs at high temperatures is not due either directly or indirectly to this electrolysis, for the magnitude of the emission at a given temperature is the same whether the current through the material is great or small. It is therefore concluded that the emission from lime in a Wehnelt cathode is not due to the re-combination of electrolytically separated calcium and oxygen, as has been suggested.

Owing to the increasing chemical activity of substances at high temperatures, and to the impossibility of removing all traces of impurities from the discharge tube, it is very difficult to prove conclusively that any known case of electron emissions is purely a thermal effect; but, on the other hand, there is no satisfactory evidence that ionisation is ever produced by chemical action alone. It would seem, therefore, that the theory that thermal agitation is by itself sufficient to cause the emission of electrons from matter could only be disproved by reducing the thermionic current from a metal at the highest attainable temperature to zero.

The increased ionisation from a glowing cathode which occurs when a chemically active gas is allowed to enter the discharge tube may arise in several ways:—

- (1) It may be directly due to chemical action;
- (2) It may be indirectly due to chemical action, arising as a consequence of an increase of temperature resulting therefrom, or from the product of the action having a greater activity than the original substance of the cathode;
- (3) It may be due to an alteration of the surface conditions at the cathode in such manner as to make it easier for the electrons to escape, for instance, by the formation of an electric double layer;
- (4) It may be due to ionisation by collisions.

It seems probable that the cause of the increased activity is different in different cases.

The author wishes to acknowledge his indebtedness to the Government Grant Committee of the Royal Society for the means of purchasing some of the apparatus used in these experiments, and also to Prof. Sir J. J. THOMSON for his advice and interest in the research, which was carried out in the Cavendish Laboratory.



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ON THE LUNAR DIURNAL VARIATION OF THE EARTH'S
MAGNETISM AT PAVLOVSK AND POLA (1897-1903).

BY

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*By S. CHAPMAN, B.A., D.Sc., Fellow of Trinity College, Cambridge, and
Chief Assistant at the Royal Observatory, Greenwich.*

Communicated by the Astronomer Royal, F.R.S.

Received June 19,—Read June 25, 1914.

§1. THE present paper contains an account of the methods and results of an investigation into the lunar diurnal magnetic variations at the observatories of Pavlovsk and Pola (1897-1903). This work is part of a larger undertaking, now in progress, by which I hope to obtain the data necessary for the discussion of the lunar diurnal magnetic variations over the whole earth, using the method of the Gaussian potential in a manner similar to that in which SCHUSTER applied it to the solar diurnal magnetic variations. For this purpose data will be computed from at least three or four other observatories. The labour of computation involved is very large, and in the present instance has been executed partly by the aid of a grant from the Government Grant Committee of the Royal Society, and partly by assistance put at my disposal by the Astronomer Royal and Dr. SCHUSTER. While the data thus obtained will, I trust, throw much new light on the theory both of the lunar and solar diurnal magnetic variations, the investigation has been undertaken not with any thought of finality in itself, but rather as an incentive to the provision of data for a more accurate and detailed discussion by subsequent workers. The reduction of the observations to the required form is, in fact, almost too great a labour for a single person to undertake if it is to be done on an adequate scale and in a reasonable time. It is in the hope, therefore, that a number of directors of magnetic observatories may be induced to carry out the reductions for their own stations that I have, in this paper, indicated what seems to me to be the simplest and most suitable method of computation for the purpose, and also some of the lines along which the discussion of the results from a single observatory should proceed. In a later paper some further points will be dealt with, in addition to the final discussion of the collected results from several observatories; for a general review of the subject, reference may be made to my recent discussion of the previously existing data for Batavia, Bombay,

and Trevandrum.* A few of its main points are summarized below for the better understanding of the object of the present paper.

§ 2. The magnetic elements undergo regular variations with the period of a lunar day, just as they do in the course of a solar day; the latter variations are much the larger of the two. The lunar diurnal changes are the simpler in character, however, taking the form, when averaged over a whole lunation, of a purely semi-diurnal wave. But if the variation is computed from a number of days all at the same lunar phase (*i.e.*, when there is a definite angular distance between the sun and moon), the solar diurnal variation having first been eliminated from the observations, harmonic components of other frequencies in the lunar day are also found to be present. The epoch of the component of frequency 2 (the semi-diurnal term) remains constant throughout the lunation, but the epochs of the terms of frequency 1, (2), 3, 4, ..., n change regularly during the lunation by amounts -2π , (0), 2π , 4π , ..., $2(n-2)\pi$ respectively. Moreover, the epochs of the various components generally seem to be nearly or quite the same at the time of new moon, when the sun and moon are on the same meridian. This law of phase change was discovered and verified by a study of the first four harmonic components; it explains the disappearance of all components of frequency other than 2 from the variation computed from a whole lunation. The phenomenon clearly suggests a solar action modifying a regular semi-diurnal lunar variation. In the paper referred to, I have shown that the law according to which the epochs of the several harmonic components change during a lunation is in accordance with the theoretical consequences of such a solar action. While the proof of this theorem might easily be stated in terms which imply no special hypothesis as to the manner in which the action takes place, it was actually given in terms of a particular theory, *viz.*, that the effect is due to a variation, periodic in a solar day, of the electrical conductivity of the medium in which flow the currents to whose magnetic potential is attributed the variable field put in evidence by the observations. The dependence of the electrical conductivity upon solar time is accounted for if we suppose that all or part of this conductivity is due to some ionizing influence from the sun. So far as regards the facts thus mentioned, this assumption fits in naturally with a view that the lunar magnetic variations arise from the lunar tide in the earth's atmosphere† in the same way as, according to SCHUSTER's well-known theory, the solar diurnal magnetic changes arise from the ordinary daily atmospheric motions‡ which are indicated by the barometer.

In two important memoirs SCHUSTER has proved that the solar-diurnal magnetic

* 'Phil. Trans.,' A, vol. 213, pp. 279-321. This paper contains fairly full references to the previous work on the subject.

† This has been detected and measured in the barometric records of some Tropical observatories, *i.e.*, St. Helena and Batavia. It is, as was to be expected, purely semi-diurnal. SABINE, 'Phil. Trans., 1847; 'Batavian Observations,' 28, 1905.

‡ Mainly of solar-thermal origin.

variations originate in currents flowing mainly above the earth's surface,* and has shown that these may very probably be ascribed to the effect of electromotive forces produced by the periodic motions of the atmosphere across the earth's permanent magnetic field, the electrical conductivity necessary for the passage of the currents being found in the rarified upper atmosphere, which is assumed to be ionized by some solar action such as ultraviolet radiation.† This theory is so valuable and successful in many ways that it is desirable to examine carefully the points not yet elucidated by it, among which the chief are the ratios of the amplitudes of the several harmonic components in the magnetic variation (as compared with the ratios of the corresponding terms in the barometric variation), and the seasonal changes in these components. If the natural assumption that the mechanism of the lunar and solar diurnal magnetic variations is similar is correct, an investigation of the former should throw much light on the latter; since the lunar diurnal variation arises from an atmospheric oscillation of a single frequency there is good reason to suppose that the relative amplitudes of the various components of the lunar magnetic variation (computed for the separate lunar phases and reduced to the time of new moon, say) will enable the daily variation of the atmospheric electrical conductivity to be determined.

The chief point of novelty and importance in the investigation for which the data here presented will be employed lies in the use of the harmonic components of the lunar magnetic variation of frequencies other than 2, as well as the main semi-diurnal component. In particular, as is explained in the memoir already referred to,‡ the fourth harmonic term has a special significance for our purpose.

The Method of Computation.

§ 3. The following is the method of computing the lunar diurnal magnetic variation from the published hourly values of the magnetic elements which, after much consideration, has appeared to me to be the simplest and best. It resembles that used by J. A. BROWN in his reduction of the Trevandrum observations of declination,§ more than any other method hitherto used.

The monthly mean solar diurnal variation, as ordinarily calculated from a calendar month's hourly observations, may be considered free from any lunar term, since in this period|| the lunar disturbance characteristic of any lunar hour affects in turn each

* SCHUSTER, 'Phil. Trans.,' A, vol. 180, p. 467.

† SCHUSTER, 'Phil. Trans.,' A, vol. 208, p. 163.

‡ 'Phil. Trans.,' A, vol. 213, p. 306.

§ 'Trevandrum Magnetical Observations,' vol. I., p. 113, 1874.

|| More strictly, in an exact lunation, but the two are so nearly equal that in view of the relative smallness of the lunar variation the difference may be neglected.

group of 28 to 31 equal solar hours in like degree. If, then, we subtract from each hourly value the monthly mean value for that hour, the remainders will be due to causes not regularly solar, *i.e.*, to irregular disturbances from whatever cause, and to any lunar action which may be at work. These remainders, when re-grouped, in sufficient quantity, according to their lunar time, enable the magnetic variations depending on lunar time to be determined either for whole lunations, or for any particular lunar phase, since the regular solar variations are already eliminated, and the accidental irregularities, independent of lunar time, average out.*

In eliminating the solar variation it is too troublesome to subtract from the hourly values on each day the monthly mean values for the twenty-nine days exactly centred at that day, yet, on the other hand, it is perhaps hardly accurate enough to assume that the monthly mean values for the ordinary calendar month properly represent the solar variation for all the days in that month—especially during seasons of rapid change in the solar variation. The simple plan which I have adopted is to take the days of a calendar month in six groups each containing five days (except that the last group contains the days from the 26th to the end of the month), and to take the same monthly mean hourly values for all the days in each group; these hourly values are interpolated according to simple rules (embodied in a table for the computer's convenience) from the monthly mean hourly values of the given calendar month and of those preceding and following it, each set being assumed to apply to the centre of its respective month. The differences "actual—mean" are written down on the sheets containing the printed hourly values, alongside the latter; it is convenient, where possible, to place positive differences to the left, and negative differences to the right of the printed values. These differences need not be written down more accurately than to one-tenth of a minute of arc in the case of declination, or to 1γ (0.00001 C.G.S. unit) in the case of force.

The re-grouping of these differences according to their lunar time is effected by copying them on to "lunar" sheets ruled into 27 columns and 40 or 45 rows, on which have been written out, in the second column, the local civil times of upper transit of the moon (lunar hour 0) for the successive days in the calendar month to which each sheet is devoted. The times of lunar hour 0 will not generally fall at an exact solar hour, and the nearest solar hour to the correct time must be used instead; the resulting error will generally be small, and is assumed to disappear in the mean from a large number of days. These times, for the meridian of Greenwich, can be taken from the Nautical Almanac or, perhaps, more conveniently, from the "*Companion to The Observatory*" (12^h must be added to obtain *civil* instead of astronomical time). They may be corrected for difference of longitude for an observatory on another meridian, but I have found it convenient not to do so, using the same civil times of

* It may be noted here that in the computation of the lunar magnetic variation many small errors of solar diurnal periodicity which occasionally occur in magnetic observations, such as those arising from a faulty temperature correction, are eliminated from the observations with the regular solar variation.

lunar hour 0 for all observatories; the necessary correction for this is indicated later.*

The length of a lunar day is not quite constant, but its mean value is approximately $24^{\text{h}} 50^{\text{m}}$ in solar time; for the present purpose it will be assumed to equal 25^{h} of solar time (which seems much preferable to the adoption of 24^{h} , as has often been done†). As the hourly differences to be copied out are given at intervals of one solar hour, the lunar day will, for the first stage of the work, be regarded as subdivided into 25 equal periods of duration of one solar hour, and the last 25 columns of the "lunar" sheets should be headed 0^{h} to 24^{h} . In copying out the hourly differences, therefore, the first value written down on any row (each of which relates to a single lunar day) is that one which corresponds to the local civil time written in the second column of the row; after this the next 24 consecutive hourly differences are written in order. The times of lunar hour 0 in the second column generally differ by successive amounts of 25^{h} , and where this is so the hourly differences run on consecutively from one row to the next; occasionally the difference is only 24^{h} , however, and in this case one hourly value has to be set down twice, once under column 24^{h} of the former row, and again under 0^{h} of the following row. To ensure that the right set of hourly differences is being copied out, the time of the last entry in each row of the lunar sheets should be checked. In this re-arrangement it is important, for convenience and accuracy, that the columns of the lunar sheets should be wide enough to allow of the preservation of the separation of the positive and negative values of the hourly differences, the former on the left and the latter on the right hand of the columns; this facilitates the formation of monthly and group sums, and obviates the use of signs. Each lunar sheet should contain the lunar days of a single calendar month, occupying about 30 rows; a few hours at the beginning or end of the month may be taken from the preceding or following calendar month in order to make each lunar day complete.

§ 4. When all the hourly differences for the period under investigation have been copied out on to the lunar sheets these should be carefully scanned for the purpose of picking out any very disturbed days for rejection. This assists also in the detection of any mistakes which have been made in the previous part of the work, as serious mistakes in forming the hourly differences in copying is generally apparent from the "run" of the numbers. As regards the rejection of disturbed days, I have adopted a very conservative policy; the proceeding must, in any case, be of an arbitrary nature, and a criterion which will mark off a given proportion of days must vary in different years, at different observatories, and with different elements (even at the same time and place). The period covered by this investigation (1897-1903) was intentionally chosen during a sunspot minimum, when quiet magnetic conditions prevailed, so that in a more disturbed period it would be necessary to reject a rather larger proportion

* P. 302. It is assumed that the hourly values are tabulated, as is usual, according to the local civil time of the observatory.

† As, for instance, at Batavia, Bombay, and in all SABINE'S work on the lunar magnetic variation.

of days. The average total number of days rejected for each element at each observatory, out of nearly 2500 lunar days (7 years) was 50 or 60*, or about 2 per cent. of the whole. The rejected days were not always the same for different elements and observatories. At first a still smaller number, hardly more than a dozen days were deleted; the further rejection of another 40 days improved the accordance of the results from the several lunar phases, and was therefore considered to be worth while, although it hardly altered the final means from all the phases.

The number of days thus rejected is very much smaller than was customary amongst those previous investigators who adopted SABINE's method of "separating values"; CHAMBERS,† for example, in treating 25 years' Bombay observations, rejected about 20 per cent. of all the days, instead of 2 per cent. as here. FIGEE,‡ at Batavia, has stated that this procedure may seriously affect the values derived for the lunar magnetic variation, and it seems desirable, therefore, for definiteness and uniformity, to restrict the number of rejected days within very close limits.

The disturbed days, chosen according to the principles thus laid down, were ruled out on the lunar sheets, as also any lunar days with incomplete record. The monthly sums were then formed at the foot of the several columns of hourly differences—the corresponding number of days used being written in the second column of the same row. Further, by reference to the Nautical Almanac, each month was divided up into eight groups of days (there being from three to five days in each group), these being distributed as evenly as possible around the times§ of new moon, one-eighth phase, first quarter, and so on, as centres. In eight rows, below the monthly sums, should be written down the sums of the hourly differences for each of these groups of days in order (new moon being phase 1, first-eighth phase 2, and so on), the number of days in each group being set down in the second column. The sum of the eight group sums in any column should equal the monthly sum in that column, and the additions should be checked by verifying that this is the case.

§ 5. At this stage the utmost freedom is possible in the choice of further grouping of the results for discussion. In general, several months must be grouped together in order to obtain an accurate determination of the variation under any given conditions, as its total amount is always very small. The groups of days taken together should therefore be as large as is consistent with the exhibition of those features of the variation which it is desired to bring out. For the present purpose the eight lunar

* As an indication of the sort of criterion which serves to separate about 60 days for rejection, in the three elements declination, horizontal force, and vertical force at Pola, the separating values were respectively $10' \cdot 0$ of arc, 100γ , and 50γ in the total range of the hourly differences on any day. These limits might be different at other places and times.

† CHAMBERS, 'Phil. Trans.,' A, vol. 178, pp. 1–48 (1887).

‡ FIGEE, 'Batavian Observations,' vol. 26 (1903).

§ If the Greenwich civil times of lunar hour 0 are written in the second column of the lunar sheets, instead of the local civil times, the Greenwich civil times of the lunar phases should also be used in dividing up the months. This plan is recommended.

phases must, of course, be kept separate, and to illustrate the seasonal changes I have divided the twelve months of the year into three groups, summer (May to August), the equinoxes (March, April, September, October) and winter (November to February)—a plan which seemed better, on the whole, than the adoption of four groups of three months each.

During the seven years there were 28 group sums for each lunar phase during each season, and these were added together, the final sum representing about 100 days in each case (approximately $7 \text{ (years)} \times 4 \text{ (months)} \times 4 \text{ (days)}$). The monthly sums were also added together for the seven years, so as to show the monthly change in the semi-diurnal component of the variation; by grouping these monthly sums in seasons a check on the sum of the eight seasonal sums for the separate phases was obtained for verification of the computing.

The twenty-four sets of inequality sums for the eight lunar phases during three seasons were then plotted, and 24 ordinates at equal intervals were read off from each of these curves (without smoothing), in order to replace the 25 values at intervals of one solar hour by 24 values, starting at lunar hour 0, at intervals of a lunar hour, or $\frac{2.5}{4}$ solar hours; this was done because the Fourier analysis of 24 values of a periodic function is arithmetically much simpler than that of 25. This analysis, as far as the first four harmonic components, was then effected. Only at this stage were means, and not sums, used, the resulting values of a_1 , b_1 , &c., being divided by the number of days corresponding to the sums whose succession of values was harmonically analysed. This plan, besides being very economical of time, permits of more convenient checks being applied to the routine computations (as already mentioned) than would be possible if means were used in an earlier stage of the work.

The eight values of a , b in the formula

$$\sum_{n=1}^4 (a_n \cos nt + b_n \sin nt)$$

deduced from the magnetic variation for each lunar phase were then all reduced to the corresponding values at new moon, by allowing for the progressive change of epoch of amount $2(n-2)\pi$ which the n^{th} harmonic undergoes during each lunation.* The change of epoch in the interval (one-eighth of a lunation) between one phase and the next is $\frac{1}{4}(n-2)\pi$, or -45° , 0 , $+45^\circ$, $+90^\circ$ for the first four harmonics in order. We will denote by a' , b' the new values of a , b after this change of epoch has been allowed for by reducing a , b , to their equivalent values at new moon. Writing k for $\frac{1}{\sqrt{2}}$, for convenience, the table on p. 302 is useful for the conversion of a , b to a' , b' .

Except for accidental error, and a small effect due to the varying distance and declination of the moon, the values of a' or b' in any one column should be the same. Their mean gives the resultant determination of the various components of the lunar

* 'Phil. Trans.,' A, vol. 213, p. 287.

Lunar phase.	a'_1 .	b'_1 .	a'_2 .	b'_2 .	a'_3 .	b'_3 .	a'_4 .	b'_4 .
1	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4
2	$k(a_1 + b_1)$	$k(-a_1 + b_1)$	a_2	b_2	$k(a_3 - b_3)$	$k(a_3 + b_3)$	$-b_4$	a_4
3	b_1	$-a_1$	a_2	b_2	$-b_3$	a_3	$-a_4$	$-b_4$
4	$k(-a_1 + b_1)$	$k(-a_1 - b_1)$	a_2	b_2	$k(-a_3 - b_3)$	$k(a_3 - b_3)$	b_4	$-a_4$
5	$-a_1$	$-b_1$	a_2	b_2	$-a_3$	$-b_3$	a_4	b_4
6	$k(-a_1 - b_1)$	$k(a_1 - b_1)$	a_2	b_2	$k(-a_3 + b_3)$	$k(-a_3 - b_3)$	$-b_4$	a_4
7	$-b_1$	a_1	a_2	b_2	b_3	$-a_3$	$-a_4$	$-b_4$
8	$k(a_1 - b_1)$	$k(a_1 + b_1)$	a_2	b_2	$k(a_3 + b_3)$	$k(-a_3 + b_3)$	b_4	$-a_4$

variation for the particular season, over the period for which data have been used, except for small corrections to be applied (i) to the amplitudes of the components, on account of the fact that the epochs are changing continuously, while we have treated the group of days corresponding to each lunar phase as though each of the days of the group was centred precisely at that phase (this makes the computed amplitude smaller than it should be); and (ii) to the epochs also, in case the civil times adopted for lunar hour 0 and the phases of new moon, &c., have been the times appropriate to the meridian of Greenwich instead of to the meridian of the observatory dealt with.

The former correction, to the amplitudes, is effected by multiplying the computed amplitudes C_1 , C_3 , C_4 , or the corresponding a , b coefficients, by 1.02, 1.02, and 1.11 respectively.*

§ 6. The correction to the epochs is found as follows: we consider a place in West longitude L° , for which the civil times of lunar hour 0 and of new moon first-eighth phases, &c., appropriate to Greenwich have been used instead of the *local* times. The error in the local civil time of lunar hour 0 is equal to the gain in longitude of the moon upon the sun during the time taken for the moon to traverse the angle L ; this time is, with accuracy sufficient for our purpose, equal to $L^\circ/29$ in lunar time. On this account an angle $nL^\circ/29$ must be subtracted from the deduced epoch θ_n . Next as regards the hour of new moon, &c., since these phenomena have no relation to any special meridian, the adopted times are in error by the full amount L° in solar time. This results in an error of amount $(n-2)L^\circ/29$ in the reduction of the phases to the true epoch of new moon. Combining the two corrections it appears that the epochs of all the components must be corrected by the same amount, viz., $-2L^\circ/29$. This correction is so simple to apply, and the adoption of one set of times for all observatories is so convenient, that the plan can be recommended, certainly for all observatories for which no direct ephemeris is published.

Having applied these corrections, it is desirable that the results from all observatories should be given in the following form: the elements should be transformed into the geographical components of force, to North, West, and the nadir, the unit being conveniently taken as 10^{-7} C.G.S.; the time zero should be the local

* Cf. 'Phil. Trans.,' A, vol. 213, p. 309, for the deduction of these factors.

time of upper culmination of the moon at the epoch of new moon, and the a , b and C , θ coefficients should both be given.

The Lunar Diurnal Magnetic Variations at Pavlovsk and Pola.

§ 7. In undertaking the present investigation it was necessary to limit, as far as possible, the quantity of observations dealt with, and this made it desirable to choose a period of years which were magnetically "quiet," in order to minimise the irregular deviations of the magnetic needle which have to be averaged out in determining the lunar diurnal variations. The period adopted was that from 1897 to 1903 inclusive, comprising seven years during the sunspot minimum immediately preceding the present one.

The choice of observatories to be first dealt with fell on Pavlovsk and Pola; both of these publish the hourly values of the magnetic elements, and their latitudes rendered them suitable for the main purpose of this work. The prior examination of these rather than of other observations was due to the greater novelty of results relating to the lunar magnetic variation in fairly high latitudes; the phenomenon has been studied in detail only at tropical observatories hitherto.

The following particulars concerning Pavlovsk and Pola are important for our purposes:—

	Pavlovsk.	Pola.
Latitude	59° 41' 13" N.	44° 51' 49" N.
Longitude	30 29 15 E.	13 50 46 E.
Declination (1900) . . .	0° 37' E.	9° 25' W.
Horizontal Force (1900) .	16550 γ .	22210 γ .

The correction to the epochs of the harmonic components of the variations, described in § 6, is therefore $+2^{\circ}1$ in the case of Pavlovsk, and $+1^{\circ}0$ in the case of Pola.

§ 8. The results from these two observatories are exhibited in Tables I. to VI. Tables IA. and IB. give a summary of the semi-diurnal components of the lunar diurnal magnetic variation for the three elements and three seasons (as well as for the mean of the whole year), in the form of hourly inequalities. These are obtained from the mean of a number of whole lunations, so that any component of period other than half a lunar day is to be regarded as merely accidental. As may be readily seen, the results exhibit very clearly in most cases the true semi-diurnal character of the constant term in the lunar variation.

The inequalities are given for twenty-five times, separated by intervals of one solar hour, there being twenty-five solar hours in a lunar day. The small correction to the epoch, mentioned in § 6, has not been applied, as this table is given only to illustrate the main feature of the lunar diurnal variation, *i.e.*, its semi-diurnal character. It will be noticed that the lunar variation is inverted during the winter months in the case of declination, and partially so in the case of horizontal force.

§ 9. The next six tables, II. to IV. (A and B) are more important, being derived by the separation of the lunar phases, enabling us, as already explained, to determine the other harmonic components, of changing epoch, in the lunar variation. The values of the coefficients a_n and b_n in the formula $\Sigma (a_n \cos nt + b_n \sin nt)$, for the first four components, are given after reduction to the same lunar phase (new moon). No corrections to amplitude or epoch (§§ 5, 6) have been made at this stage, nor are the results transformed into the geographical components of force, as the purpose of these tables is to show how far the results from the different lunar phases are in agreement. Apart from accidental error any eight corresponding values of a or b should be equal (except for the small effect of the varying distance and declination of the sun and moon). Judged by ordinary standards the agreement of the numbers in any one column is in general not at all good, even in the case of the larger amplitudes; but for work of this kind, where the effect sought after is so small in comparison with the superposed regular and irregular variations which have to be eliminated in its determination, the results may be regarded as satisfactory considering (i) the extreme smallness of the unit, and (ii) the short period (on the average equal to 100 days) from which each of the values in Tables II. to V. is determined. Occasionally there are largely outstanding values, and it would seem necessary to use a much larger series of observations if close accordance of the values for the separate lunar phases were desired. I hope, however, that the mean coefficients deduced from the combination of the eight phases are reasonably free from accidental error—the parallelism of the results from the two observatories, which will be pointed out later, confirms this to some extent.

§ 10. In order, however, to examine this point more closely, I have derived the values of the probable errors of the mean values of a_n and b_n in each case from the discordances from the means in Tables II. to IV.* The results are given in Table V., those for declination (Tables IIIA. and IIIB.) having for convenience of comparison been converted into force units. The essential features of the table may be summarized thus: the sum of all the probable errors for all the elements in summer, equinox, and winter are 293,276 and 333 respectively, so that the season has no great effect upon the magnitude of the probable error; but as the variations are generally greatest in summer, the values of a, b for this season are in general proportionately the most accurate. Summing the errors for all seasons and components, but keeping the results for each element and observatory separately, we find the following:—

	Horizontal force.	Transverse force.	Vertical force.
Pola	171	150	108
Pavlovsk	163	191	119

* The probable error of the mean of n observations is taken to equal $0.845 / \sqrt{n-1}$ times the mean discordance from the mean.

Hence the probable errors of the results for vertical force are distinctly less than those for the other two elements, but here again, an account of the much smaller magnitude of the variations in vertical force, these are *proportionately* the least accurately determined.* These differences between the probable errors are, however, small compared with those between the errors relating to the different harmonic components. The means of the successive columns of Table V. (after the application of the small amplitude corrections of § 5) are

a_1 .	b_1 .	a_2 .	b_2 .	a_3 .	b_3 .	a_4 .	b_4 .
11·7	9·9	6·7	6·6	4·4	4·3	4·2	3·4

Hence the probable errors of either resultant of the harmonic components of frequency 1, 2, 3, 4, are respectively 10·8, 6·6, 4·3 and 3·8, in units of 10^{-7} C.G.S. These results apply directly to Table VIB.

§ 11. In Tables VIA. and VIB. are given the results of this paper in their complete form, the necessary corrections to amplitude and epoch having been applied to the mean values of a_n and b_n in Tables II. to IV., after which these were converted into the geographical components of force, with a common unit 10^{-7} C.G.S. The C, θ and a , b results are given separately in Tables VIA. and VIB. The latter is the more useful for the ultimate purpose of this work, viz., for the investigation of the potential of the lunar diurnal variations; the C, θ table, however, gives a better idea of the relations between the various components at a single observatory, and between corresponding components at the two observatories.

* The following comparison with the values deduced for the a_2 , b_2 coefficients of the lunar diurnal variations at Pavlovsk and Pola by VAN BEMMELEN ('Meteorologische Zeitschrift,' May, 1912) may also be of interest. VAN BEMMELEN divided his data into the summer and winter half-years, and therefore the only comparison that can be made is between the mean variations throughout the whole year. Moreover, as he did not separate the different lunar phases, but took the mean of a number of whole lunations, the values of a_1 , b_1 which he gives represent accidental error and are no true lunar effect (as he fully recognized); the real diurnal lunar component simply disappears from the result when whole lunations are dealt with. Further, the method of computation used by him was different, as also the period of observations dealt with (1894-1903 in the case of Pavlovsk, and 1899-1906 in the case of Pola). While exact agreement is not to be looked for, therefore, the following comparison has some interest in the present connection:—

Element.	Pavlovsk.				Pola.			
	a_2 .		b_2 .		a_2 .		b_2 .	
	VAN BEMMELEN.	CHAPMAN.	VAN BEMMELEN.	CHAPMAN.	VAN BEMMELEN.	CHAPMAN.	VAN BEMMELEN.	CHAPMAN.
North force . . .	11	0	15	48	52	42	59	56
West force . . .	53	61	- 2	7	74	61	9	30
Vertical force . . .	10	0	12	0	-20	-27	37	27

First considering the phase angles, a general tendency to equality is evident between the values of θ for the four components of a single element and season; this result is obscured to some extent by irregularities, which are probably mainly accidental; it is what would be expected on the grounds of general theory (*cf.* § 23 of my former paper already cited). A comparison of corresponding phase angles at the two observatories reveals considerable similarity, though by no means exact agreement between the two sets of values. Both exhibit the more or less partial inversion, during the winter, of the variations in the horizontal force components, which does not appear in the case of vertical force (the vertical force variations for Pavlovsk are so small in amplitude that but little reliance can be placed upon the phase angles).

As regards the amplitudes, the second harmonic is generally the greatest, though the first harmonic (which is usually the next greatest) sometimes equals or even exceeds it. The fourth harmonic is generally the smallest of all, often being quite inappreciable. The first three harmonics are as a rule much greater in summer than in the other seasons, the winter values usually being the smallest. A remarkable feature, however, brought out by Table VIA. is that the fourth harmonic is always greatest at the equinoxes, both absolutely and (*à fortiori*) in proportion to the other components; this is shown even in the case of Pavlovsk vertical force. The equinoctial amplitude is frequently one-third that of the second harmonic, and sometimes greater. As I have previously shown (§ 25, p. 306, *loc. cit.*), the fact that the fourth harmonic is appreciable at all indicates that the diurnal variation in the atmospheric electrical conductivity cannot be represented by a simple diurnal wave $a + b \cos t$, but that terms of higher frequency in the solar day must be present. The occurrence of these large values of C_4 at the equinoxes should therefore have an interesting physical bearing; as yet, however, I have not entered into these theoretical questions, pending the acquisition of data from further observatories, and also the completion of an extension, by improved methods, of the mathematical theory of my former paper, both of which are now in hand.

In conclusion it is a pleasure to acknowledge the assistance which has been placed at my disposal in carrying out the computations of this paper by the Government Grant Committee of the Royal Society, by the Astronomer Royal, and by Dr. SCHUSTER. The calculations have been performed mainly by Miss A. M. PALMER, and by Messrs. D. J. R. EDNEY and H. FURNER; their careful and accurate work has materially lightened my task of supervision.

TABLE IA.—Lunar Diurnal Magnetic Variations at Pola Determined from the Mean of a Number of Whole Lunations.*

Twenty-five hourly values are given, separated by intervals of one solar hour, there being twenty-five solar hours in a lunar day.

The unit is 10^{-7} C.G.S. for force, and 0'001 for declination; + denotes force increasing to North, West, and the Nadir.

Lunar hour.	Horizontal force.			Declination.			Vertical force.		
	Summer.	Equinox.	Winter.	Year.	Summer.	Equinox.	Winter.	Year.	
0	+ 97	+ 21	+ 28	+ 49	+ 278	+ 67	+ 24	+ 41	
1	109	63	10	54	221	101	22	24	
2	123	52	31	48	169	54	18	10	
3	74	80	46	36	58	17	28	36	
4	58	68	41	28	69	0	11	38	
5	20	32	41	10	179	32	6	40	
6	89	15	30	45	257	66	4	24	
7	154	43	23	73	201	96	13	14	
8	188	78	9	92	165	75	3	14	
9	183	55	32	90	75	59	9	5	
10	72	27	0	33	28	65	11	10	
11	22	25	13	5	171	75	13	32	
12	66	60	25	50	237	78	3	24	
13	130	86	15	77	212	38	7	18	
14	156	85	22	88	152	75	8	15	
15	169	105	24	99	46	57	17	8	
16	122	44	15	60	65	21	27	17	
17	63	10	19	31	135	32	38	34	
18	6	16	8	6	190	42	33	39	
19	62	58	6	42	235	44	21	35	
20	112	121	9	81	198	88	9	28	
21	110	129	27	71	142	60	26	6	
22	110	106	37	60	25	46	31	5	
23	70	61	34	32	126	35	38	15	
24	21	15	37	14	188	5	32	40	

* See § 8.

TABLE IB.—Lunar Diurnal Magnetic Variations at Pavlovsk Determined from the Mean of a Number of Whole Lunations.*

Twenty-five hourly values are given, separated by intervals of one solar hour, there being twenty-five solar hours in a lunar day.

The unit is 10^{-7} C.G.S. for force, and 0'001 for declination; + denotes force increasing to North, West, and the Nadir.

Lunar hour.	Horizontal force.				Declination.				Vertical force.			
	Summer.	Equinox.	Winter.	Year.	Summer.	Equinox.	Winter.	Year.	Summer.	Equinox.	Winter.	Year.
0	+ 28	+ 30	+ 21	+ 26	+ 320	+ 117	+ 29	+ 136	+ 6	+ 4	+ 3	+ 4
1	80	22	21	41	344	56	37	121	9	27	3	7
2	112	29	3	46	188	27	36	42	10	18	1	2
3	134	34	25	48	66	108	33	3	9	9	11	4
4	35	31	35	10	122	120	56	62	12	24	21	19
5	34	32	24	14	204	10	78	39	18	0	11	10
6	9	17	1	2	282	57	52	96	6	14	13	2
7	49	0	11	13	273	19	42	84	6	27	3	6
8	100	16	13	34	106	76	38	73	12	46	8	14
9	78	42	31	30	17	6	22	15	32	33	3	1
10	69	30	39	46	93	71	47	39	23	27	15	4
11	49	10	27	29	214	103	67	83	3	25	16	2
12	11	25	36	7	214	119	67	89	22	1	2	7
13	59	61	24	32	232	103	87	83	20	8	5	5
14	79	48	33	31	144	92	0	79	26	13	5	6
15	93	34	16	37	70	43	86	66	21	9	24	12
16	120	52	14	62	162	93	145	25	22	14	25	11
17	79	38	38	52	227	48	60	72	16	14	22	8
18	15	14	38	13	262	78	56	95	3	14	23	9
19	16	39	38	20	234	93	7	107	2	1	12	3
20	77	59	43	31	285	32	77	131	3	6	4	4
21	94	66	1	54	163	89	63	63	2	10	3	4
22	96	63	16	48	16	35	0	6	5	6	1	3
23	80	67	31	39	159	40	72	16	2	4	2	4
24	119	57	18	65	290	31	109	71	29	2	11	13

* See § 8.

TABLE IIA.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations,
Reduced to the Epoch of New Moon.

Pola. Horizontal Force.

The unit of force is 10^{-7} C.G.S.; the small corrections mentioned in §§ 5, 6 have not been applied at this stage, *cf.* § 9; + denotes increasing horizontal force.

Season.	Lunar phase.	a'_1 .	b'_1 .	a_2 .	b_2 .	a'_3 .	b'_3 .	a'_4 .	b'_4 .
Summer . . .	1	14	91	85	132	78	69	22	11
	2	— 3	150	62	103	86	67	29	36
	3	119	31	44	119	35	64	19	13
	4	— 3	109	59	139	83	71	3	4
	5	56	74	167	95	87	41	54	12
	6	8	178	107	193	95	80	— 11	20
	7	109	10	108	118	65	61	4	— 9
	8	17	110	43	83	24	43	— 24	8
	Mean . .	40	94	84	123	69	62	12	12
Equinox . . .	1	— 59	26	84	114	50	42	19	— 16
	2	13	92	16	85	53	45	25	38
	3	26	80	59	72	36	25	28	— 13
	4	50	16	21	66	42	72	14	— 4
	5	28	26	60	50	43	69	14	— 4
	6	81	— 5	16	60	39	48	33	29
	7	3	102	71	65	95	51	9	28
	8	— 36	94	51	76	38	66	7	23
	Mean . .	13	54	47	73	50	52	19	10
Winter . . .	1	50	— 40	— 21	— 14	— 16	— 32	— 14	— 10
	2	— 97	51	— 34	27	14	16	10	6
	3	— 42	— 54	7	— 10	5	— 6	6	22
	4	— 15	— 57	33	— 17	13	— 10	— 21	— 17
	5	64	— 86	40	6	12	23	— 4	— 19
	6	2	— 41	23	— 23	— 18	16	38	11
	7	82	— 83	52	— 69	— 2	— 26	3	12
	8	139	— 53	52	3	9	— 43	— 9	— 16
	Mean . .	23	— 45	19	— 12	2	— 8	1	— 1

TABLE IIB.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations,
Reduced to the Epoch of New Moon.

Pavlovsk. Horizontal Force.

The unit of force is 10^{-7} C.G.S. ; the small corrections mentioned in §§ 5, 6 have not been applied at this stage, *cf.* § 9 ; + denotes increasing horizontal force.

Season.	Lunar phase.	a'_1 .	b'_1 .	a_2 .	b_2 .	a'_3 .	b'_3 .	a'_4 .	b'_4 .
Summer . . .	1	- 3	90	- 2	103	- 8	61	- 22	9
	2	19	107	12	104	- 14	82	- 22	- 2
	3	- 30	6	- 25	80	- 39	23	- 19	1
	4	62	55	23	77	- 13	79	- 22	7
	5	43	26	36	141	10	75	- 35	- 2
	6	29	106	- 6	167	27	69	- 69	0
	7	53	2	30	71	21	46	5	7
	8	4	51	22	88	- 10	50	0	23
	Mean . .	22	55	11	104	- 3	61	- 23	5
Equinox . . .	1	- 102	- 2	40	76	- 5	60	23	13
	2	- 55	66	- 41	11	18	61	- 34	15
	3	90	84	11	66	13	5	6	9
	4	42	- 1	- 1	73	- 33	79	- 22	26
	5	- 49	46	- 17	- 8	- 10	77	- 10	34
	6	15	43	- 48	5	1	46	- 5	56
	7	10	38	36	78	50	99	- 29	27
	8	- 27	48	26	86	- 5	68	1	7
	Mean . .	- 10	40	1	48	4	62	- 9	23
Winter . . .	1	45	- 39	- 29	- 32	- 1	- 28	13	- 1
	2	- 77	43	- 49	4	- 16	10	- 1	- 11
	3	- 57	0	- 54	8	- 30	- 7	12	- 3
	4	- 18	- 55	- 23	0	- 34	- 6	12	2
	5	9	- 30	8	- 20	- 40	- 2	- 34	7
	6	- 35	- 23	3	- 17	- 42	1	- 18	13
	7	49	- 27	- 16	- 31	10	- 12	- 11	20
	8	90	1	11	28	6	- 17	29	- 9
	Mean . .	1	- 16	- 19	- 8	- 18	- 8	0	2

TABLE IIIA.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations,
Reduced to the Epoch of New Moon.

Pola. Declination West.

The unit is 0'001; the small corrections mentioned in §§ 5, 6 have not been applied at this stage, *cf.* § 9; + denotes increasing transverse force.

Season.	Lunar phase.	a'_1 .	b'_1 .	a_2 .	b_2 .	a'_3 .	b'_3 .	a'_4 .	b'_4 .
Summer . .	1	206	– 35	235	27	124	– 25	– 42	– 21
	2	153	– 134	249	44	85	40	– 16	14
	3	199	– 50	201	74	103	30	– 16	– 17
	4	62	– 174	147	17	123	– 68	– 17	– 20
	5	46	– 89	270	– 67	114	– 39	14	– 6
	6	192	– 206	185	4	92	8	– 12	26
	7	69	– 135	259	16	109	– 20	27	17
	8	131	– 158	263	20	91	– 4	– 10	– 26
	Mean . .	132	– 123	226	17	105	– 10	– 9	– 4
Equinox . .	1	108	– 42	58	– 18	37	– 55	49	– 46
	2	46	– 109	40	– 6	69	25	44	– 12
	3	74	– 72	21	15	7	– 57	– 26	– 59
	4	64	– 47	11	82	47	5	22	13
	5	161	– 47	52	112	72	– 16	12	0
	6	114	– 40	89	56	131	5	60	5
	7	25	– 98	180	– 16	38	– 121	88	– 19
	8	91	– 164	119	6	80	– 13	– 6	10
	Mean . .	85	– 77	71	27	60	– 28	30	– 14
Winter . .	1	– 48	– 43	– 108	77	– 1	54	– 61	51
	2	– 136	– 52	– 14	27	– 25	36	– 9	– 5
	3	– 46	74	– 42	169	7	11	39	11
	4	– 32	– 112	– 13	31	– 22	2	18	7
	5	– 64	– 52	– 58	29	– 1	10	– 16	– 20
	6	– 75	– 63	– 52	88	– 29	17	17	21
	7	– 98	– 38	– 9	25	– 56	66	– 14	– 31
	8	89	– 60	– 94	20	5	– 38	19	– 3
	Mean . .	– 51	– 43	– 49	58	– 15	20	– 1	4

TABLE IIIB.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations,
Reduced to the Epoch of New Moon.

Pavlovsk. Declination West.

The unit is 0'001; the small corrections mentioned in §§ 5, 6 have not been applied at this stage, *cf.* § 9; + denotes increasing transverse force.

Season.	Lunar phase.	a'_1 .	b'_1 .	a_2 .	b_2 .	a'_3 .	b'_3 .	a'_4 .	b'_4 .
Summer . . .	1	339	20	249	- 16	95	84	- 18	24
	2	308	- 221	209	- 103	47	19	26	11
	3	333	- 96	180	86	- 20	54	- 11	- 14
	4	157	- 106	95	116	73	- 26	70	5
	5	117	- 53	446	- 9	52	- 13	- 9	21
	6	309	- 237	291	- 61	150	- 57	- 11	- 3
	7	174	- 209	268	16	- 10	- 53	16	- 54
	8	180	- 23	395	- 52	75	24	- 36	15
	Mean . .	240	- 116	267	- 3	58	5	3	1
Equinox . . .	1	183	- 150	84	- 3	18	- 23	82	- 13
	2	201	42	4	- 166	76	31	31	24
	3	127	- 163	- 78	2	2	4	22	- 98
	4	206	7	- 91	117	60	- 43	- 9	9
	5	193	- 39	126	115	- 37	65	100	- 13
	6	370	66	54	34	88	- 13	44	- 20
	7	- 203	- 137	283	- 66	- 2	- 89	- 78	- 25
	8	49	- 268	175	11	6	33	4	31
	Mean . .	141	- 80	70	5	26	- 4	25	- 13
Winter . . .	1	- 5	- 21	- 60	121	- 35	44	2	- 107
	2	- 85	14	- 22	- 17	- 91	32	29	- 58
	3	8	32	- 134	144	39	- 14	17	- 24
	4	- 61	- 113	13	70	- 34	69	37	10
	5	- 17	18	- 82	35	22	49	- 10	- 35
	6	- 33	- 93	- 17	33	- 91	7	1	40
	7	- 43	- 139	5	- 7	- 107	26	69	- 38
	8	*(183)	- 76	- 143	13	24	- 16	12	62
	Mean . .	- 34	- 47	- 55	49	- 34	25	20	- 19

* This value has been rejected in taking the mean.

TABLE IVA.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations,
Reduced to the Epoch of New Moon.

Pola. Vertical Force.

The unit of force is 10^{-7} C.G.S. ; the small corrections mentioned in §§ 5, 6 have not been applied at this stage, *cf.* § 9 ; + means increasing downward force.

Season.	Lunar phase.	a'_1 .	b'_1 .	a_2 .	b_2 .	a'_3 .	b'_3 .	a'_4 .	b'_4 .
Summer . . .	1	-49	49	-44	35	-18	31	-2	9
	2	12	-17	-48	36	-21	10	9	-4
	3	11	9	-46	36	-43	16	-10	-10
	4	-54	16	-51	47	-24	33	-0	6
	5	4	11	-42	48	-8	22	-6	0
	6	-18	28	-47	71	-14	22	-9	0
	7	23	-21	-50	33	-26	31	-0	10
	8	-26	-2	-48	11	-21	-2	-17	11
	Mean . .	-12	9	-47	40	-22	20	-4	3
Equinox . . .	1	-52	50	17	25	13	9	5	7
	2	-28	-27	-32	7	-29	10	5	6
	3	-71	56	-21	14	6	5	7	15
	4	52	-2	-13	54	-3	21	-10	19
	5	-1	-57	-15	5	-25	10	-15	14
	6	-69	31	7	56	-3	-9	-27	26
	7	38	-20	32	20	27	45	22	11
	8	15	-19	-25	5	-8	17	-1	13
	Mean . .	-14	2	-6	23	-3	13	-2	14
Winter . . .	1	-30	-23	-7	-10	-10	-8	-13	-10
	2	53	48	-34	40	3	7	15	10
	3	-68	2	-55	19	-29	1	2	-21
	4	-10	51	-46	27	-1	-13	-15	-2
	5	-18	-54	-24	31	1	3	6	-2
	6	-4	31	-18	7	-6	-5	8	1
	7	-0	19	-25	25	-5	0	3	5
	8	-21	-13	-27	-7	-5	12	-1	19
	Mean . .	-12	8	-30	17	-7	0	1	0

TABLE IVB.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations,
Reduced to the Epoch of New Moon.

Pavlovsk. Vertical Force.

The unit of force is 10^{-7} C.G.S.; the small corrections mentioned in §§ 5, 6 have not been applied at this stage, *cf.* § 9; + means increasing downward force.

Season.	Lunar phase.	a'_1 .	b'_1 .	a_2 .	b_2 .	a'_3 .	b'_3 .	a'_4 .	b'_4 .
Summer . .	1	39	38	-15	31	11	4	3	-10
	2	-76	28	4	29	16	-2	-19	8
	3	-34	-20	6	-2	4	5	-4	-9
	4	-70	24	10	30	10	-4	9	7
	5	55	88	-1	19	37	-9	3	-9
	6	7	-14	34	-4	13	-1	-1	9
	7	-10	17	13	-5	5	-8	-5	-15
	8	-63	78	-6	-3	-15	-6	9	10
	Mean . .	-19	30	6	12	10	-3	-1	-1
Equinox . .	1	10	34	-9	-30	8	-10	10	8
	2	58	-20	31	-19	1	1	-9	4
	3	-53	-47	-28	-17	-13	17	4	7
	4	-26	55	-8	13	9	3	-12	5
	5	-9	-13	-16	-30	15	0	7	-11
	6	82	-67	3	-5	-2	-28	15	3
	7	-54	-46	-10	-34	-22	-6	13	8
	8	-84	35	-3	-8	-1	12	1	-1
	Mean . .	-10	-9	-5	-16	-1	-1	4	3
Winter . .	1	33	34	-4	11	-6	-8	-14	0
	2	19	29	30	39	4	17	3	-1
	3	12	-60	-26	-1	17	12	-7	6
	4	-57	14	-8	41	-12	10	3	-1
	5	85	30	4	0	-1	0	-6	3
	6	38	-40	-6	-18	4	3	-6	-3
	7	29	-38	-10	-1	21	-25	-1	-14
	8	49	-28	3	-28	-10	-11	6	-9
	Mean . .	26	-7	-2	5	2	0	-3	-2

TABLE V.—Probable Errors of the Mean Values of α_n and b_n in Tables II.–IV.*

The results are in every case reduced to force, the unit being 10^{-7} C.G.S.

Observatory and element.	Season.	a_1 .	b_1 .	a_2 .	b_2 .	a_3 .	b_3 .	a_4 .	b_4 .
Pola. Horizontal force	Summer	13	14	10	8	7	3	6	3
	Equinox	10	12	7	4	4	4	2	6
	Winter	19	8	8	6	3	6	4	4
Pavlovsk. Horizontal force	Summer	8	11	5	8	5	5	5	2
	Equinox	16	7	9	11	6	6	5	4
	Winter	15	7	6	6	6	3	5	2
Pola. Transverse force	Summer	11	10	7	5	2	6	3	4
	Equinox	7	7	9	8	6	8	6	4
	Winter	9	6	6	8	4	5	5	4
Pavlovsk. Transverse force	Summer	13	12	13	9	6	6	4	3
	Equinox	15	15	15	10	5	6	6	4
	Winter	8	9	5	7	7	4	3	6
Pola. Vertical force	Summer	8	5	1	4	2	3	2	2
	Equinox	13	10	6	5	4	3	4	1
	Winter	7	10	4	5	2	2	2	3
Pavlovsk. Vertical force	Summer	13	9	3	5	3	1	2	3
	Equinox	14	12	4	4	3	3	2	1
	Winter	8	11	3	6	3	3	2	1

* Cf. § 10.

TABLE VIA.—Fourier Coefficients of the Lunar Diurnal Variations of the Geographical Components of Magnetic Force at Pavlovsk and Pola,* in the Formula

$$\Sigma C_n \sin (nt + \theta_n).$$

The time $t = 0$ is the local time of upper culmination of the moon at the phase of new moon. The unit of force is 10^{-7} C.G.S.; + denotes an increase of force to the North, West, or to the Nadir.

n.	North force.				West force.				Vertical force.			
	Pavlovsk.		Pola.		Pavlovsk.		Pola.		Pavlovsk.		Pola.	
	C_n .	θ_n .	C_n .	θ_n .	C_n .	θ_n .	C_n .	θ_n .	C_n .	θ_n .	C_n .	θ_n .
Summer.												
1	61	23°·8	111	14°·5	131	117°·9	113	125°·8	36	329°·5	15	308°·1
2	105	8°·3	133	27°·2	128	92°·7	161	79°·9	13	27°·1	62	311°·4
3	62	0°·9	86	43°·3	28	87°·3	80	88°·2	11	106°·6	30	313°·3
4	26	295°·2	20	47°·5	2	—	—	4† (91)	1	—	5 ₅	(308)
Equinox.												
1	42	11°·2	63	4°·6	79	120°·7	71	126°·2	13	230°·5	14	278°·7
2	48	3°·1	80	31°·0	82	88°·0	60	62°·1	17	199°·0	24	346°·5
3	63	5°·4	71	39°·3	13	100°·3	48	102°·3	2	—	14	348°·1
4	27	18°·2	21	54°·9	15	119°·6	26	109°·2	—	5† (233)	14	352°·8
Winter.												
1	17	176°·6	50	145°·8	28	218°·0	46	220°·5	28	107°·9	15	304°·8
2	20	250°·2	30	127°·8	36	313°·8	45	322°·2	6	341°·0	35	300°·4
3	20	249°·6	11	161°·2	21	308°·4	15	321°·7	2	—	7	270°·8
4	2	—	2	—	—	15†	315°·6	3 (351)	4	(231)	1	—

* Cf. § 11.

† The coefficients C_n of these components are reversed in sign in order to exhibit the accordance between the values of θ_n for different values of n .

TABLE VI.B.—Fourier Coefficients of the Lunar Diurnal Variations of the Geographical Components of Magnetic Force at Pavlovsk and Pola,* in the Formula

$$\Sigma (a_n \cos nt + b_n \sin nt).$$

The time $t = 0$ is the local time of upper culmination of the moon at the phase of new moon; the unit of force is 10^{-7} C.G.S.; + denotes an increase of force to the North, West, or to the Nadir.

n.	North force.				West force.				Vertical force.			
	Pavlovsk.		Pola.		Pavlovsk.		Pola.		Pavlovsk.		Pola.	
	a_n .	b_n .	a_n .	b_n .	a_n .	b_n .	a_n .	b_n .	a_n .	b_n .	a_n .	b_n .
Summer.												
1	25	56	28	107	116	-61	91	-66	-18	31	-12	9
2	15	104	61	119	128	-6	158	28	6	12	-46	41
3	-1	62	59	62	28	1	78	2	10	-3	-22	21
4	-25	7	14	13	2	1	-4	0	-1	-1	-4	3
Equinox.												
1	-8	41	5	63	68	-40	57	-42	-10	-8 ₅	-14	2
2	3	48	41	68	82	3	53	28	-5 ₅	-16	-6	23
3	6	63	45	55	13	-2	47	-10	-1	-1	-3	13
4	-9	26	17	12	13	-7	24	-8	4	3	-2	14
Winter.												
1	0	-17	28	-41	-17	-22	-30	-35	26	-8 ₅	-12	8
2	-19	-7	24	-18	-26	25	-28	35	-2	5 ₅	-30	18
3	-19	-7	4	-10	-16	13	-9	12	2	0	-7	0
4	0	2	1	-2	11	-11	0	3	-3	2 ₅	1	0

* Cf. § 11.



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PHILOSOPHICAL TRANSACTIONS
OF THE
ROYAL SOCIETY OF LONDON.

SERIES A, VOL. 214. Pp. 319-357.

ON THE CAPACITY FOR HEAT OF METALS AT
LOW TEMPERATURES.

BY

E. H. GRIFFITHS, Sc.D., F.R.S.,

AND

EZER GRIFFITHS, M.Sc.,

FELLOW OF THE UNIVERSITY OF WALES.



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The Capacity for Heat of Metals at Low Temperatures.

By E. H. GRIFFITHS, *Sc.D., F.R.S.*, and EZER GRIFFITHS, *M.Sc., Fellow of the University of Wales.*

Received June 16,—Read June 25, 1914.

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Introduction.

IN two previous papers* accounts have been given of experimental investigations into the variation with temperature of the specific heats of certain metals, the range of temperature covered being, in general, from 0° C. to 100° C., or, in the case of sodium, to nearly 140° C.

The present paper is a further contribution to the same subject, the temperature range being continued down to about -160° C.

As pointed out in the previous papers, determinations of the true specific heats throughout this range are needed to bridge the large gap between NERNST's observations at liquid hydrogen and liquid air temperatures and the observations above referred to at higher temperatures.

For determinations in the vicinity of fixed points at low temperatures the method employed by NERNST is an admirable one and his experimental values carry great weight.

He has given, in the cases of silver and lead, the results of investigations at higher temperatures (*e.g.*, at -78° C.), but these determinations cannot, for several reasons, be regarded as satisfactory. The validity of this conclusion is indicated by the fact that individual values at the same temperature differ by about 4 per cent.

PART I.

EXPERIMENTAL METHODS AND RESULTS.

(1) *Description of Apparatus.*

Our method of investigation demanded a constant temperature enclosure, and the apparatus described in succeeding pages was designed for this purpose.

We abandoned the use of liquid air on account of the uneconomical nature of the processes that would be involved, and utilised the Joule-Thomson cooling effect in a direct manner.

Fig. 1 is a sectional view of the apparatus. Compressed air† at a pressure of 2000 to 3000 lbs. per square inch entered the interchanger by the pipe A. This interchanger BB was constructed of ordinary solid drawn copper tubing, $\frac{1}{8}$ " bore, coiled in the form of flat spirals. Successive layers of the coil were separated by strips of cardboard and the entire coil was inserted into a length of motor tyre tubing which fitted it closely. From the interchanger coils the air was carried to the valve C, by means of which an observer controlled the flow, excess of air being discharged at the safety valve on the compressor. Although not shown in the figure, a Hook's joint was inserted in the spindle controlling the valve.

* 'Phil. Trans.,' A, 500, vol. 213, p. 119, 1913; 'Roy. Soc. Proc.,' A, vol. 89, p. 561, 1914.

† Freed from moisture and CO_2 .

After expansion the air circulated through the coil of lead tubing, D, and then back over the surface of the interchanger coils.

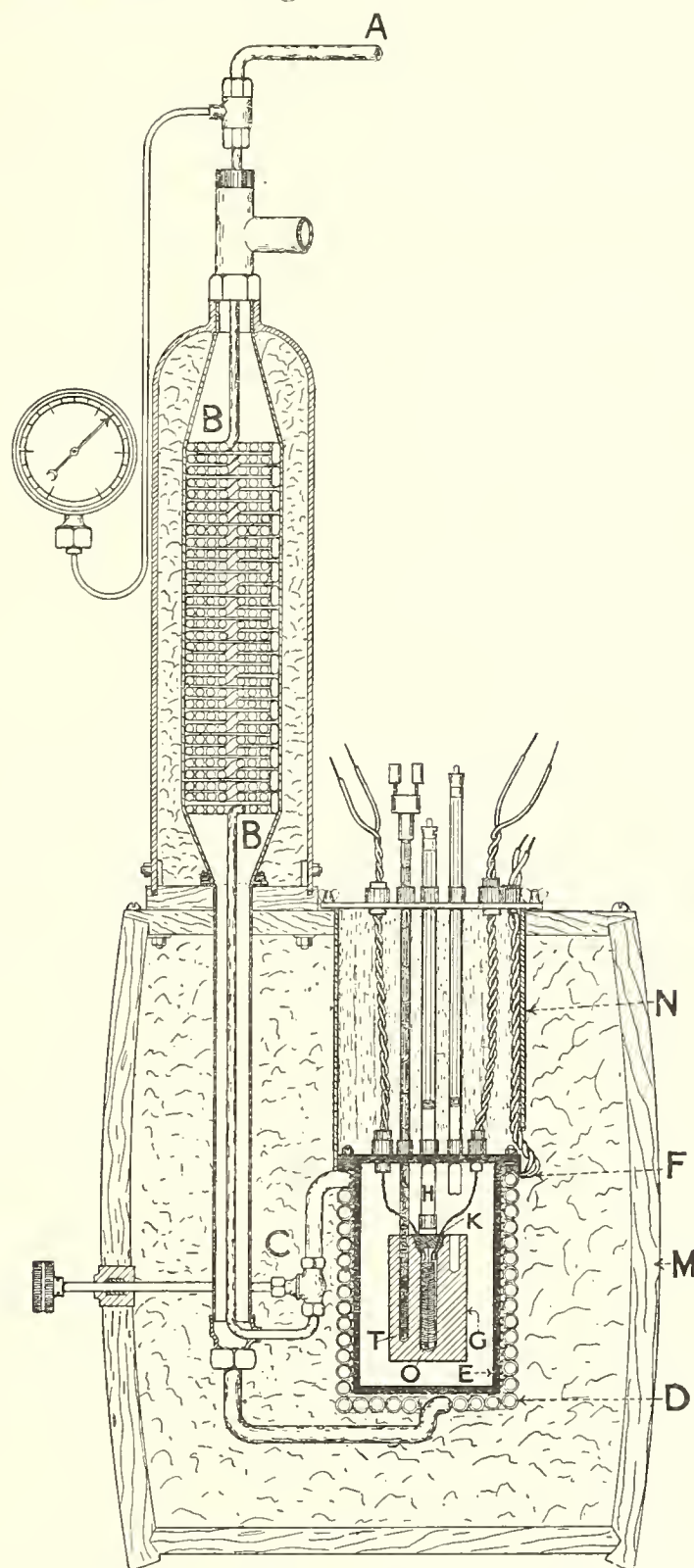


Fig. 1. BB, interchanger coils ; C, valve ; D, coil piping around enclosure ; E, thick copper enclosure ; F, insulated copper wire around enclosure ; G, metal block ; H, supporting tube ; K, taper plug closing central hole in block ; O, heating coil ; T, platinum thermometer ; N, cardboard tube to keep slag wool in position.

On the exterior surface of the thick walled copper enclosure, E, was wound a layer of insulated copper wire, F, which served as a resistance thermometer. Variations in

temperature of the walls of this enclosure were rendered visible by the movements of a galvanometer spot. By controlling the flow of air the oscillations of the spot could be kept within narrow limits and, under normal conditions, the oscillations did not exceed a hundredth of a degree.

The interior of the wooden vessel, M, was packed with slag wool, the passage for the withdrawal of the copper enclosure being kept clear by a cylindrical tube of cardboard, N.

The space between the top of the enclosure and the outer lid was filled by wrapping felt matting around the glass tubes and leads.

The block of metal, G, was suspended within the enclosure by a single glass tube, H.

The central hole contained the heating coil, O, of manganin wire wound on a mica rack and immersed in a light paraffin, usually petrol.

The heating coil was fixed to a short taper plug of copper, K, which closed the central hole. Many such coils and plugs were used in the course of the work, since it was rarely possible to withdraw a plug, after it had once been firmly inserted, without doing considerable damage to the heating coil. The usual resistance for the heating coil was about 26 ohms.

In our preliminary experiments we used many modifications of the above arrangements: *e.g.*, the central hole was but one half the diameter and was closed by a rubber washer pressed down by the lid; the heating coil was of 12 ohms resistance with leads either of manganin or of copper. The defect of this arrangement was that the rubber disintegrated and, with such a volatile liquid as pentane, it was necessary to ensure that no evaporation could take place during the interval (prior to an experiment) that the block was at room temperature.

The platinum thermometer was inserted in the cylindrical hole, T, the annular gap between the stem and the walls being closed by a packing of asbestos thread. The thermometer was connected to a Callendar and Griffiths resistance bridge which was carefully recalibrated during the course of the work.

It may be remarked that, in the preliminary experiments referred to above, a thermometer with a much larger size coil was used. The results obtained were in complete agreement with the values subsequently found by the modified arrangements, using a smaller sized thermometer (Q) the coil of which was constructed of platinum wire of a higher degree of purity.

The same samples of metals were used in this as in the previous work, their dimensions being reduced to the required size by machining.

In the case of sodium, this procedure could not be followed, so another copper sheath was constructed and filled in the manner described below.

(2) *Construction of the Sodium Block.*

The block of sodium was constructed on a different plan* from that described in the

* We are indebted to Mr. EDGAR A. GRIFFITHS for this design.

previous work and, as the method is generally applicable to substances of low melting point, a brief description may be of use.

A section of the block complete is shown in fig. 2. The heating element consisted of 300 cm. of double cotton covered manganin wire, No. 36, enclosed in a fine metal tube* of about 1 mm. bore; the total resistance of the coil was about 38 ohms.

The wire was dipped in hot paraffin wax before insertion in the tubing and the insulation resistance between wire and tube was found to be entirely satisfactory.

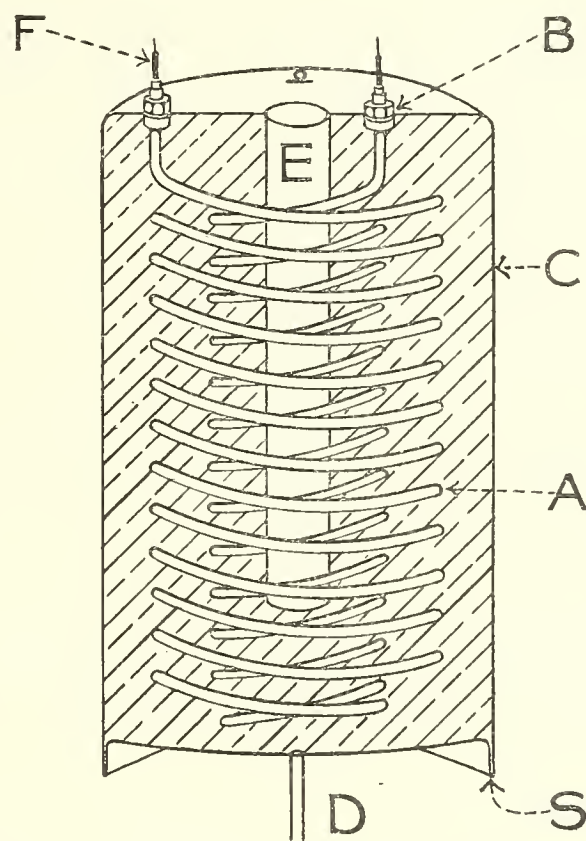


Fig. 2. A, tubing containing insulated manganin wire; B, hollow steel bolts brazed on to tubing to make joint with copper case; C, copper case silver-soldered at S; D, tube used for filling case with sodium, afterwards sealed off; E, copper tube closed at lower end to contain Pt thermometer; F, copper leads to heating coil.

The tubing was wound in the form of a double helix and spaced so as to be distributed as uniformly as possible through the volume of the sodium.

(This would be a point of considerable importance when working with poor conductors of heat.)

The other details of the construction will be understood from the figure.

One further point, however, deserves mention, namely, the soldering on of the bottom of the case after the insertion of the helix.

It was found that hard soldering was absolutely necessary and, since this involved

* This solid drawn tubing was composed of copper alloyed with a small proportion of brass to strengthen it.

a fairly high temperature, precautions had to be taken to shield the spiral, otherwise the insulation would have suffered.

This protection was effected by drawing the spiral up compactly against the top with thin wire loops passing out through the bolt holes. The case was fixed in an inverted position in water with about 1 cm. projecting, the ends of the fine tubing being capped with rubber tubing to prevent access of water to the manganin wire.

The spiral, being totally immersed, was thus effectively shielded, while the bottom was soldered on with an oxy-coal gas blowpipe and, when completed, the spiral was released from without and the nuts screwed on to the ends of the tubing.

Filling the case with sodium was effected *in vacuo*, the electrically heated iron vessel containing the sodium being supported by a tripod, the tube from the case projecting up through the bottom of the iron vessel.

After sealing, the sodium was annealed by slow cooling from the molten state.

The block was suspended within the enclosure by two threads, the platinum thermometer being inserted into the central copper tube.

Two experiments near 0°C . were performed by the method described later in this paper, and the resulting mean value of the specific heat, namely, 0.2826 at -2.5°C ., agrees extremely well with the value at 0°C . for the annealed state, previously obtained,* namely, 0.2829 .

It will be observed that in the present arrangement there were no supporting tubes of quartz; the thermometer was in the centre, while the heating coil was distributed throughout the volume of the metal and had a resistance of about twice that of the coil used previously.

The relative proportions of copper and sodium were very different, as well as the geometrical form of the blocks. When we consider the complete change, both of apparatus and of method, the close correspondence between the results affords a strong proof of the validity of the assumptions underlying both methods.

(3) *Measurement of Mass, Time, Electromotive Force, &c.*

Details concerning the measurement of the above quantities have already been given† and all the precautions there dealt with have been observed.

The method of determining the correction for the heating effect of the current on the resistance of the coil was slightly different from that previously described, inasmuch that the temperature of the block, while the electrical supply was on, was maintained constant by lowering the temperature of the enclosure until losses by radiation,‡ &c., balanced the electrical supply for each rate.

* 'Roy. Soc. Proc.,' A, vol. 89, 1914, p. 567.

† *Loc. cit.*, p. 126.

‡ To save repetition, losses by radiation, convection, and conduction will be referred to as losses by radiation.

The procedure in other respects was the same as before. The rise in temperature of the wire above the mean temperature of the block is given by the relation

$$\theta = 0.10n^2,$$

where n is the number of standard cells balanced at the ends of the heating coil.

Thus, from the temperature coefficient of resistance at any temperature, the correction could be calculated.

Our practice of determining the resistance of the heating coil on the completion of an experiment, when the temperature of the block was a few degrees above the surroundings, rendered the correction of small magnitude and in most cases it could be neglected.

In the case of sodium it was found that the increase of resistance due to increase of current was not appreciable.

(4) *Measurement of Temperature.*

The calibration of a platinum thermometer for use at low temperatures presents some difficulty, since the parabolic formula

$$t - pt = \delta \left\{ \left(\frac{t}{100} \right)^2 - \frac{t}{100} \right\}$$

does not hold true below about -40°C .

At first we endeavoured to approximate to the true scale below 0°C . by means of an expression of the form

$$R = R_0 (1 + \alpha t - \beta t^2 + \gamma t^3),$$

where the constants α , β , and γ are determined by standardisation in liquid oxygen, ice, steam and vapour of boiling sulphur.

Recently, however, HENNING* has effected a direct comparison down to liquid air temperature of some platinum thermometers with the hydrogen gas thermometer, and as our standardisations enabled us to utilize the data obtained by HENNING for the purpose of a reduction to the gas scale, we base our calculations on the results obtained in that comparison. He found that thermometers constructed of wires having different values of δ † do not give consistent results at low temperatures, their indications differing by as much as half a degree.

If, however, the platinum temperatures pt' and pt deduced from observations with two thermometers are compared at the same true temperature t , then the following relationship holds good

$$pt' - pt = cpt(pt - 100), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where c is a constant determined by a comparison of the two thermometers at one temperature, as for example, in a liquid air bath.

* 'Annalen d. Physik,' vol. 4, p. 635, 1913; vol. 10, p. 1064, 1913.

† HENNING determined δ by the freezing-point of zinc ($419^\circ.40$).

While it was not possible to compare our thermometers with a standard, the observations in liquid oxygen supplied data for calculating c , taking the boiling-point of liquid oxygen on the gas scale as $-182^{\circ}97$ C., thus connecting our scale with that of the platinum thermometer which HENNING had calibrated by means of the gas thermometer.

Table I. below summarises some of the fundamental quantities for the various thermometers :—

Column I.—The letters designate the various thermometers.

Column II.—The mean coefficient between 0° C. and 100° C., denoted by α .

Column III.—The constant of the parabolic formula

$$t-pt = \delta \left\{ \left(\frac{t}{100} \right)^2 - \frac{t}{100} \right\}$$

as determined by the sulphur point.

Column IV.—Pt (oxygen), the boiling-point of oxygen on the platinum scale.

Column V.—The difference between the boiling-point of oxygen by the gas thermometer and the value deduced from the parabolic formula given above.

Column VI.—The constant c of equation 1, connecting the thermometer with HENNING'S standard.

TABLE I.

I.	II.	III.	IV.	V.	VI.
Thermometer.	$\alpha \times 10^2$.	δ .	Pt (oxygen).	Difference.	$c \times 10^5$.
Q	0.38598	1.514	-192.37	1.56	+0.07
AA	0.38591	1.534	-192.60	1.68	-0.34
AB	0.38899	1.508	-192.63	1.85	-0.39
CD	0.38888	1.501	-192.71	1.99	-0.53

HENNING'S standard platinum thermometer gives a difference of $1^{\circ}74$ C. from the gas scale at the oxygen point.

The values of the constant c for the different thermometers examined by him vary from -0.68 to $+0.11$.

The quantity with which we were most concerned, however, was not the exact

temperature on the gas scale at which the values of the specific heat were determined, but the value of $\partial(t)/\partial(pt)$ at each temperature.

For this purpose it was more convenient to work with the curve representing the relation between $(t-pt)$ and pt , and in case any revision should be necessary in future, we have inserted below a table of the values of $\partial(t)/\partial(pt)$ used in the reduction of our observations.

TABLE II.

pt° .	$t-pt$.	$\partial(t)/\partial(pt)$.
- 20°	0·33	0·980
- 40°	0·78	0·974
- 60°	1·38	0·967
- 80°	2·12	0·960
- 100°	2·98	0·952
- 120°	4·00	0·944
- 140°	5·21	0·934
- 160°	6·62	0·924
- 180°	8·27	0·912
- 190°	9·17	0·906

(5) *Details of the Standardisation of the Thermometers.*

Four thermometers were standardised at the same time; thermometer AA was used in the preliminary experiments, Q in the final determinations, while AB and CD were used in subsidiary work with the ice calorimeter.

At 0° C., 100° C., and 444°·6 C. (boiling-point of sulphur), the standardisations were performed with the customary form of apparatus and the results call for no remark.

For the low temperature point -182°·97 C. (boiling-point of liquid oxygen), a two-litre vacuum flask was filled with commercial liquid oxygen. The thermometer bulbs were well immersed in the liquid and the mouth of the flask lightly stoppered with cotton wool.

The observations were continued at intervals, as long as the level of the oxygen was sufficient for total immersion of the coils and portion of the stem, with the object of ascertaining how far the constancy was affected by the presence of traces of nitrogen in the liquid oxygen.

Three different values of the battery current were used, and Table III. below summarises one series for thermometer Q, with a battery current giving 0·003 amperes through the wire of the coil.

The quantities in brackets [] are the corrections necessary to reduce the resistance to mean box units ; temperature of box, 17° C., and barometer 760 mm.

The value of $\frac{\partial p}{\partial t}$ for liquid oxygen was taken as 77 mm. Hg per degree Centigrade.

The abnormally high value of R indicated by the asterisk (*) is probably due to superheating of the liquid and was also shown by another thermometer standardised at the same time.

TABLE III.—Thermometer Q in Oxygen.

Time.	Plugs.	Bridge wire.	Temperature box.	Barometer.	Resistance (after correction).
h. m.			° C.		
Date : October 14th.					
12 32	330·116	3·212 [- 0·029]	18·09 [+ 0·078]	767·41 [- 0·526]	332·851
3 21 3 22		5·447 5·510 [- 0·043]	18·485 18·49 [- 0·109]	766·54 [- 0·464]	335·196*
8 43 8 45 8 50		3·209 3·186 3·176 [- 0·029]	18·83 18·85 18·85 [+ 0·138]	765·81 [- 0·412]	333·004
Date : October 15th.					
10 37 10 39 10 41 10 43 10 46	330·116	3·430 3·515 3·630 3·593 3·550 [- 0·032]	17·97 17·98 17·985 [+ 0·069]	767·22 [- 0·513]	333·184

(6) *Heat Capacity of Subsidiary Substances.*

The mean values of the specific heats of glass, petrol, alloy, &c., between 0°C. and liquid air temperatures were determined by means of the ice calorimeter.

The arrangement was very simple; a vessel with a central tube was fixed above the calorimeter and the annular space was filled with liquid air.

The substance was suspended in the central tube alongside a platinum thermometer, and when it had settled to an approximately constant temperature it was dropped directly into the calorimeter.

An approximate value of the coefficient of variation of specific heat with temperature was obtained by determining the mean values of the specific heats over another range, usually 0°C. to 80°C. , and assuming a linear variation the values given in Table IV. were deduced.

Since the heat capacity of these substances entered only as a small correction, the approximate values were amply sufficient for our purpose.

TABLE IV.

Substance.	Specific heat at 0°C.	Specific heat at -150°C.
Glass (soft soda).	0.180	0.092
Petrol (density 0.74). . .	0.455	0.358

The calorimeter constant* was taken as 15.486 mgr. mercury per mean calorie.

(7) *Method of Experiment.*

The constant attention necessary to maintain the surrounding enclosure at a fixed temperature imposed considerable strain on the observer attending to this part of the apparatus and precluded the possibility of experiments such as those described in our previous work.

The procedure adopted in the present investigation was as follows:—

The temperature of the enclosure was lowered progressively by utilizing the full supply from the compressor and controlling the flow so as to produce a steady pressure drop through the valve of 120 to 150 atmospheres.

The temperature of the block would fall at a steady rate by radiation to the enclosure walls, and when its temperature had nearly reached the desired point the compressor was stopped.

The temperature of the enclosure would then rise rapidly by conduction from without and soon pass that of the block which, in consequence of the slow transmission of heat by radiation, would lag behind that of the walls.

* 'Proc. Phys. Soc.,' London, vol. 26, p. 1, 1913.

For $\overline{\theta + \theta_i} = \theta_0$ the radiation term vanishes, hence, if $\partial\theta_N/\partial t$ denotes the value of the ordinate at this point,

$$\frac{\partial\theta_N}{\partial t} = \frac{n^2 E^2}{JRf(MS + ms)}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (4)$$

which enables the value of the specific heat to be deduced.

As an illustration of the method, one experiment in the case of zinc at -64° Pt may be quoted.

All temperatures are expressed on the platinum scale, the conversion being effected by the factor $\frac{\partial(t)}{\partial(pt)} (\equiv f)$ in the final reduction.

TABLE V.

I. Warming up under Radiation.

Temperature.	$\frac{\partial\theta}{\partial t} \times 10^4.$	Temperature.	$\frac{\partial\theta}{\partial t} \times 10^4.$
-66.75	3.87	-66.63	3.81
-66.71	3.84	-66.59	3.77
-66.67	3.99		

II. Electrical Supply on.

Temperature. ($\theta + \theta_i$).	$\frac{\partial\theta}{\partial t} \times 10^4.$	Temperature. ($\theta + \theta_i$).	$\frac{\partial\theta}{\partial t} \times 10^4.$
-65.44	35.78	-63.03	33.09
-65.04	35.44	-62.63	32.51
-64.64	34.98	-62.23	32.13
-64.23	34.31	-61.87	31.59
-63.83	34.01	-61.47	31.15
-63.43	33.57		

III. Cooling down by Radiation.

Temperature.	$\frac{\partial\theta}{\partial t} \times 10^4.$	Temperature.	$\frac{\partial\theta}{\partial t} \times 10^4.$
-61.35	2.65	-61.47	2.54
-61.39	2.66	-61.51	2.53
-61.43	2.60		

The data for computing III. in the above table were obtained from the following observations of temperature and time, after switching off the electrical supply :—

IV. Cooling Curve.

(Time measured from instant of switching off, and temperature difference from the last transit.)

Time.	Temperature difference.	Time.	Temperature difference.
secs.		secs.	
246	+ 0·120	698	0
331	0·100	786	- 0·020
396	0·080	854	- 0·040
481	0·060	942	- 0·060
545	0·040	1011	- 0·080
633	0·020		

If the results of IV. are plotted on sectional paper and the line prolonged to cut the temperature axis for $t = 0$, we obtain the "rise above" after switching off, which quantity is denoted by θ_i in the equations above. In this particular case $\theta_i = 0·185^\circ \text{C}$.

Referring to fig. 3, the observations at "A" are the rates of rise by radiation in the above experiment, when the temperature of the block was below that of the surroundings; those at "B" correspond to the rate of cooling under radiation after the block has been heated up above the surroundings by the electrical supply.

In some experiments the electrical supply was switched off; a group of "radiation" observations taken; then the electrical supply continued and the experiment completed in the usual way.

The intermediate group of "radiation" points thus obtained always fell on the straight line joining the extreme groups and justified the application of NEWTON'S law for the small temperature differences employed.

The observations distributed about the line "CD" correspond to the period of the electrical supply. It will be observed that, as required by theory, "CD" is parallel within the limits of experimental error to the line "AB."

If desired, the radiation group at "A" could be dispensed with and the line drawn through the mean of group "B" parallel to "CD." Occasionally this plan was adopted when the observations "A" were very irregular on account of the conditions not being sufficiently steady when the observations were commenced.

The point "N" on "AB" corresponds to the temperature at which there is no loss or gain by radiation and "NQ" to $\partial\theta_N/\partial t$ of equation 4.

In the case of several metals, experiments were performed near 0°C . by this method and also by the "intersection" method described in the previous paper, with entire accordance between the results.

(8) Experimental Results.

The results obtained in this and the previous investigation are summarised below. Where more than one experiment has been performed at any temperature, the mean

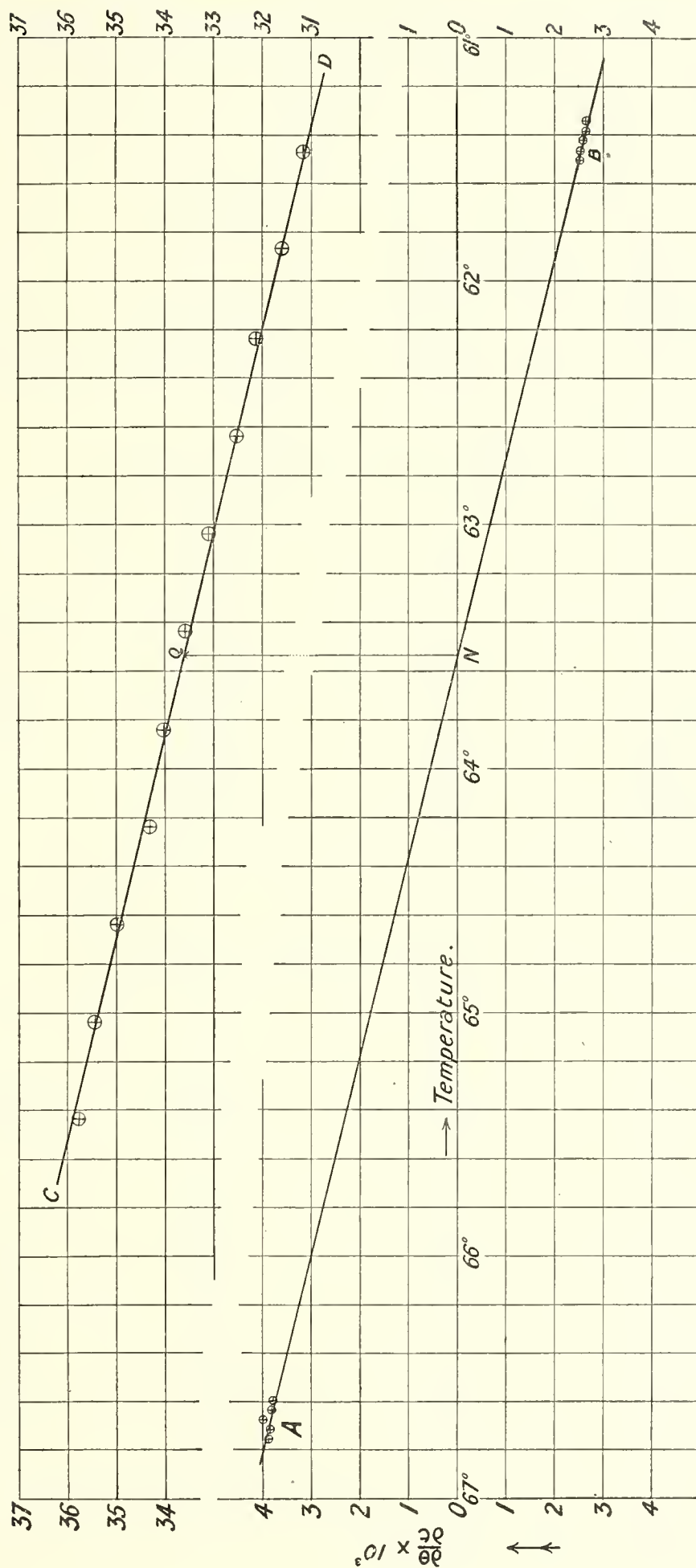


Fig. 3.

Line CD, rate of rise due to electrical supply and radiation; Group A, rate of rise due to radiation when temperature of block below surroundings; Group B, rate of fall due to radiation when temperature of block above surroundings.

NOTE.—Temperatures are in degree μt below 0°C .

value is given. The atomic heats were plotted on a large scale against absolute temperature and the mean curves drawn through the observations, the lines being prolonged through the experimental values of NERNST* or DEWAR† at very low temperatures.

Explanation of the Tables VI.–XIII.

Column I.—The absolute temperature, T.

Column II.—The specific heat at this temperature.

Column III.—The atomic heat under constant pressure, which is denoted by C_p .

Column IV.—Value of C_p at this temperature, as read from the smoothed curve.

Column V.—Difference, Column III. minus Column IV.

TABLE VI.—Aluminium. (Atomic Weight : 27.1.)

I.	II.	III.	IV.	V.
T.	Specific heat.	C_p (experimental).	C_p (curve).	Difference.
158.7	0.1709	4.63	4.62	+ 0.01
191.5	0.1867	5.06	5.06	—
273.1	0.2096	5.68	5.68	—
301.5	0.2147	5.82	5.82	—
324.1	0.2184	5.92	5.92	—
370.6	0.2248	6.09	6.09	—

TABLE VII.—Iron. (Atomic Weight : 55.85.)

I.	II.	III.	IV.	V.
T.	Specific heat.	C_p (experimental).	C_p (curve).	Difference.
140.3	0.0770	4.30	4.29	+ 0.01
214.3	0.0967	5.40	5.40	—
267.0	0.1044	5.83	5.82	+ 0.01
273.1	0.1045	5.84	5.86	− 0.02
274.0	0.1045	5.84	5.87	− 0.03
283.1	0.1060	5.92	5.93	− 0.01
294.1	0.1077	6.02	6.00	+ 0.02
297.6	0.1080	6.03	6.02	+ 0.01
323.4	0.1105	6.17	6.16	+ 0.01
339.4	0.1112	6.21	6.23	− 0.02
370.6	0.1137	6.35	6.35	—

* Berlin, 'Sitz. Ber.,' p. 262, 1910; pp. 306, 316, 494, 1911; p. 1160, 1912.

† 'Roy. Soc. Proc.,' A, vol. 89, p. 158, 1913.

TABLE VIII.—Copper. (Atomic Weight: 63.57.)

I.	II.	III.	IV.	V.
T.	Specific heat.	C_p (experimental).	C_p (curve).	Difference.
138	0.07766	4.94	4.92	+0.02
149.5	0.07942	5.05	5.05	—
171.4	0.08235	5.23	5.23	—
204.9	0.08593	5.46	5.46	—
273.1	0.09088	5.78	5.78	—
301.6	0.09230	5.87	5.87	—
336.7	0.09365	5.95	5.96	-0.01
340.5	0.09387	5.97	5.97	—
370.5	0.09521	6.05	6.04	+0.01

TABLE IX.—Zinc. (Atomic Weight: 65.37.)

I.	II.	III.	IV.	V.
T.	Specific heat.	C_p (experimental).	C_p (curve).	Difference.
145.5	0.08421	5.50	5.50	—
211.1	0.08898	5.82	5.82	—
273.1	0.09176	6.00	6.01	-0.01
294.6	0.09265	6.06	6.06	—
323.6	0.09412	6.15	6.14	+0.01
370.5	0.09521	6.22	6.22	—
396.5	0.09570	6.26	6.26	—

TABLE X.—Silver. (Atomic Weight: 107.88.)

I.	II.	III.	IV.	V.
T.	Specific heat.	C_p (experimental).	C_p (curve).	Difference.
158.1	0.05210	5.62	5.62	—
187.4	0.05362	5.78	5.78	—
273.1	0.05560	6.00	6.00	—
301.5	0.05613	6.05	6.05	—
340.5	0.05680	6.13	6.13	—
370.6	0.05737	6.19	6.19	—

TABLE XI.—Cadmium. (Atomic Weight: 112.4.)

I.	II.	III.	IV.	V.
T.	Specific heat.	C_p (experimental).	C_p (curve).	Difference.
108.3	0.04907	5.52	5.52	—
181.8	0.05287	5.94	5.94	—
273.1	0.05475	6.15	6.17	-0.02
301.5	0.05554	6.24	6.24	—
327.6	0.05616	6.31	6.31	—
370.8	0.05714	6.42	6.42	—

TABLE XII.—Lead. (Atomic Weight: 207.1.)

I.	II.	III.	IV.	V.
T.	Specific heat.	C_p (experimental).	C_p (curve).	Difference.
118.6	0.02867	5.94	5.93	+0.01
166.3	0.02903	6.01	6.02	-0.01
254.4	0.02989	6.19	6.22	-0.03
273.1	0.03020	6.25	6.26	-0.01
301.5	0.03053	6.32	6.33	-0.01
324.1	0.03073	6.36	6.38	-0.02
340.5	0.03102	6.42	6.41	+0.01
370.6	0.03127	6.48	6.48	—

TABLE XIII.—Sodium. (Atomic Weight: 23.)

I.	II.	III.	IV.	V.
T.	Specific heat.	C_p (experimental).	C_p (curve).	Difference.
Annealed.				
123.3	0.2466	5.67	5.67	—
155.2	0.2586	5.95	5.93	+0.02
179.3	0.2619	6.02	6.06	-0.04
210.5	0.2707	6.23	6.23	—
270.6	0.2826	6.51	6.51	—
273.1	0.2829	6.51	6.52	-0.01
305.9	0.2910	6.69	6.69	—
322.4	0.2952	6.79	6.79	—
340.9	0.3019	6.94	6.94	—
352.2	0.3083	7.09	7.09	—
358.7	0.3171	7.29	7.28	+0.01
363.1	0.3209	7.38	7.39	-0.01
368.6	0.3258	7.49	7.49	—
Molten State.				
373.9	0.3234	7.44	7.44	—
376.2	0.3232	7.43	7.43	—
390.1	0.3217	7.40	7.40	—
400.6	0.3205	7.37	7.37	—
411.6	0.3189	7.34	7.34	—

In Part II., where the various theoretical formulæ are examined, to avoid confusion between experimental errors and systematic divergences, the values from the mean curves will be taken as the basis of comparison.

Fig. 4 exhibits the results graphically over the temperature range which we have investigated.

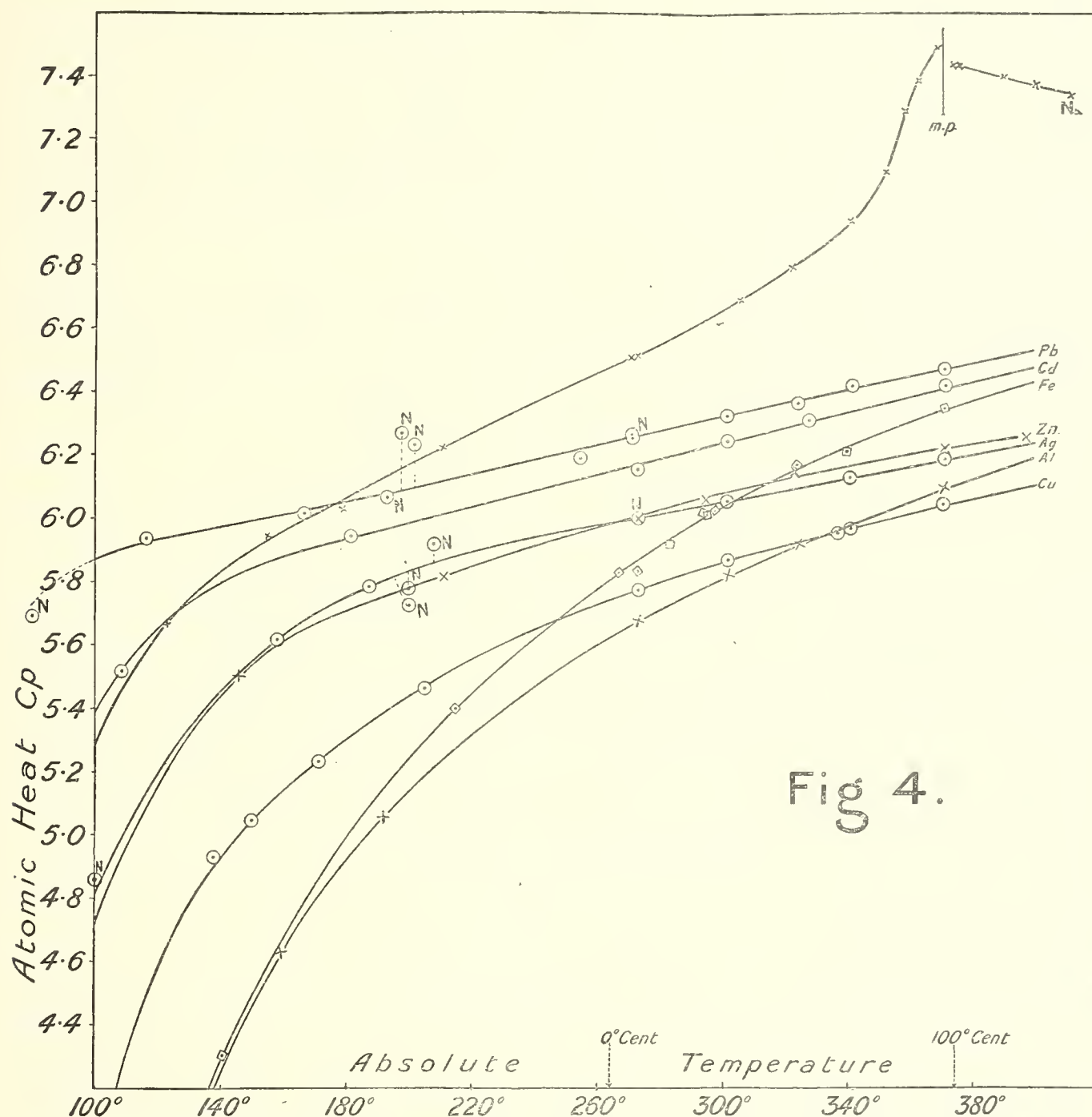


Fig 4.

The letter N denotes the observations of NERNST in the case of silver and of lead.

(9) NERNST'S Method.

The work of NERNST at low temperatures has already been referred to and since some of his results are quoted below, it is desirable to indicate briefly the method by which they have been obtained.

The block of metal under examination was suspended in an exhausted glass vessel immersed in liquid air or hydrogen. The heat was supplied electrically by a platinum wire which also served as a resistance thermometer. It was essential that the vacuum should be extremely high, since no correction was applied for losses or gains by radiation, &c., and for this reason his values at the lowest temperatures should be the most reliable.

In the cases of silver and lead NERNST has also recorded observations at the temperatures of solid CO_2 and ice by the same method.

In fig. 4 these values are indicated by the letter N, and it will be observed that in both cases the smoothed curves through our observations are in general accordance with NERNST'S *mean* values, though his individual observations show marked discrepancies.

For the same two metals at liquid air temperatures, the mean of several values at closely adjacent temperatures is given in Table XIV.

In the lead group the values are not very concordant, some of the observations differ from the smoothed curve by more than 3 per cent.; the probable error, however, is much less than this. Lead, from our own experience, appears to be a metal with which it is difficult to obtain concordant results.

TABLE XIV.—NERNST'S Values at Low Temperatures.

	T.	32°·4.	35°·1.	83°.	86°.	88°·3.		
Al	C_p	0·25	0·33	2·41	2·52	2·62		
	T.	23°·5.	27°·7.	33°·4.	87°.	88°.		
Cu	C_p	0·22	0·32	0·54	3·33	3·38		
	T.	40°.	60°.	80°.				
Zn*	C_p	1·77	3·15	4·09				
	T.	35°.	39°·1.	44°·2.	52°·6.	65°.	77°.	85°·3.
Ag	C_p	1·58	1·90	2·36	2·85	3·74	4·07	4·37
	T.	23°.	28°·3.	37°·3.	80°.			
Pb	C_p	2·96	3·92	4·41	5·69			

* In the case of zinc the actual figures are not recorded, and the above figures have been obtained from the curves given in the paper.

For iron, cadmium, and sodium, which NERNST has not investigated, the mean values obtained by DEWAR over the range from liquid nitrogen to liquid hydrogen have been utilized for the purpose of extending the curves to low temperatures.

Whilst DEWAR's method is of great use for obtaining comparative values, especially of organic substances and poor heat conductors, it cannot, when applied to metals, claim to be capable of giving *absolute values* with the same precision as the electrical methods, and it is noticeable that DEWAR used a mass of only 1 gr. of lead as compared with 400 gr. by NERNST and 2200 gr. in the investigation recorded in this paper.

Moreover it should be remembered that, where the curvature is marked, the mean value over a range may depart appreciably from the real value at the mid point; and, for this reason also, values obtained by an electrical method are to be preferred.

DEWAR'S Mean Values at 50° C. Absolute.

C_p	Fe, 0·98	Cd, 3·46	Na, 3·50
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(10) *Discussion of the Results.*

The question of the variation of atomic heat with temperature has recently received a considerable amount of attention, chiefly on account of its bearing on PLANCK'S "quantum" theory.

The theoretical formulæ are concerned with C_v (the atomic heat at constant volume), while the quantity experimentally investigated is C_p , the atomic heat when the solid is free to expand with temperature.

It is necessary to know the values of the compressibility and expansion coefficients, in order to apply the well-known thermodynamical relation

$$C_p - C_v = \alpha^2 \epsilon v t (5)$$

where α is the coefficient of cubical expansion, ϵ is the coefficient of volumetric elasticity, v is the reciprocal of the density, and t the absolute temperature.

The values of ϵ^* obtained by RICHARDS have been assumed, but, on account of lack of data, we have been compelled to disregard the variation of the compressibilities with temperature.

The coefficient of expansion of metals decreases with temperature, as shown by the work of FIZEAU, DORSEY,† AYRES,‡ and others.

* 'Journ. Chem. Soc.,' 1911.

† 'Phy. Rev.,' vol. xxv., p. 97, 1907.

‡ 'Phy. Rev.,' vol. xx., p. 38, 1905.

With the exception of sodium, the smoothed curves have been drawn through the experimental results of DORSEY and of AYRES in the range 0°C. to -180°C. , and the curves extrapolated in order to deduce the value of $a\epsilon v \int_0^t a dt$ at various temperatures.

For sodium the value 0.000224, determined by us in some preliminary experiments by a new method, has been utilized.

This method, which will be described in a later paper, did not indicate any marked change in the coefficient over the range 0°C. to 95°C.

For low temperature a coefficient of decrease of the same order as that of cadmium was assumed, a procedure which is justified by the fact that the coefficient of expansion is proportional to the atomic heat, as shown by GRUNEISEN, and at low temperature the atomic heat curves of sodium and cadmium are similar.

NERNST and LINDEMANN* adopt a different procedure, basing their method on GRUNEISEN's observation that the coefficient of expansion is proportional to the atomic heat; they obtain the approximate relation

$$C_p - C_v = C_p^2 TA. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where A is a constant characteristic of the substance which can be deduced from measurements of the compressibility and coefficient of expansion made at one temperature.

In the cases of Al, Ag, Pb, and Cu the values so obtained are compared with the values calculated by means of equation (5), which are utilized in all subsequent comparisons of theory and experiment in this paper.

It is shown in Part II. of this paper that no one of the theoretical formulæ hitherto published is capable of representing the atomic heat-temperature curves over the entire range. Since data based on the experimental observations are necessary to test any theory, we summarise below the values, at convenient intervals of temperature, of the atomic heat read from curves which we believe represent the most probable paths through the observations of NERNST at liquid air and hydrogen temperatures; of DEWAR at 50°C. abs. and through our own numbers† from $100^{\circ}\text{C. abs.}$ to $400^{\circ}\text{C. abs.}$

* 'La Théorie du Rayonnement et les Quanta,' p. 265, 1912.

† *Ide* Tables VI. to XIV. and fig. 4.

TABLE XV.

Column I.—Absolute temperature.

Column II.—Atomic heat C_p , from the smoothed curves.

Column III.—Difference between the atomic heat when the metal is free to expand and the hypothetical value at constant volume.

For Al, Cu, Ag, and Pb the values deduced from equation (5) are given under (1) and those from equation (6) under (2).

I.	II.	III.		I.	II.	III.		I.	II.	III.	
T.	C_p .	$C_p - C_v$.		T.	C_p .	$C_p - C_v$.		T.	C_p .	$C_p - C_v$.	
		(1.)	(2.)			(1.)	(2.)			(1.)	(2.)
Aluminium.											
° 32.4	0.25			° 120	3.74	0.05	0.05	° 300	5.81	0.24	0.21
35.1	0.33			160	4.64	0.08	0.08	320	5.90	0.26	0.23
50	0.91			200	5.14	0.12	0.11	340	5.98	0.29	0.26
80	2.27	0.01	0.01	230	5.40	0.15	0.14	360	6.05	0.31	0.28
100	3.08	0.03	0.03	250	5.54	0.17	0.16	380	6.13	0.34	0.30
Iron.											
° 50	0.98			° 170	4.85	0.03		° 290	5.98	0.09	
70	1.86			190	5.13	0.04		310	6.09	0.10	
90	2.74			210	5.36	0.04		330	6.19	0.12	
110	3.47	0.01		230	5.54	0.06		350	6.28	0.13	
130	4.05	0.01		250	5.70	0.07		370	6.35	0.15	
150	4.50	0.02		270	5.85	0.08		390	6.41	0.17	
Copper.											
° 23.5	0.22			° 150	5.05	0.04	0.06	° 290	5.83	0.15	0.14
27.7	0.32			170	5.23	0.06	0.07	310	5.89	0.16	0.15
50	1.32			190	5.37	0.07	0.08	330	5.95	0.18	0.16
70	2.43		0.01	210	5.50	0.08	0.09	350	5.99	0.20	0.17
90	3.48		0.02	230	5.61	0.10	0.10	370	6.04	0.22	0.18
110	4.31	0.02	0.03	250	5.70	0.11	0.11	390	6.09	0.23	0.19
130	4.78	0.03	0.05	270	5.77	0.13	0.13				
Zinc.											
° 30	0.94			° 160	5.61	0.09		° 290	6.05	0.23	
50	2.54			190	5.75	0.12		310	6.10	0.25	
80	4.09	0.01		210	5.82	0.14		330	6.15	0.28	
100	4.72	0.03		230	5.88	0.16		350	6.19	0.30	
120	5.15	0.05		250	5.95	0.18		370	6.22	0.33	
140	5.44	0.07		270	5.99	0.20		390	6.25	0.36	

TABLE XV. (continued).

I.	II.	III.		I.	II.	III.		I.	II.	III.	
T.	C_p .	$C_p - C_v$.		T.	C_p .	$C_p - C_v$.		T.	C_p .	$C_p - C_v$.	
		(1.)	(2.)			(1.)	(2.)			(1.)	(2.)
Silver.											
°				°				°			
35	1.58			150	5.55	0.08	0.11	280	6.02	0.21	0.25
40	1.99		0.01	180	5.75	0.11	0.15	300	6.05	0.23	0.27
60	3.32	0.01	0.02	200	5.83	0.13	0.17	320	6.09	0.25	0.29
80	4.24	0.02	0.04	220	5.90	0.15	0.19	340	6.12	0.27	0.31
100	4.83	0.04	0.06	240	5.94	0.17	0.21	360	6.17	0.30	0.34
130	5.33	0.06	0.09	260	5.98	0.19	0.23	380	6.21	0.32	0.36
Cadmium.											
°				°				°			
50	3.46	0.02		180	5.93	0.15		300	6.24	0.32	
70	4.58	0.04		200	5.99	0.18		320	6.29	0.35	
100	5.37	0.07		220	6.04	0.20		340	6.33	0.38	
120	5.65	0.08		240	6.09	0.23		360	6.39	0.42	
140	5.79	0.11		260	6.14	0.26		380	6.44	0.45	
160	5.87	0.13		280	6.19	0.29					
Lead.											
°				°				°			
23	2.96	0.01	0.01	140	5.97	0.15	0.14	280	6.28	0.35	0.34
28	3.58	0.01	0.01	160	6.01	0.17	0.17	300	6.32	0.39	0.37
37	4.50	0.02	0.02	180	6.05	0.20	0.19	320	6.37	0.43	0.40
50	5.14	0.04	0.04	200	6.10	0.23	0.22	340	6.41	0.47	0.43
80	5.72	0.07	0.07	220	6.14	0.26	0.25	360	6.45	0.51	0.46
100	5.87	0.10	0.09	240	6.18	0.29	0.28	380	6.49	0.55	0.49
120	5.93	0.12	0.11	260	6.23	0.32	0.31				
Sodium (Annealed).*											
°				°				°			
50	3.50	0.04		180	6.06	0.22		330	6.85	0.48	
70	4.43	0.07		210	6.22	0.27		350	7.05	0.51	
90	5.04	0.09		240	6.36	0.32		360	7.33	0.53	
120	5.62	0.13		270	6.50	0.37		368	7.48	0.55	
150	5.90	0.17		300	6.66	0.43					

* The changes in the atomic heat consequent on variations in the heat treatment will be considered in a later paper.

PART II.

COMPARISON OF THE EXPERIMENTAL RESULTS WITH THOSE DEDUCED FROM FORMULÆ BASED ON THE "QUANTUM" THEORY.

(1) *Theoretical Formulæ for C_v .*

In order to appreciate the limitations of the various theoretical formulæ which have been given to represent the variation of atomic heat with temperature, it is necessary to review briefly the fundamental assumptions on which they are based.

(a) EINSTEIN'S *Theory*.—EINSTEIN* was the first to show that if we regard the atoms of solid substances as charged with electricity and oscillating about fixed positions, we may consider them as equivalent to a number of PLANCK'S resonators.

On this theory each atom is regarded as possessing three degrees of freedom, and consequently the total vibrational energy of an atom is three times that of a linear oscillator. From these considerations he deduces the expression

$$C_v = 3R \frac{\left(\frac{h}{k} \frac{\nu}{T}\right)^2 e^{\frac{h}{k} \frac{\nu}{T}}}{\left(e^{\frac{h}{k} \frac{\nu}{T}} - 1\right)^2} \quad \dots \dots \dots (7)$$

for the atomic heat of any elementary substance, where

R is the gas constant, 1.9876,

h is the quantum constant, 7.10×10^{-27} erg sec,†

k , BOLTZMANN'S constant, 1.47×10^{-16} erg,

T , the absolute temperature,

ν , the monochromatic frequency of vibration—a constant characteristic of the substance.

In the case of salts, the quantity ν is the frequency found by means of the *reststrahlen*, but, in the case of the metals, it cannot be obtained directly. EINSTEIN showed that if the elastic forces which maintain the atoms in their equilibrium position are the same as those which oppose a diminution of volume when the solid is compressed, and if we assume that these forces are due to the mutual action of adjacent atoms spaced in the form of a lattice, then

$$\nu = 2.54 \times 10^7 \times \frac{V^{1/6}}{m^{1/2} k^{1/2}}, \quad \dots \dots \dots (8)$$

where V is the atomic volume,

m is the atomic weight,

k is the coefficient of compressibility.

* 'Ann. d. Phys.,' vol. 22, pp. 180, 800, 1907.

† 'Ann. d. Phys.,' vol. 38, p. 41, 1912.

MAGNUS found that while formula (7) accounted qualitatively for the decrease in atomic heat with temperature, it gave a too rapid decrease with temperature to represent accurately the form of the experimental line.

(b) NERNST and LINDEMANN.*—NERNST† at first endeavoured to effect a better agreement between the formula and his experimental results by the addition of an empirical term‡ of the form $bT^{3/2}$ to equation (7).

In a later memoir NERNST and LINDEMANN showed that the expression

$$C_v = \frac{3R}{2} \left[\frac{\left(\frac{\beta\nu}{T}\right)^2 e^{\frac{\beta\nu}{T}}}{(e^{\frac{\beta\nu}{T}} - 1)^2} + \frac{\left(\frac{\beta\nu}{2T}\right)^2 e^{\frac{\beta\nu}{2T}}}{(e^{\frac{\beta\nu}{2T}} - 1)^2} \right], \quad \dots \dots \dots (9)$$

where $h/k = \beta$, more nearly expressed the experimental results than (7).

Here again ν , in the case of salts, is the frequency found by means of the *reststrahlen*, but in no case has an infra red band been observed whose frequency is $\frac{1}{2}\nu$.

LINDEMANN suggested another method for the indirect determination of ν .

He assumed that fusion was produced when the amplitude of the atomic vibrations became of the same order of magnitude as the distance between adjacent atoms, and thus obtained the expression

$$\nu = 2.12 \times 10^{12} \sqrt{\frac{T_s}{mV^{2/3}}}, \quad \dots \dots \dots (10)$$

where T_s is the absolute temperature of the melting-point,

V , the atomic volume,

m , the atomic weight.

Expression (9) is empirical, but appears to have been a step in the right direction, as it takes into account more than one frequency, although NERNST was of opinion that the second term represented potential energy.

It is difficult to conceive the atoms in solids, such as metals, as executing monochromatic vibrations; the wide range over which absorption takes place indicates that the vibrations are more complex and, were it possible to apply FOURIER'S theorem, we should expect frequencies distributed over a continuous spectrum.

(c) DEBYE'S *Theory*.§—DEBYE has attempted the solution of the problem when the atomic vibrations are not regarded as monochromatic. He considers the possible vibrations of the substance as a whole, and proceeds on the supposition that the heat vibrations depend on the elastic forces in such a way that the longest heat waves are

* Berlin, 'Sitz. Ber.,' p. 494, 1911.

† 'Journal de Physique,' vol. 9, p. 721, 1910.

‡ EINSTEIN had indicated that such a term was necessary to complete the expression on account of the difference between C_p and C_v .

§ 'Annalen d. Physik,' vol. 39, p. 789, 1912.

identical with elastic vibrations. By applying the classical theory of elasticity to the vibrations of a homogeneous isotropic sphere in the case where the pressures on the surface vanish,* DEBYE determined the number of characteristic types of vibrations up to or below a given frequency ν , and obtained the result that

[illegible]

where

z is the number of sets of "standing waves" as in the analogous problem in "radiation,"

v , the volume of the sphere,

$\mathbb{F}$, a quantity calculated from the elastic constants and given by

$$F = \frac{4\pi}{3} \left[\left(\frac{1}{a_1} \right)^3 + 2 \left(\frac{1}{a_2} \right)^3 \right], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where

a_1 is the velocity of dilatational waves,

a_2 „ „ torsional waves.

Lord RAYLEIGH† has shown that a coupled system of N mass points with $3N$ degrees of freedom may be regarded as giving rise to $3N$ types of vibrations.

Taking the N atoms in unit mass of a solid as such a system, DEBYE ascribes to each of these types of vibration the mean energy $\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$ of a linear oscillator.

The next assumption is the characteristic one of the entire theory. He applies equation (11) deduced for a continuous medium in which an infinite number of vibrations is possible to the system of N atoms in which only $3N$ vibrations are possible.

Since the form of the body considered has no influence on the result expressed in equation (11) for unit volume, we have

[illegible]

where ν_m is the limit frequency.

And the number of “standing waves” in the interval $d\nu$ denoted by dz is given by

$$dz = 3\nu^2 F d\nu. \quad (14)$$

Hence the total energy of the N atoms is

$$E_N = \frac{9N}{\nu_m^3} \int_0^{\nu_m} \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \cdot \nu^2 d\nu, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

* A condition which is regarded as corresponding to the case of a sphere in thermal equilibrium with the outer surroundings, since no energy crosses the surface.

† "Theory of Sound," vol. 1, p. 129, 1877.

from which the following expression is obtained for the atomic heat in which for simplicity ν_m is inserted as ν and β for h/k as before,

$$C_v = 3R \left[\frac{4\pi^4}{5} \left(\frac{T}{\beta\nu} \right)^3 - \frac{3\frac{\beta\nu}{T}}{e^{\frac{\beta\nu}{T}} - 1} - 12 \frac{\beta\nu}{T} \sum_{n=1}^{n=\infty} e^{-\frac{n\beta\nu}{T}} \left(\frac{1}{n\frac{\beta\nu}{T}} + \frac{3}{n^2 \left(\frac{\beta\nu}{T} \right)^2} + \frac{6}{n^3 \left(\frac{\beta\nu}{T} \right)^3} + \frac{6}{n^4 \left(\frac{\beta\nu}{T} \right)^4} \right) \right]. \quad (16)$$

The great point of difference between the theory of EINSTEIN and that of DEBYE lies in the fact that the latter takes into account all frequencies from 0 to ν_m .

The distribution of the different frequencies given by equation (14) appears to be capable of deduction from different standpoints, as shown by the researches* of BORN and V. KÀRMÀN.

CALLENDAR has drawn our attention to one important difficulty of the quantum theory which is mentioned by EINSTEIN, namely, that for large values of $e^{\beta\nu/T}$, but a small proportion of the atoms could have even one quantum. When $\beta\nu/T = 3$ (as for iron at $T = 130^\circ \text{C. abs.}$, but one molecule in twenty could have a single quantum. Even at 100°C. it would be only one in three. PLANCK has endeavoured to avoid this by a more recent and artificial proof which has not received much support.

Before proceeding to consider the other theories of specific heat, we will briefly compare the experimental results with the formulæ based on PLANCK's "quantum" theory. DEBYE's equation (16) is acknowledged, for the representation of the facts at low temperature, to be superior to the formulæ of EINSTEIN and of NERNST and LINDEMANN, but we find that no single value of the limit frequency will make the formula fit the experimental results over the complete range.

It is at very low temperatures that the assumption made by DEBYE, namely, that the vibrations whose frequency is greater than ν_m are negligible, can best be justified, since at such temperatures the slow heat vibrations would be the most important. Hence, in the following comparison, the value of ν_m is so chosen that DEBYE's equation fits the results over the lowest portion of the range.

It is of but little use to calculate the appropriate values of ν_m from the elastic constants of the metals, since these constants are considerably influenced by the nature of the previous heat treatment and by the temperature.

The values calculated from the elastic constants are in accordance with those required by the atomic heat results, as will be shown subsequently.

* 'Phys. Zeitschr.,' vol. 14, pp. 15 and 65, 1913.

TABLE XVI.

(2) Comparison of DEBYE'S Formula with the Experimental Results when the Limit Frequency inserted is such that the Locus coincides with the Atomic Heat Curve over the Lowest Portion of the Temperature Range.

T.	C_p (obs.).	C_p (calc.).	Difference.	T.	C_p (obs.).	C_p (calc.).	Difference.
Aluminium, $\beta\nu = 394$.							
°				°			
32.4	0.25	0.25	—	200	5.14	5.06	0.08
35.1	0.33	0.31	0.02	250	5.54	5.45	0.09
80	2.27	2.26	0.01	300	5.81	5.72	0.09
100	3.08	3.08	—	340	5.98	5.87	0.11
120	3.74	3.72	0.02	380	6.13	5.99	0.14
Iron, $\beta\nu = 385$.							
°				°			
50	0.98	0.92	0.06	210	5.36	5.10	0.26
70	1.86	1.86	—	250	5.70	5.38	0.32
90	2.74	2.75	-0.01	310	6.09	5.63	0.46
110	3.47	3.46	+0.01	350	6.28	5.74	0.54
150	4.50	4.40	0.10	390	6.41	5.85	0.56
Copper, $\beta\nu = 325$.							
°				°			
27.7	0.32	0.28	0.04	190	5.37	5.23	0.14
50	1.32	1.34	-0.02	250	5.70	5.60	0.09
70	2.43	2.46	-0.03	290	5.83	5.75	0.08
90	3.48	3.35	0.13	350	5.99	5.91	0.09
130	4.78	4.47	0.31	390	6.09	5.99	0.10
Zinc, $\beta\nu = 230$.							
°				°			
30	0.94	0.91	0.03	190	5.75	5.67	0.08
50	2.54	2.50	0.04	250	5.95	5.90	0.05
80	4.08	4.08	—	310	6.10	6.06	0.04
100	4.72	4.68	0.04	350	6.19	6.14	0.05
140	5.44	5.29	0.15	390	6.25	6.21	0.04
Silver, $\beta\nu = 215$.							
°				°			
40	1.99	1.93	0.06	240	5.94	5.91	0.03
60	3.32	3.39	-0.07	280	6.02	6.00	0.02
100	4.83	4.82	0.01	300	6.05	6.05	—
150	5.55	5.47	0.08	340	6.12	6.12	—
200	5.83	5.76	0.07	380	6.21	6.19	0.02

TABLE XVI. (continued).

T.	C_p (obs.).	C_p (calc.).	Difference.	T.	C_p (obs.).	C_p (calc.).	Difference.
Cadmium, $\beta\nu = 168$.							
°				°			
50	3.46	3.60	-0.14	260	6.14	6.10	0.04
70	4.58	4.58	—	300	6.24	6.19	0.05
100	5.37	5.26	0.11	340	6.33	6.27	0.06
140	5.79	5.66	0.13	380	6.44	6.36	0.08
200	5.99	5.94	0.05				
Lead, $\beta\nu = 92$.							
°				°			
23	2.96	3.01	-0.05	160	6.01	6.03	-0.02
28	3.58	3.68	-0.10	200	6.10	6.10	—
37	4.50	4.48	0.02	240	6.18	6.21	-0.03
80	5.72	5.66	0.06	300	6.32	6.32	—
120	5.93	5.91	0.02	360	6.45	6.46	-0.01
Sodium, $\beta\nu = 180$.							
°				°			
50	3.50	3.41	0.09	250	6.41	6.14	0.27
70	4.43	4.43	—	290	6.60	6.26	0.34
90	5.04	5.01	0.03	330	6.85	6.35	0.50
130	5.74	5.57	0.17	350	7.06	6.40	0.66
190	6.12	5.94	0.18	365	7.43	6.44	0.99

In the table below the values of ν_m , which have been obtained from the atomic heat formulæ, are compared with the values deduced by DEBYE from the elastic constants of the metals.

$$\nu_m \times 10^{-12}.$$

Metal.	Al.	Fe.	Cu.	Ag.	Cd.	Pb.
ν_m (atomic heat) . . .	8.2	8.0	6.7	4.5	3.5	1.9
ν_m (elastic constants) .	8.3	9.7	6.8	4.4	3.5	1.5

Attention may be drawn to some of the conspicuous features shown by the above comparisons.

In the first place it will be observed that, with the possible exception of lead, the calculated values are too low at the higher temperatures; also that the departure is most pronounced where the curvature is greatest; in copper for example, at 130° C. abs. and in zinc at 150° C. abs.

Much weight should not be attached to the fact that the calculated values are slightly higher than the observed ones in the case of lead at about 200° C. abs., since there is an uncertainty in the value of $C_p - C_v$, due to lack of knowledge of the variation in the coefficient of compressibility with temperature. At 80° C. abs., the calculated value for this metal is also too low.

Only for the metals iron and sodium does the difference between the observed and the calculated values increase considerably with the temperature.

The divergence in the case of iron must, to some extent, be associated with its ferromagnetic properties.

From the work of WEISS and BECK* it appears that about 5 per cent. of the specific heat of iron at room temperature is due to the potential energy of demagnetization of an internal field which, according to the theory advanced by WEISS, exists in ferromagnetic bodies. The magnitude of this term increases rapidly in the neighbourhood of the temperature at which spontaneous magnetization disappears (about 750° C.), amounting there to nearly 40 per cent. of the total.

In the case of sodium, as the melting point is approached, molecular changes† take place and the theories of EINSTEIN and DEBYE are not sufficient.

(3) *Comparison of the Experimental Results with the Formulæ of EINSTEIN, NERNST and LINDEMANN and DEBYE, when the Frequencies Inserted give Coincidence at a Temperature of about 125° C. abs.*

If, instead of requiring the formulæ to fit the experimental curves at the lowest temperatures, we take as basis of comparison the range of temperature through which our investigation extends, we find that there is a closer approximation of the theoretical values to the experimental results, and it is notable that the various formulæ give results in fair agreement among themselves over this range.

In the comparison below, values of the frequencies have been chosen to bring the calculated values into coincidence with the experimental values at about 125° C. abs., which, on the average, was the lowest point to which we carried our experiments.

It will be observed that, generally, near the boiling-point of liquid hydrogen, EINSTEIN'S formula gives values which are too low; from NERNST and LINDEMANN'S the values are too high; while DEBYE'S formula gives values which are in fair agreement for Al, Ag, Zn, and Pb, and, in the other metals, agrees with the experimental values better than either EINSTEIN'S or NERNST and LINDEMANN'S. At liquid air temperatures all three formulæ give values which are too high.

* 'Journ. de Phys.,' col. 7, p. 249, 1908.

† A. E. OXLEY ('Proc. Cam. Phil. Soc.,' vol. XVII., p. 450, 1914), has suggested that a magnetic energy term similar to that of WEISS and BECK could explain abnormally high values of the specific heat near the melting point.

TABLE XVII.

Comparison of the Experimental Results with the Formulæ of EINSTEIN, NERNST and LINDEMANN and DEBYE.

Column I.—T, absolute temperature.

Column II.— C_p , from the smoothed curves previously referred to.

Column III.— C_p , calculated value from EINSTEIN's formula (E.).

Column IV.— C_p , calculated value from NERNST and LINDEMANN's formula (N. & L.).

Column V.— C_p , calculated value from DEBYE's formula (D.).

The values of $\beta\nu$ are given at the top of the columns.

T.	C_p (obs.).	C_p (E.).	C_p (N. and L.).	C_p (D.).
Aluminium.				
		$\beta\nu = 292.$	$\beta\nu = 385.$	$\beta\nu_m = 385.$
°				
35	0.33	0.12	0.37	0.35
80	2.27	2.18	2.46	2.35
140	4.26	4.26	4.26	4.28
200	5.14	5.12	5.10	5.10
250	5.53	5.49	5.47	5.47
300	5.81	5.75	5.74	5.74
380	6.13	6.02	6.01	6.01
Copper.				
		$\beta\nu = 222.$	$\beta\nu = 285.$	$\beta\nu_m = 286.$
°				
33.4	0.54	0.36	0.82	0.71
88	3.38	3.60	3.74	3.70
120	4.58	4.52	4.58	4.57
200	5.44	5.46	5.47	5.47
280	5.80	5.78	5.80	5.80
360	6.02	5.96	6.01	6.01
Zinc.				
		$\beta\nu = 160.$	$\beta\nu = 210.$	$\beta\nu_m = 210.$
°				
30	0.95	0.84	1.30	1.13
80	4.09	4.33	4.33	4.33
130	5.31	5.32	5.31	5.31
200	5.78	5.76	5.77	5.77
280	6.02	6.00	6.01	6.00
360	6.21	6.19	6.19	6.19

TABLE XVII. (continued).

T.	C_p (obs.).	C_p (E.).	C_p (N. and L.).	C_p (D.).
Silver.				
		$\beta\nu = 157.$	$\beta\nu = 207.$	$\beta\nu_m = 207.$
°				
35	1.58	1.40	1.80	1.64
85	4.42	4.56	4.53	4.53
120	5.20	5.23	5.21	5.21
200	5.84	5.73	5.78	5.78
280	6.01	6.01	6.01	6.01
360	6.16	6.16	6.16	6.15
Cadmium.				
		$\beta\nu = 112.$	$\beta\nu = 143.$	$\beta\nu_m = 144.$
°				
50	3.46	4.02	4.10	4.10
115	5.60	5.59	5.60	5.60
160	5.87	5.85	5.86	5.86
200	5.99	5.97	5.99	5.99
360	6.39	6.33	6.34	6.34
Lead.				
		$\beta\nu = 68.$	$\beta\nu = 92.$	$\beta\nu = 94.$
°				
23	2.96	3.03	3.06	2.94
80	5.72	5.75	5.65	5.64
120	5.93	5.93	5.91	5.91
200	6.10	6.13	6.13	6.13
280	6.28	6.28	6.28	6.28
360	6.45	6.45	6.45	6.45
Sodium.				
		$\beta\nu = 119.$	$\beta\nu = 152.$	$\beta\nu_m = 152.$
°				
50	3.50	3.87	3.98	3.95
120	5.62	5.62	5.64	5.64
200	6.17	6.02	6.04	6.02
320	6.78	6.36	6.36	6.36
360	7.32	6.43	6.43	6.43
Iron.				
		$\beta\nu = 286.$	$\beta\nu = 376.$	$\beta\nu_m = 376.$
°				
50	0.98	0.65	1.10	0.98
140	4.28	4.28	4.29	4.29
220	5.45	5.24	5.21	5.21
300	6.03	5.62	5.61	5.61
380	6.37	5.82	5.82	5.82

TABLE XVIII.

(4) *Comparison of Frequencies obtained by Calculation from the Physical Constants with the Values assumed in the above Tables.*

Frequencies $\times 10^{-12}$.

Metal.	EINSTEIN'S formulæ.		NERNST and LINDEMANN'S formulæ.		DEBYE'S formulæ.	
	Specific heat.	Calculated.	Specific heat.	Calculated.	Specific heat.	Calculated.
Al	6.1	6.7	8.0	5.8	8.0	8.3
Cu	4.6	5.7	5.9	5.1	5.9	6.8
Zn	3.3	3.7	4.4	3.3	4.4	6.5
Ag	3.3	4.1	4.3	3.3	4.3	4.4
Cd	2.3	2.7	3.0	2.1	3.0	3.5
Pb	1.4	1.9	1.9	1.4	1.9	1.5

The above comparison of the frequencies shows that the values obtained from the specific heat equations are in fair agreement with those calculated from the elastic constants and that the limit frequency ν_m of DEBYE is comparable with the monochromatic frequency of EINSTEIN.

It was found that no progressive change with temperature in the value of the frequency assumed would bring the results into better agreement in all cases.*

The work of ROSENHAIN and HUMFREY† and others give considerable support to the view that a cast metal consists of an agglomeration of crystals cemented together by an amorphous modification and that the elastic constants are intimately connected with this intercrystalline film. Now the glassy or amorphous forms of quartz, benzophenone, and betol possess higher specific heats than the corresponding crystalline forms.

The presence of an amorphous modification would make the problem a more complex one than is taken into consideration in the preceding theories, since a single value of the frequency or the limit frequency would not suffice for the representation of the thermal agitation of the two different forms.

It seems probable, however, that the proportion of an amorphous form present under ordinary conditions is very small.

* EUCKEN ('Verh. d. D. Phy. Ges.,' 15, p. 571, 1913) has calculated the probable frequency values at low temperatures, taking into account the change in the elastic constants with temperature, but the values so obtained were not in agreement with the atomic heat curves.

† 'Journ. Inst. Metals,' p. 653, Part II., 1913.

If the assumptions underlying the theoretical equations are valid, then we must conclude that other energy than that required for the three degrees of freedom ascribed to the atom is necessary to account for the atomic heat.

JEANS* has shown that if the atoms in a crystal do not rotate, then they must be kept in their direction by couples under whose influence they could carry out vibrations.

It is possible that at the higher temperatures the energy associated with these vibrations contributes to the specific heat, but hitherto no quantitative expression has been given which would enable this hypothesis to be verified.

(5) *Theories of Atomic Heat which are not Based on the Quantum Theory.*

CALLENDAR has proposed a different theory† of radiation; according to his view, equipartition of energy is not possible in full radiation, because the higher frequencies are being continually degraded by the Doppler effect in isothermal emission under equilibrium conditions. The formula‡ obtained for the representation of the atomic heat is applicable, at present, only to those substances possessing a single frequency and further hypotheses are necessary before it can be applied to the metals.

Prof. CALLENDAR has very kindly permitted us to include the following brief summary of his views in our paper.

CALLENDAR'S THEORY OF SPECIFIC HEAT.

“The total heat $E + PV$ per gramme atom of an isotropic substance, all the atoms of which have the same frequency ν , should be of the form $E + PV = 3RT(1+z)e^{-z}$ (where $z = b\nu/T$) when the substance is in equilibrium with full radiation in which the partial pressure p per unit range of frequency is given by RAYLEIGH'S formula,

$$p d\nu = C\nu^2 T e^{-z} d\nu.$$

In this case the latent heat of emission or absorption per unit range and volume (in order to agree with WIEN'S displacement law and the Doppler effect in isothermal emission), must be of the form ('Phil. Mag.,' May, 1914, p. 874),

$$T(dp/dT)_\nu = C\nu^2 T(1+z)e^{-z},$$

which agrees better with experiments on radiation than PLANCK'S formula, and has the advantage of being consistent with the classical thermodynamics.

* 'La Théorie du Rayonnement et les Quanta,' p. 63, 1912.

† 'Phil. Mag.,' May, 1914.

‡ 'Phil. Mag.,' October, 1913.

The atomic heat, *as measured experimentally* for a solid or liquid in equilibrium with full radiation at a negligible external pressure, is of the form (Curve I., dotted)

$$d(E+PV)/dT = 3R(1+z+z^2)e^{-z},$$

which gives good agreement in the case of rock salt and sylvin, when the *reststrahlen* frequencies are inserted, in the neighbourhood of the temperature given by $z = 2.732$, which corresponds to the maximum of the radiation curve plotted on a frequency base.

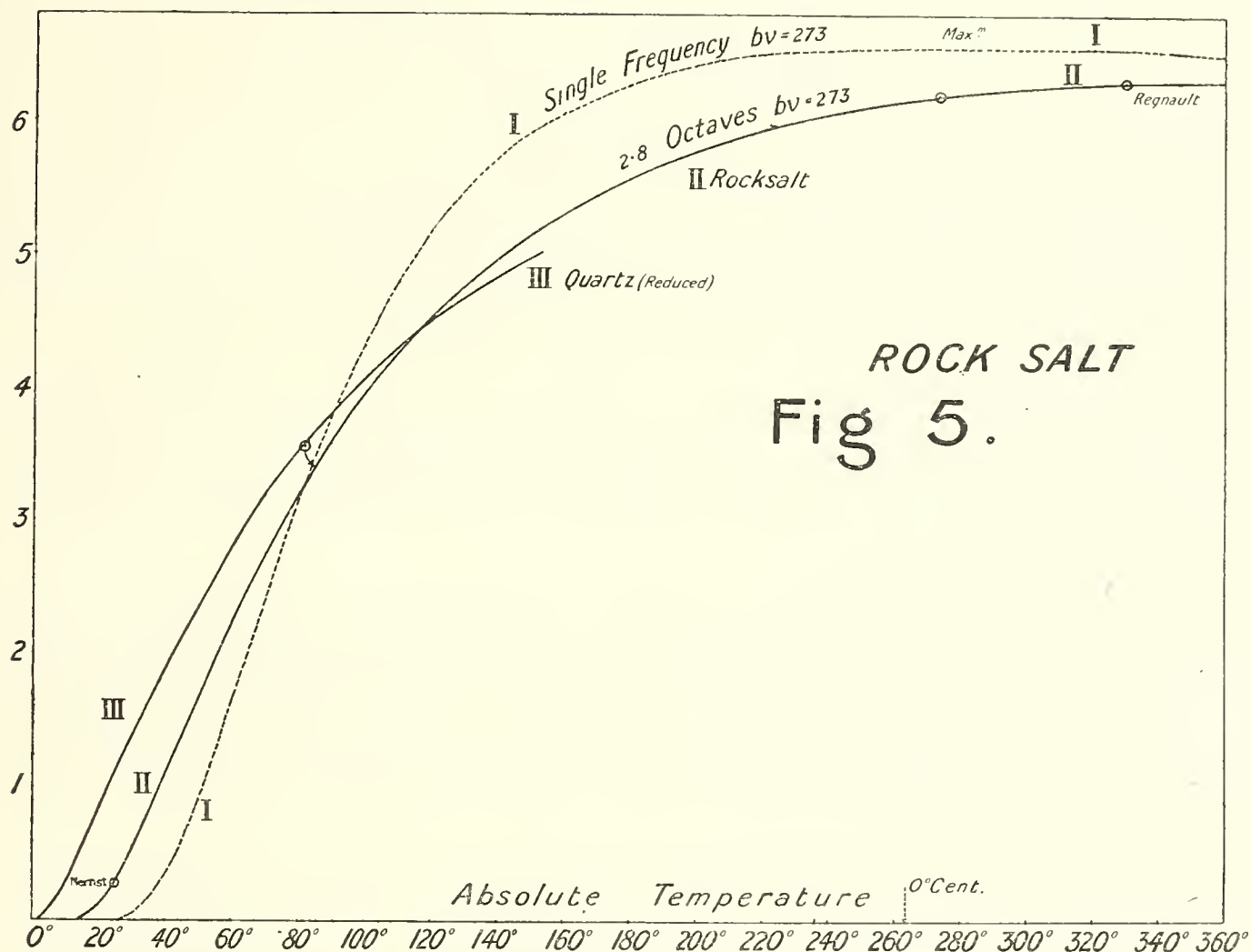
In the case of an actual solid, it is impossible that all the molecules should simultaneously have the same frequency. Observation shows that the absorption and emission bands widen considerably with increase of density, so that rock salt, for instance, is opaque over a region extending approximately from 20 to 130 microns, the mean wave-length of the *reststrahlen*, according to RUBENS, being 51.2 microns. Below a certain limit of frequency the substance again becomes transparent. It would appear that resonance generally extends with diminishing amplitude for an equal *interval* of frequency on either side of the mean, and may be represented by supposing that the number of molecules having frequencies included in a given interval is the same for equal intervals on either side of the natural frequency.

In the case of rock salt the probable distribution of frequencies can be approximately inferred from the absorption band and the dispersion formula, and is equivalent to a uniform distribution through a range of a little more than an octave on either side of the mean. The effect of this distribution on the specific heat is shown by the full Curve II., which represents the actual variation of the specific heat of rock salt as closely as can be expected from liquid hydrogen to the ordinary temperature. It will be seen that the agreement would be improved by adopting a slightly lower value for the mean frequency, and a smaller range of resonance. It is evident that the method employed by RUBENS must give too small a value for the wave-length if the resonance is not perfectly sharp, and it is possible that the value 55 microns deduced from the specific heat at $z = 2.732$ may be more accurate for the mean wave-length.

The simple theory above sketched could not be expected to apply except in the case of an isotropic substance possessing a single absorption band. The specific heats of quartz, ice, and benzol show a different type of curve, No. III., as shown for quartz, on a reduced scale of temperature (1/6). Such a curve can be fairly represented by assuming two absorption bands, one agreeing with the position indicated by the *reststrahlen*, wave-length 8.85 microns for quartz, and the other about two octaves lower. From our knowledge of absorption spectra it is extremely likely that most substances possess two or more bands of different intensities, and it is possible that the variation of specific heat could be consistently represented if we had all the required data. But a hypothetical selection of frequencies and intensities, though illustrating the possibility of the theory, would not serve as an experimental verification.

It seems fairly certain that the curves often differ in type, and cannot possibly be

represented by a single parameter (frequency or characteristic temperature) as in DEBYE's theory, even with the assistance of the correction from constant volume to constant pressure. In a special class, like the metals, the variations of type are not so great, but there is here endless room for speculation with regard to the appropriate frequencies, and if these fail, the magnetons and electrons can be invoked. There are, no doubt, relations between the specific heats and the optical, electrical, and mechanical properties of substances, but qualitative agreement may be obtained on so



many different and mutually contradictory hypotheses that very little stress can be laid on such agreement as a verification of any particular hypothesis unless every step in the argument can be proved and tested by independent experimental evidence."

WALKER,* in a note to a paper entitled "A Suggestion as to the Origin of Black Body Radiation," states that, by similar reasoning to that of EINSTEIN and NERNST, he found that if the radiation function varied as

$$\theta^5 \left(\frac{\lambda \theta}{\lambda^2 \theta^2 + \alpha^2} \right)^4$$

* 'Roy. Soc. Proc.,' A, vol. 89, p. 398, 1914.

the typical term in the expression for the atomic heat varies as

$$\left(\frac{\theta^3}{\theta^2 + \theta_0^2}\right)^4 \left\{1 + \frac{8\theta_0^2}{\theta^2 + \theta_0^2}\right\},$$

where θ_0 is some definite temperature.

We have been unable to find any value of θ_0 which would reconcile the locus of this curve with our data.

Summary.

1. In Part I. a description is given of a new method of maintaining an enclosure at any required temperature from that of the room down to about -165°C. , by a direct application of the Joule-Thomson cooling effect on expansion of air.

2. Specific heat determinations have been made at various points in the above range for Al, Cu, Zn, Ag, Cd, Pb, Fe, and Na, and data obtained for these metals which determine the *loci* of the atomic heat temperature curves from about 120°C. abs. to 400°C. abs.

3. It is shown that in the case of silver and lead there is no difference greater than the probable errors of experiment between the values obtained by NERNST and those given by the methods described in this paper. Hence, for Al, Cu, Zn, Ag, and Pb it is possible to extend the atomic heat curves by means of the data given by NERNST for these metals at the temperatures of liquid air and hydrogen.

4. The most probable values of the atomic heat at various temperatures are tabulated in Table XV., together with the calculated difference between C_p and C_v .

5. In Part II. a brief review is given of the various theoretical formulæ and the assumptions upon which they are based.

6. It is shown that none of the formulæ are capable of representing the experimental results over the entire range of temperature.

The formula of DEBYE appears to give the nearest approximation to the experimental values over the lowest portion of the range, but systematic divergences occur at the higher temperatures. This may be due either to the assumptions made by DEBYE being valid only at very low temperatures, or that additional heat energy to that involved in the three degrees of freedom, ascribed to the atom, must be taken into account.

7. Above liquid air temperatures the three formulæ above referred to are capable of approximately representing the results, but in some cases the discrepancies are considerable.

At these higher temperatures the difference between C_p and C_v is considerable, and for some of the metals the increase in C_v is small.

We take this opportunity of expressing our great indebtedness to Mr. D. A. THOMAS for his gift of a complete liquid air plant. It was through his generosity that we were able to continue the investigation to low temperatures.

Our thanks are due to the Court of the University of Wales for a contribution towards the expenses of this investigation, also to Mr. EDGAR A. GRIFFITHS, student in Engineering at this College, for constructing the special apparatus required and for his skilful management of the plant during the course of the work.



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SERIES A, VOL. 214. Pp. 359-371.

[PLATES 6-7.]

THE SPARK SPECTRUM OF NICKEL UNDER
MODERATE PRESSURES.

BY

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XI. *The Spark Spectrum of Nickel under Moderate Pressures.*

By E. G. BILHAM, *A.R.C.Sc., B.Sc., D.I.C., Research Student at the Imperial College of Science and Technology.*

Communicated by A. FOWLER, *F.R.S.*

Received July 9, 1914.

[PLATES 6-7.]

1. *Introductory.*

THE work of HUMPHREYS, DUFFIELD, GALE and ADAMS, ROSSI and others has shown that an increase of pressure round a source of light results in a general small displacement of the spectrum lines in the direction of longer wave-length, together with striking changes in the character of the lines themselves. Thus all lines have been found to broaden to a greater or less extent, either symmetrically or unsymmetrically, and many lines are reversed. In the present investigation the effect of pressures up to ten atmospheres above the normal atmospheric pressure, on the spark spectrum of nickel, has been studied with a view to obtaining accurate measurements of the displacements over that range of pressure which has been found to possess the greatest interest from an astrophysical standpoint. The chief problem which has to be considered in this connection is that of the pressure of the solar reversing layer. It is well known that wave-lengths of spectrum lines, determined from measurements in the solar absorption spectrum, exhibit small discrepancies when compared with the same lines derived from terrestrial sources. The solar wave-lengths are in general slightly greater than the terrestrial.

Ascribing the solar displacements to a difference of pressure, HUMPHREYS, MOHLER, and JEWELL* deduced a mean pressure of seven to eight atmospheres in the reversing layer. Later determinations of wave-lengths by FABRY and BUISSON† using the interferometric method have indicated a somewhat lower figure, namely five to six atmospheres.

* 'Astrophysical Journal,' III. (1896), p. 138.

† 'Astrophysical Journal,' XXXI. (1910), p. 97.

These conclusions have recently been strongly questioned by EVERSHED* who has deduced a zero pressure and ascribes the solar displacements to Döpler effects. The elucidation of the problem depends upon the comparison of the solar displacements with pressure displacements of comparable magnitude. It was felt, therefore, that a study of the effect of pressures up to about ten atmospheres above the normal was likely to yield the most useful results in relation to solar problems.

2. *Apparatus and Method.*

The Pressure-Chamber.—This was made from a T-shaped piece of steam-piping, 4 inches in external, and $2\frac{1}{2}$ inches in internal diameter. The three limbs terminated in flanges to which the covers were bolted, leather washers being interposed to make the joints airtight. One of the electrodes was fixed in position and well insulated by means of a cylindrical block of ebonite. The other electrode was movable so that the spark-gap could be adjusted to any desired distance, by turning a nut attached to the outside of the cylinder. A thick leather collar acted as a substitute for a stuffing box and worked very satisfactorily under the pressures employed.

The vertical limb of the T was closed by a cover having a glass window 1 inch in diameter. The cover could be easily removed and replaced, and was always removed while the exposures on the normal spark at atmospheric pressure were being made. The apparatus was further provided with a safety valve and pressure-gauge, and the pressure was applied by means of a cylinder of compressed nitrogen. The nickel rods were cut from sheet nickel, and were held in clips attached to the electrodes. The pressure-chamber was supported in a horizontal position on a wooden stand provided with levelling-screws.

Method of Excitation.—In order to obtain a spark of sufficient brilliancy to keep the exposures within reasonable limits, a Resonance Transformer, designed by Mr. A. EAGLE, was used. A motor-alternator supplies an alternating E.M.F. of 50 cycles per second to the primary coil, at any desired voltage up to 200, and produces a potential difference up to about 100,000 volts at the spark-gap. A large adjustable parallel-plate condenser is connected in parallel with the secondary. Since the movable electrode was in metallic connection with the case of the pressure-chamber and the gas-holder the corresponding secondary terminal was carefully earthed. In air at atmospheric pressure a very brilliant spark is produced with a spark-gap of two or three millimetres. Under a pressure of eleven atmospheres a gap of about one millimetre gave the best results. The behaviour of the nickel spark under pressure displayed some points of interest. Although such a small spark-gap was employed, the spark was exceedingly brilliant and noticeably whiter in colour than the normal spark. The electrodes were rapidly disintegrated, and although the design of the

* 'Kodaikanal Obs. Bulletin,' No. 36.

pressure-chamber was such that the window was seven inches from the spark-gap the window soon became clouded and had to be cleaned after each exposure. This effect was still more strongly marked when the nickel electrodes were replaced by others of a nickel-steel containing 25 per cent. of nickel.

The Spectrograph.—The 10 ft. 6 in. Rowland concave grating spectrograph in the Spectroscopic Laboratory of the Imperial College of Science and Technology was used for this work. The grating is mounted in the Littrow manner described by EAGLE.*

One of the greatest advantages of this method of mounting is that the whole apparatus is completely enclosed in a double-walled box, so that spurious shifts due to temperature variations are much less likely to give trouble than would be the case with the usual Rowland mounting. The importance of this feature in work involving the measurement of very small displacements will be sufficiently obvious.

The method of taking the photographs was as follows:—A horizontal spark was employed, as it was found that in this position the wandering of the spark seldom caused the image to leave the slit, thus effecting a considerable economy in the time of exposure. The astigmatism of the grating served to give sufficient length to the spectrum lines. In all the plates intended for measurement, the third order spectrum was employed, giving a linear dispersion of about $1.7 \text{ \AA.U.}$ per millimetre.

Each plate was exposed in three strips, the pressure spectrum being in the middle with a normal comparison spectrum just touching it on either side. One of the comparison spectra was photographed before the pressure spectrum and one after, so that temperature-shifts, if any, could be easily detected and allowed for. An iron arc spectrum was also photographed on the same plate for purposes of identification and for convenience in determining the dispersion-factor of the measuring apparatus.

The region investigated for purposes of measurement was included between $\lambda 3450$ and $\lambda 4600$, the exposures ranging from 5 to 20 minutes on Lumière ultra-rapid plates. The changes in the character of the lines have been investigated as far as $\lambda 6100$ in the first order.

3. *The Measuring Apparatus.*

A Hilger measuring-machine, of the usual type in which the microscope is driven forward by means of a finely cut screw, was employed. The ordinary eyepiece was, however, replaced by one in which the cross-wires could be made to travel across the field of view by means of a micrometer screw.

The arrangement of cross-wires is shown in fig. 1. Three parallel wires, a , b_1 , b_2 are attached to the sliding frame, the two latter being at such a distance apart that the image of a line of average intensity nearly fills the space between them. The third wire a is about four times as far away and is only used when measurements are

* 'Astrophysical Journal,' XXXI. (1910), p. 120.

being made on lines which reverse under pressure. The head of the micrometer screw is divided into 60 divisions, and whole turns are read off on the comb d .

In measuring displacements the following procedure was adopted. The micrometer was first adjusted so that the wires a , b_1 , b_2 were at right angles to the direction of travel of the microscope. The plate was then placed on the stage of the measuring machine so that the line of separation of the pressure spectrum and one of the normal comparison spectra bisected the field of view, and so that the spectrum lines were parallel to the cross-wires. Having brought the line to be measured into the middle of the field of view, the final settings were made by means of the eye-piece micrometer. The normal and displaced lines were made to bisect the space between the wires b_1 and b_2 alternately, six settings being made on each. In the case of reversed lines greater precision is obtained by using a single thread. Consequently,

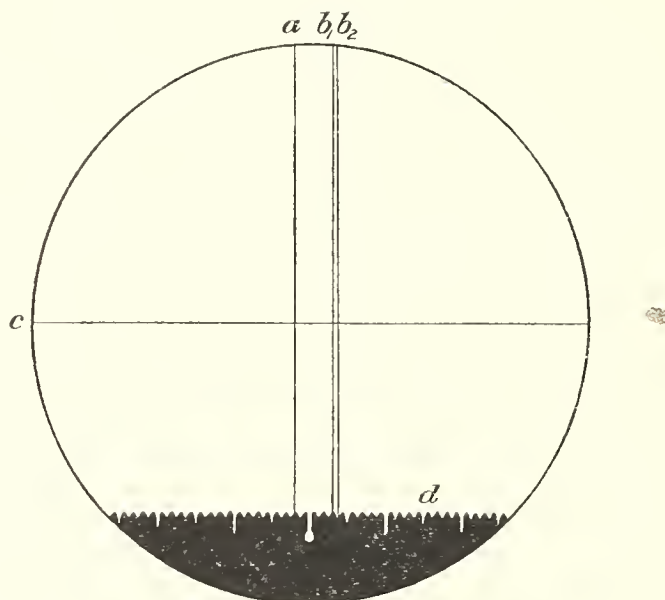


Fig. 1.

in such cases, the wire a was brought into requisition, the normal line being measured as before. A constant difference is, of course, introduced corresponding to the distance in scale divisions between a and a line midway between b_1 and b_2 .

Determinations were made using each comparison spectrum and measuring in both directions. Four sets of readings were thus obtained from each plate. The mean displacement determined in this way had to be multiplied by a dispersion-factor to convert scale divisions into Ångström units. The factor was obtained by measuring the separation of suitable lines in the iron arc spectrum on the same plate, using Dr. K. BURNS' values for the wave-lengths. One division of the micrometer head corresponded to about 0.0027 Å.U. , the factor varying slightly in different parts of the spectrum, being smaller towards the red. Readings were taken to tenths of a division, but the final results are only expressed to 0.001 Å.U. In the case of good lines the error is probably not more than two units in the third place of decimals.

It will be seen that the constant difference, say D , referred to above, does not affect the final result. Thus if d_1, d_2, d_3, d_4 are the means of the differences between the settings for the normal and displaced lines in the four positions, the corresponding shifts are given by

$$\left. \begin{aligned} s_1 &= d_1 - D \\ s_2 &= d_2 - D \end{aligned} \right\} \text{plate direct,}$$

$$\left. \begin{aligned} s_3 &= D - d_3 \\ s_4 &= D - d_4 \end{aligned} \right\} \text{plate reversed,}$$

whence the mean shift

$$= \frac{s_1 + s_2 + s_3 + s_4}{4} = \frac{1}{4} \{ (d_1 + d_2) - (d_3 + d_4) \}.$$

The value of D need not therefore be determined.

4. *Changes in Character of the Lines.*

One of the most obvious results of the increase of pressure is the very striking alteration which takes place in the character of the lines. A great variety of behaviour is to be observed, and it has been found convenient to divide the lines into five groups following the classification of GALE and ADAMS* :—

Class I.—Lines which reverse symmetrically.

Class II.—Lines which reverse unsymmetrically.

Class III.—Lines which remain bright and fairly narrow.

Class IV.—Lines which remain bright but are very much broadened symmetrically.

Class V.—Lines which become very much broadened unsymmetrically towards the red.

All the stronger arc lines of wave-length shorter than λ 3900 are included in classes I. and II., the majority being unsymmetrically reversed. In all cases in which the reversal is unsymmetrical, the reversal lies on the violet side of the middle of the emission line, at least so far as nickel lines are concerned. The same conclusion has been reached by DUFFIELD in the case of the iron arc. The rule does not appear to be universally true, however, for the K and H lines of calcium which appear on some plates as impurity lines, are unsymmetrically reversed in the opposite sense. The enhanced lines of nickel all belong to class IV. and exhibit no tendency to reverse.

* 'Astrophysical Journal,' XXXV. (1912), p. 15.

For most of the lines of wave-length shorter than λ 3600 it is impossible to tell from examination of the eleven-atmosphere plates whether they belong to classes I. or II. The appearance of these lines under pressure is such that this region looks like a continuous spectrum crossed by absorption lines. On the six-atmosphere plates, however, the emission lines are visible.

Class V. displays the most striking features. The lines of this type become broad bands under pressure, with wings extending considerable distances towards the red. A width of ten to twenty Å.U. is not uncommon. A large proportion of the lines of wave-length greater than λ 4400 behave in this way, so that a curious fluted appearance is imparted to the spectrum. On some plates the strongest lines of class V. are faintly reversed, so that they qualify for class II. The appearance is, however, radically different from the ordinary members of that class.

In addition to the five well-defined classes described above, there is some indication of a sixth, analogous to the fifth, except that the winging extends towards the violet. The lines 4752.58 and 5035.55 appear to behave in this way.

5. *Modifications in Intensity.*

The radical changes in the appearance of the lines, produced by increase of pressure, make it a difficult matter to draw any conclusions with regard to changes of intensity. In the case of the stronger lines there seems to be very little alteration. An examination of the fainter lines, however, has yielded the following conclusions :—

(1) Enhanced lines are decreased in intensity. In some cases very little trace of the enhanced line is to be seen in the pressure-spectrum. This is particularly noticeable for a group of enhanced manganese impurity lines in the region λ 3400 to λ 3560. The strong nickel enhanced lines 3576.91 and 3769.62, however, are not greatly affected.

(2) Lines which are relatively stronger in the arc than in the spark are increased in intensity.

(3) Lines due to oxygen and nitrogen are completely eliminated. These results bear a striking analogy to the effects of including self-induction in an ordinary spark circuit. The general conclusion is that in the spark under pressure we get an approach to the arc condition.

The following lines are also greatly reduced under pressure : 3668.35, 3744.75, 4437.17, 4437.75. The line 4520.16 is notably strengthened.

6. *Results of Measurements.*

Two series of measurements have been made upon plates taken under total pressures of eleven and six atmospheres respectively. Those at the higher pressure are probably the more reliable. Since the absolute wave-lengths of the lines are not concerned in this work the numbers quoted are taken from the tables published by EXNER and HASCHEK, and HASSELBERG, and merely serve for purposes of identification. The wave-lengths are referred to ROWLAND'S system.

Enhanced lines are indicated by the letter E.

TABLE I.

Wave-length.	Intensity.	Class.	+ 10 atmospheres.		+ 5 atmospheres.	
			$\frac{\Delta\lambda}{\text{in } \frac{1}{1000} \text{ \AA.U.}}$	No. of plates.	$\frac{\Delta\lambda}{\text{in } \frac{1}{1000} \text{ \AA.U.}}$	No. of plates.
3446.41	6	I.	17	1	—	—
3453.06	4	I.	13	1	—	—
3454.29 E.	2	IV.	15	1	—	—
3458.62	10	I.	18	1	—	—
3461.84	10	I.	25	1	—	—
3472.71	6	II.	14	1	—	—
3483.95	6	II.	15	1	—	—
3493.13	10	I.	15	1	—	—
3501.01	3	II.	15	1	—	—
3510.52	8	I.	22	1	17	1
3515.21	10	I.	27	1	14	1
3519.90	3	II.	19	1	10	1
3524.69	15	I.	30	1	14	1
3528.10	3	II.	18	1	—	—
3548.32	3	II.	21	2	25	1
3561.92	2	I.	11	2	—	—
3566.55	10	I.	20	2	15	1
3572.06	6	II.	18	2	12	1
3576.91 E.	6	IV.	82	1	51	1
3588.07	2	II.	14	2	—	—
3597.86	6	II.	21	3	—	—
3602.44	2	II.	16	3	—	—
3609.49	2	I.	16	3	—	—
3610.68	4	II.	26	3	13	1
3612.91	3	II.	16	3	14	1
3619.54	15	I.	18	2	13	1
3624.89	2	I.	7	3	—	—
3635.07	1	III.	51	2	—	—
3662.11	1	III.	35	2	—	—
3664.26	3	II.	22	3	18	1
3669.39	1	II.	12	3	—	—
3670.59	2	II.	25	3	—	—
3674.29	3	II.	19	2	13	2
3688.57	2	II.	20	3	—	—
3694.07	2	III.	25	3	—	—
3722.62	3	II.	29	3	—	—
3736.96	3	II.	10	3	—	—
3739.38	2	I.	10	2	—	—

TABLE I (continued).

Wave-length.	Intensity.	Class.	+ 10 atmospheres.		+ 5 atmospheres.	
			$\frac{\Delta\lambda}{1000}$ Å.U.	No. of plates.	$\frac{\Delta\lambda}{1000}$ Å.U.	No. of plates.
3769·62 E.	5	IV.	77	3	55	2
3775·74	5	II.	22	3	12	2
3783·67	5	I.	16	3	12	2
3807·29	7	II.	18	3	9	2
3831·87	2	I.	36	2	—	—
3849·70 E.	5	IV.	149	1	—	—
3858·50	8	I.	21	3	12	2
3889·80	3	IV.	—	—	—	—
3972·32	2	III.	28	1	28	1
3973·75	3	V.	58	1	82	1
4015·65	3	IV.	—	—	91	1
4121·48 ? Co.	2	III.	26	1	48	1
4142·47	1	III.	32	1	—	—
4288·20	5	V.	121	1	—	—
4331·83	4	III.	44	2	—	—
4359·76	2	III.	106	2	—	—
4401·77	6	V.	102	2	—	—
4459·25	6	V.	93	2	—	—
4462·65	3	V.	99	2	—	—
4470·70	6	V.	89	2	—	—
4592·76	5	V.	118	1	—	—

7. Comparison with Previous Measurements.

HUMPHREYS* has measured the displacements of ten nickel lines in the arc spectrum at pressures of $9\frac{3}{4}$, $12\frac{1}{2}$, and $14\frac{1}{2}$ atmospheres. The mean shifts per atmosphere of those lines which have also been measured at South Kensington have been calculated. The results are given in Table II.

TABLE II.

Wave-length.	Mean shift per atmosphere in $\frac{1}{1000}$ Å.U.	
	HUMPHREYS (Arc).	BILHAM (Spark).
3458·62	2·08	1·80
3461·84	1·74	2·50
3501·01	2·17	1·50
3515·21	2·67	2·75
3524·69	2·40	2·90
Means . .	2·21	2·29

* 'Astrophysical Journal,' VI., p. 204.

A few iron lines occurring as impurity in the nickel have been measured with a view to comparing the shifts with the determinations of GALE and ADAMS, and of DUFFIELD.

The following results (Table III.) have been obtained for six lines particularly suitable for measurement.

TABLE III.

Wave-length.	Mean shift per atmosphere in $\frac{1}{1000}$ Å.U.		
	GALE and ADAMS (Arc).	DUFFIELD (Arc).	BILHAM (Spark).
4045.98	2.9	3.0	3.6
4063.76	2.5	2.5	3.2
4071.91	2.6	2.6	2.3
4308.08	2.6	2.1	4.3
4325.94	2.5	2.3	3.9
4383.72	3.4	2.3	4.2
Means . .	2.8	2.5	3.6

The mean of all DUFFIELD's values, set B,* has been taken, assuming proportionality of displacement to increase of pressure. The values in the second column have been obtained from GALE and ADAMS' main series of measurements at nine atmospheres.†

It will be observed that the values given in the last column are, on the average, higher than the other two. This is to be expected from the fact that all the lines referred to are reversed in the arc under eleven-atmospheres pressure, but bright in the spark under the present experimental conditions. It has been found‡ that if a line is reversed at a given pressure its shift is very much smaller than when it is unreversed.

8. Classification of Shifts.

As the measured lines belonging to each class are mostly comprised between somewhat narrow limits of wave-length, no attempt has been made to investigate the law of variation of displacement with wave-length. Consequently the shifts have not been reduced to a standard wave-length, which is desirable in discussing their relative magnitudes.

Some light is, however, thrown on this subject by averaging the shifts of lines belonging to the same class. The results are given in Table IV. The line 4121.48, doubtfully due to nickel, has been omitted in making the calculation.

* 'Phil. Trans.,' A, vol. 208, p. 138.

† *Loc. cit.*, p. 17.

‡ DUFFIELD, *loc. cit.*, p. 155.

TABLE IV.

Class.	Number of lines.	Average shift in $\frac{1}{1000}$ Å.U. (11 atmospheres).	Ratios.
I.	17	18·9	1
II.	21	18·6	1
III.	7	45·9	2·4
IV.	4	80·8	4·3
V.	7	97·1	5·1

There appears to be some evidence that symmetrical and unsymmetrical reversed lines give the same average shifts. The ratios of the average shifts for the different classes are of interest in connection with the fact, first noted by HUMPHREYS, that for some metals the lines tended to form three groups in which the ratios of the pressure-shifts were approximately 1 : 2 : 4, when reduced to a standard wave-length. Later work has confirmed HUMPHREYS' view to a certain extent, notably in the case of the iron arc. The ratios, however, do not appear to be integral, and the grouping according to pressure-shifts is, to a great extent, independent of the classification according to behaviour. Under these circumstances it seems doubtful whether the phenomenon is anything more than a chance arrangement such as would follow from considering "small," "average," and "large" shifts. On the other hand, the relative shifts of lines whose behaviour justifies the supposition that they are of radically different types is a question of more direct interest, and the results are more likely to be capable of theoretical interpretation. Following out this view, and retaining our five original classes, we get the ratios in the last column of Table IV. Adopting a more general grouping by combining classes I. and II., IV. and V., we arrive at three main divisions, viz. :—

- (a) Reversed lines 38 lines
- (b) Unreversed lines (narrow) 7 „
- (c) Unreversed lines (broad) 11 „

giving the displacement ratios,

$$\begin{array}{ccc} a. & b. & c. \\ 1 & : & 2\cdot4 & : & 4\cdot6. \end{array}$$

If we neglect the enhanced lines and consider only arc lines we obtain

$$\begin{array}{ccc} a. & b. & c. \\ 1 & : & 2\cdot4 & : & 5\cdot1 \end{array}$$

9. *The Apparent Violet Shift of the Line 3514.*

An interesting case is furnished by the line λ 3514·10. At eleven-atmospheres pressure this line appears as a very narrow reversal superimposed upon a broad bright

line (see Plate 7, fig. 2c). Under the microscope the reversal shows a pronounced displacement towards the violet, amounting to $0\cdot023 \text{ \AA.U.}$ at eleven atmospheres and $0\cdot035 \text{ \AA.U.}$ at six atmospheres. As this was the only violet shift observed, further investigation seemed desirable. It was found that the spark line showed a displacement of $0\cdot035 \text{ \AA.U.}$ towards the red with respect to the corresponding line in the arc (Plate 7, fig. 1). This fact, and the anomalous behaviour under pressure, may be co-ordinated if we suppose that in the spark spectrum the line is really a doublet consisting of the arc line $\lambda 3514\cdot10$ together with an enhanced line, the mean wave-length of the doublet being $3514\cdot10$ *plus* $0\cdot035$. Under pressure, we should expect the arc line to reverse and the enhanced line to broaden without reversal, a conclusion in entire agreement with the appearance actually observed. Moreover if the pressure displacement of the arc line was less than $0\cdot035 \text{ \AA.U.}$, the reversal should exhibit a violet displacement with respect to the normal spark line. On this theory the arc line behaves in a perfectly normal fashion, giving a red displacement of $0\cdot035 - \cdot023$, *i.e.*, $0\cdot012 \text{ \AA.U.}$ at eleven atmospheres.

10. *Abnormal Shift of Line 3609.*

A somewhat similar example is furnished by a line of wave-length $3609\cdot02$ in the arc (possibly due to iron). This line gives a very narrow reversal under pressure which shows an abnormally large shift ($0\cdot055 \text{ \AA.U.}$ at eleven atmospheres) towards the red. (Plate 7, fig. 4.)

A comparison of the arc and spark spectra (Plate 7, fig. 3) shows that the spark line is displaced towards the violet by $0\cdot040 \text{ \AA.U.}$ with respect to the arc line.

Assuming as before that the spark line is really a doublet, the more refrangible component of which is, in this case, an enhanced line we get a true shift of $0\cdot015$ for the arc line, which is of about the right order.

11. *Qualitative Observations.*

A large number of lines have been investigated qualitatively with a view to observing their behaviour under pressure. In some cases, where the lines are very close together, such as in the doublet $4732\cdot00$ and $4732\cdot66$, it is impossible to examine the behaviour of the individual lines. Such groups have therefore been bracketed together and treated collectively.

TABLE V.

Wave-length.	Intensity.	Class.	Wave-length.	Intensity.	Class.
3467·63	2	II.	5035·55	7	[VI.]
3469·61	2	II.	5080·70	5	} IV.
3471·50 E.	2	IV.	5081·30	5	
4067·20 E.	6	IV.	5099·50	3	
4520·16	2	III.	5100·13	3	V.
4547·16	2	V.	5115·55	6	V.
4600·58	3	IV.	5137·23	4	III.
4605·20	6	V.	5142·96	2	IV.
4648·89	8	V.	5146·64	4	IV.
4667·95	6	V.	5155·92	4	IV.
4686·41	4	III.	5168·83	4	IV.
4714·59	10	V.	5265·89	1	V.
4732·00	3	} V.	5268·59	2	V.
4732·66	3		5371·64	4	V.
4752·58	3	[VI.]*	5411·50	2	IV.
4756·70	8	V.	5424·85	4	III.
4764·07	5	V.	5436·10	2	III.
4786·66	9	V.	5477·13	10	II.
4807·17	6	V.	5510·28	3	[VI.]?
4829·18	3	} V.	5578·98	2	III.
4831·30	3		5588·12	2	III.
4832·86	1		5592·44	4	V.
4855·57	6	IV.	5615·00	3	V.
4866·42	6	V.	5709·80	3	III.
4873·60	6	V.	5712·10	2	III.
4887·16	2	V.	5715·31	5	V.
4904·56	6	IV.	5748·57	1	III.
4918·53	6	V.	5754·86	4	III.
4936·02	4	V.	5761·10	3	V.
4953·34	4	V.	5805·45	2	IV.
4980·36	6	IV.	5858·03	3	IV.
4984·30	6	IV.	5893·13	3	III.
5017·75	6	V.	6116·34	4	III.

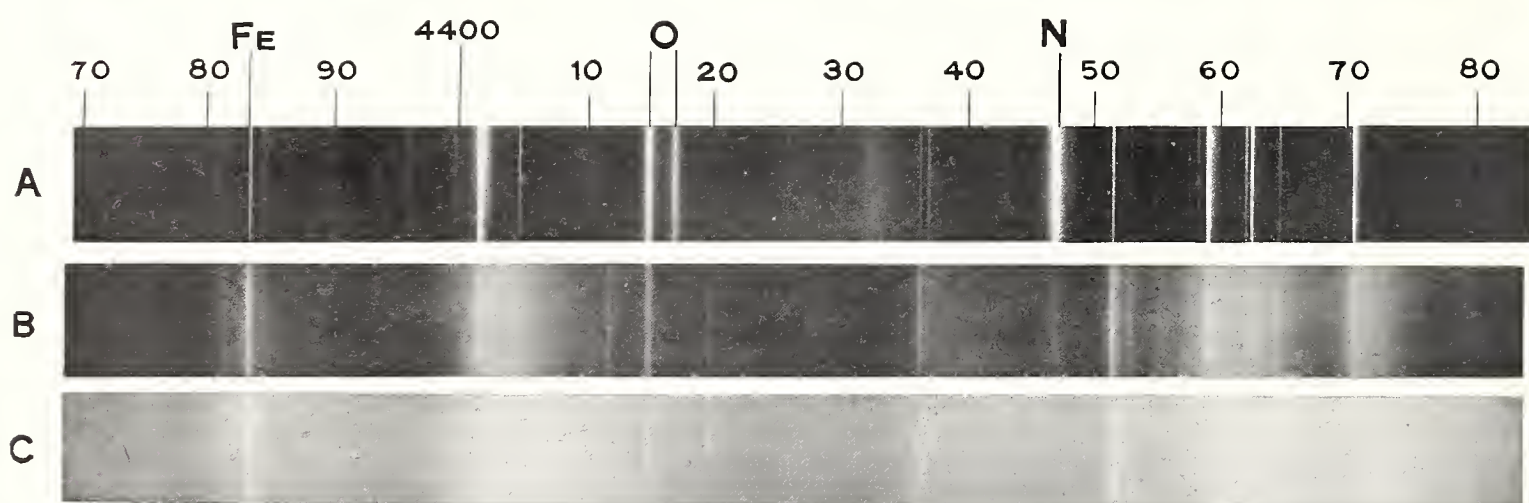
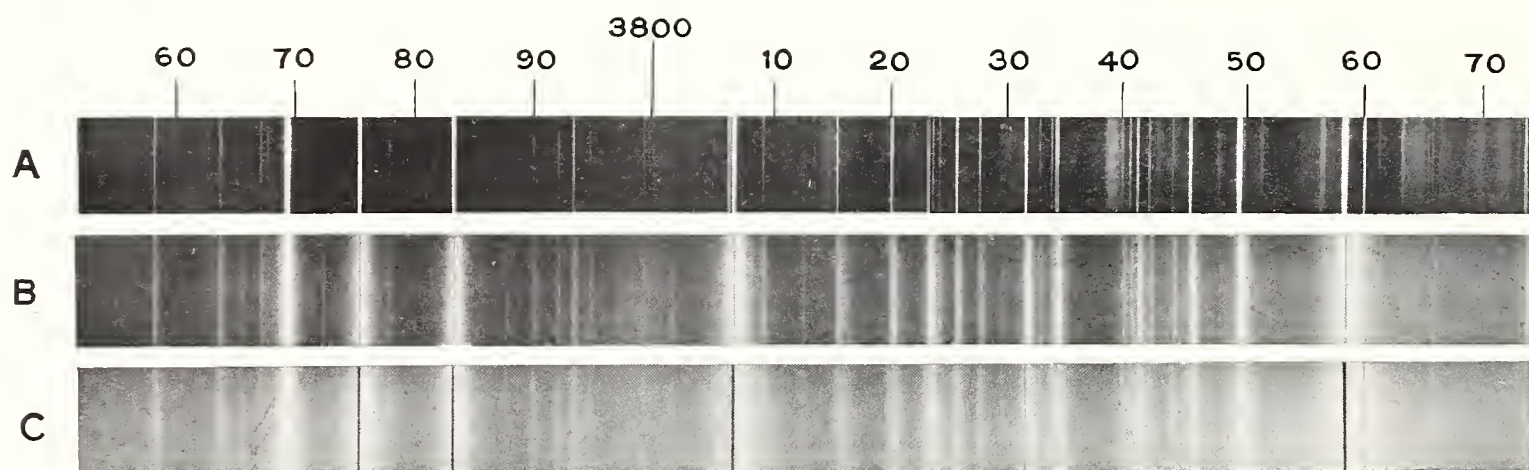
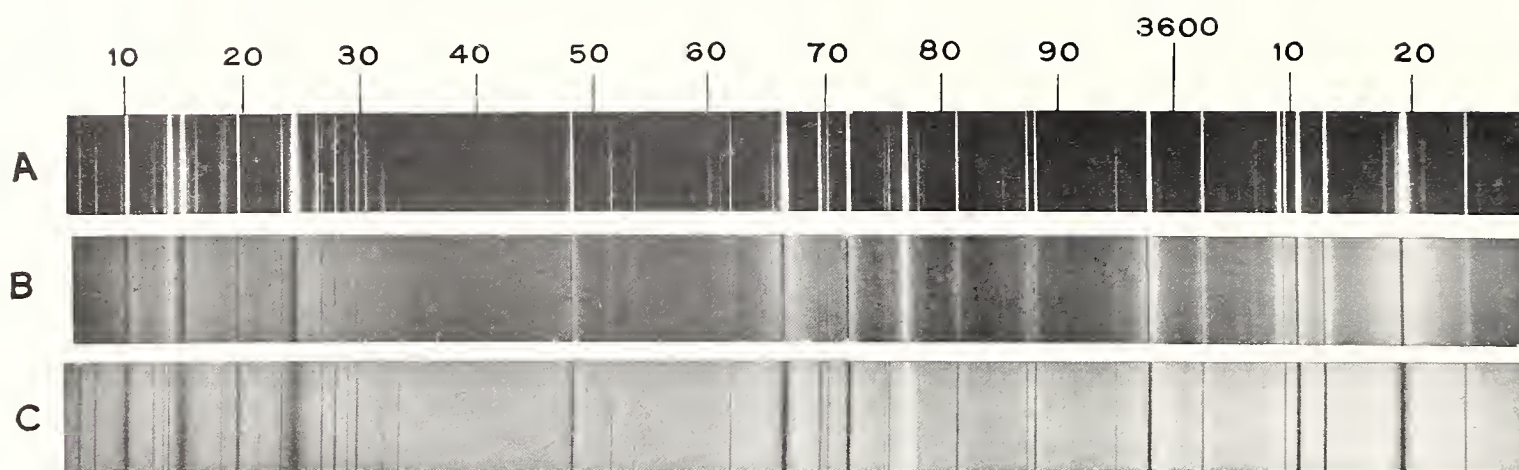
* Shaded to violet.

12. *Summary.*

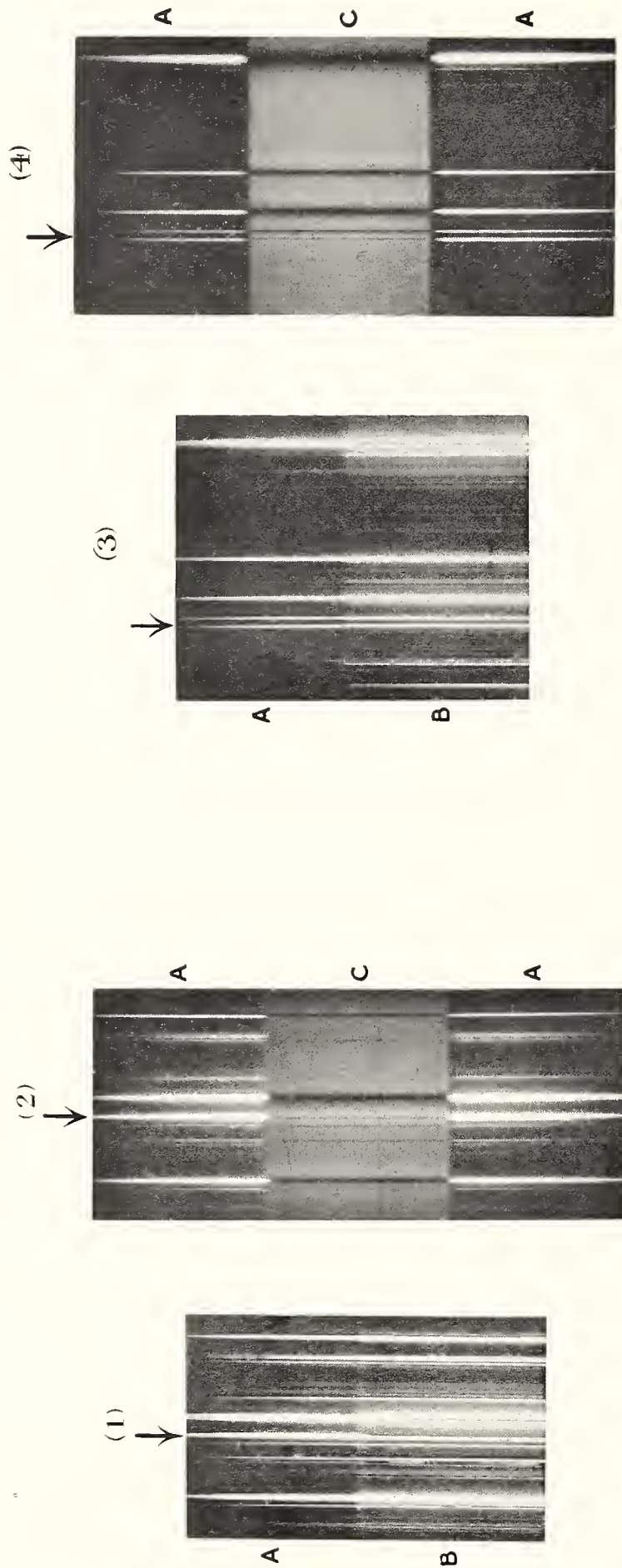
1. The nickel spark spectrum has been investigated under pressures up to eleven atmospheres.
2. The lines exhibit a remarkable variety of behaviour and may be divided into five classes, according to their types of reversal or broadening.
3. With increase of pressure the enhanced lines decrease in intensity and broaden symmetrically.
4. Gas lines disappear under pressure.
5. The general effect of pressure on the relative intensities of the lines is similar to that of including self-induction in a spark circuit.
6. All lines are displaced towards the red end of the spectrum.
7. The average shifts are the same for symmetrical and unsymmetrical reversals.



NICKEL SPARK UNDER PRESSURE



A, PRESSURE 1 ATMOSPHERE,
 B, PRESSURE 6 ATMOSPHERES,
 C, PRESSURE 11 ATMOSPHERES.



ABNORMAL BEHAVIOUR OF LINES λ 3514 AND λ 3609 IN NICKEL SPARK UNDER PRESSURE.

8. The shifts are larger for unreversed than for reversed lines and are greatest for lines broadening unsymmetrically towards the red.

9. Certain abnormalities in the behaviour under pressure indicate the existence of enhanced lines, having wave-lengths 3514·14 and 3608·98 (approximately) almost coincident with two nickel arc lines.

In conclusion I wish to express my gratitude to Prof. A. FOWLER for his helpful interest in this research, and also to Mr. COLEBROOK, under whose supervision almost the whole of the apparatus was constructed in the workshops of the Imperial College.

DESCRIPTION OF PLATES.

Plate 6.

Changes in character of lines in the spark spectrum of nickel under pressure—

- A. Pressure, 1 atmosphere.
- B. „ 6 atmospheres.
- C. „ 11 „

Plate 7.

Apparent violet displacement of λ 3514—

- | | |
|-------------------------------|------------------------------|
| (1) Spark and arc comparison. | (2) Spark under pressure. |
| A. Spark. | A. Normal spark. |
| B. Arc. | C. Pressure, 11 atmospheres. |

Abnormal shift of λ 3609.

- | | |
|-------------------------------|------------------------------|
| (3) Spark and arc comparison. | (4) Spark under pressure. |
| A. Spark. | A. Normal spark. |
| B. Arc. | C. Pressure, 11 atmospheres. |

(Scale of reproduction :—1 mm. = 0·43 Å.U.)



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[PLATE 8.]

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PART I.

MATHEMATICAL THEORY OF THE CONVECTION OF HEAT FROM A CYLINDER OF ANY FORM OF CROSS-SECTION IN A STREAM OF FLUID

Section 1. Introduction.

THE general problem of the convection of heat from bodies immersed in moving media has recently received considerable attention both from the theoretical and experimental point of view. The equation of the conduction of heat in a moving fluid was stated by FOURIER as long ago as 1820,⁽¹⁾ and a few years later was expressed by POISSON⁽²⁾ and OSTROGRADSKY⁽³⁾ in the familiar form

$$c \frac{D\theta}{Dt} = \frac{\partial}{\partial x} \left(\kappa \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial y} \left(\kappa \frac{\partial \theta}{\partial y} \right) + \frac{\partial}{\partial z} \left(\kappa \frac{\partial \theta}{\partial z} \right), \quad \dots \quad (1)$$

where θ is the temperature of the fluid at any point (x, y, z) , c the heat capacity of the fluid per unit volume, κ its thermal conductivity, and D/Dt the "mobile operator" $D/Dt = \partial/\partial t + u \partial/\partial x + v \partial/\partial y + w \partial/\partial z$ of the hydrodynamical equations.

In 1901 the problem was taken up by BOUSSINESQ,⁽⁴⁾ whose memoir on the subject in 1905 contains a great number of successful calculations of heat losses from bodies of various shapes immersed in a stream of fluid.

A full account of the theoretical development of the subject is given by RUSSELL,⁽⁵⁾ and extensive references are given to papers and memoirs relating to the convection of heat.

Section 2. BOUSSINESQ'S Transformation.

Under certain assumptions BOUSSINESQ, by an extremely elegant transformation, was able to reduce (1) to a differential equation capable of solution. Assuming a frictionless, incompressible fluid, the flow of liquid past an obstacle maps out the field in the neighbourhood of an obstacle by stream lines and equipotential surfaces which in some cases may constitute a set of orthogonal co-ordinates. If in such cases the general equation of heat conduction (1) be expressed in these co-ordinates, it takes a

⁽¹⁾ FOURIER, 'Mémoires de l'Académie,' t. 12, p. 507, 1820.

⁽²⁾ POISSON, 'Théorie Mathématique de la Chaleur,' 1835.

⁽³⁾ OSTROGRADSKY, 'St. Pét. Ac. Sc. Bll.,' t. 1, p. 25, 1836.

⁽⁴⁾ BOUSSINESQ, 'Comptes Rendus,' vol. 133, p. 257; also 'Journal de Mathématiques,' vol. 1, pp. 285-332, 1905.

⁽⁵⁾ RUSSELL, 'Phil. Mag.,' vol. 20, pp. 591-610, October, 1910.

greatly simplified form, reducing in many cases to one or other of the known partial differential equations of mathematical physics. The case of two-dimensional flow lends itself especially well to such an investigation as well as to a comparison with the results of experiment. If $\alpha = \text{constant}$ represent the stream lines and $\beta = \text{constant}$ the equipotentials obtained from the solution of the hydrodynamical problem of flow past a cylinder of any form of cross-section (fig. 1), BOUSSINESQ first showed that the general equation could be transformed to the linear form

$$\frac{\partial^2 \theta}{\partial \alpha^2} + \frac{\partial^2 \theta}{\partial \beta^2} = 2n \frac{\partial \theta}{\partial \beta}, \quad \dots \dots \dots (2)$$

where θ is the temperature at any point of the liquid, κ its heat conductivity supposed to be independent of the temperature and therefore constant throughout the liquid,

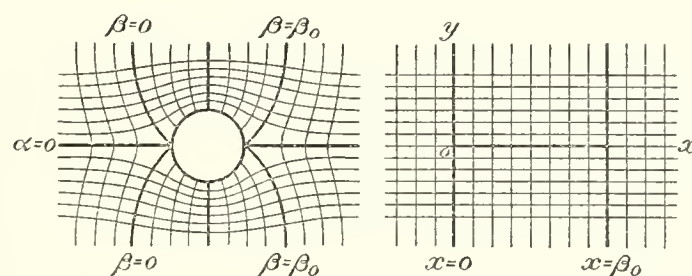


Fig. 1. BOUSSINESQ's transformation.

c is the specific heat per unit volume, and V the velocity of the stream at a great distance from the cylinder. The constant n is defined by the relation

$$2n = cV/\kappa = s\sigma V/\kappa, \quad \dots \dots \dots (3)$$

where s is the specific heat per unit mass of the fluid and σ its density.

The complete solution of (2) requires a knowledge of the conditions of heat-transfer over the interface between solid and liquid, a point which can only be settled by referring to the results of experiment. If the surface of the cylinder be the particular stream-line $\alpha = 0$, and the critical equipotentials be the curves $\beta = 0$ and $\beta = \beta_0$, the heat-flux per unit length of the cylinder is given by

$$H = - \int_0^{\beta_0} \kappa (\partial \theta / \partial \alpha)_0 d\beta, \quad \dots \dots \dots (4)$$

where the integral is taken to include both branches of the stream-line $\alpha = 0$.

It will be noticed that BOUSSINESQ's transformation reduces the problem to the simple case of calculating the temperature distribution in a uniform stream flowing parallel to the axis of x , ($\alpha = 0$), when the distribution of temperature or heat-flux is prescribed over the interval $x = 0$ to $x = \beta_0$. A solution of this simple *transformed*

of which a particular solution appropriate to a line source at the origin is seen, on transforming (6) to cylindrical co-ordinates, to be ⁽⁷⁾

$$\phi = AK_0(nr), \quad \text{where } r = |\sqrt{(x^2 + y^2)}|, \quad x = r \cos \theta, \quad y = r \sin \theta, \quad . \quad (7)$$

A is a constant of integration and $K_0(z)$ that solution of BESSEL'S equation most conveniently defined by the definite integral

$$K_0(z) = \int_0^\infty e^{-z \cosh \phi} d\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

We write down for future reference the expansions

$$K_0(z) = e^{-z} \sqrt{(\pi/2z)} \left[1 - \frac{1}{8z} + \frac{1^2 \cdot 3^2}{(8z)^2 \cdot 2!} - \dots \right], \quad . \quad . \quad . \quad . \quad . \quad (9)$$

when z is large. When z is small we may make use of the expansions

$$K_0(z) = -I_0(z) [\gamma + \log(z/2)] + \frac{z^2}{2^2} + \frac{z^4}{2^2 \cdot 4^2} (1 + \frac{1}{2}) + \frac{z^6}{2^2 \cdot 4^2 \cdot 6^2} (1 + \frac{1}{2} + \frac{1}{3}) + \dots, \quad (10)$$

where

$$I_0(z) = \pi^{-1} \int_0^\pi \cosh(z \cos \phi) d\phi = 1 + \frac{z^2}{2^2} + \frac{z^4}{2^2 \cdot 4^2} + \frac{z^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots, \quad . \quad . \quad (11)$$

and γ is EULER'S constant, $\gamma = 0.57721$.

If we denote by Q the rate at which heat is being supplied to the line source per unit length, Q should also be equal to the total flux calculated by integrating around any closed circuit enclosing the source; that is,

$$Q = \int_c [-\kappa (\partial \theta / \partial \nu) + cV \cos \epsilon \cdot \theta] ds, \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where the integral is taken around any closed circuit c enclosing the source, and $V \cos \epsilon$ is the component of velocity along the outward drawn normal ν . This remarkable property of the solution $\theta = Ae^{nr}K_0(nr)$ is easily verified by carrying out the integration around a circle of radius r , the result serving to determine the constant A . Remembering that

$$\frac{dK_0(z)}{dz} = K_1(z) \quad \text{and} \quad \frac{dI_0(z)}{dz} = I_1(z), \quad . \quad . \quad . \quad . \quad . \quad (13)$$

we have $d\theta/dr = Ane^{nr \cos \theta} [K_1(nr) + \cos \theta K_0(nr)]$, and thus from (12)

$$Q = 2r \int_0^\pi [2n\kappa \theta \cos \epsilon - \kappa (d\theta/dr)] d\epsilon = 2A\kappa nr \left[-K_1(nr) \int_0^\pi e^{nr \cos \epsilon} d\epsilon + K_0(nr) \int_0^\pi \cos \epsilon e^{nr \cos \epsilon} d\epsilon \right].$$

⁽⁷⁾ GRAY and MATTHEWS, 'Treatise on Bessel Functions,' 1895, pp. 77 and 90. The notation employed throughout is that of the above treatise.

It is not difficult to prove from (11) and (13) that

$$\int_0^\pi e^{z \cos \epsilon} d\epsilon = \pi I_0(z) \quad \text{and} \quad \int_0^\pi \cos \epsilon e^{z \cos \epsilon} d\epsilon = \pi I_1(z). \quad (14)$$

Making use of the relation⁽⁸⁾ $I_{n+1}K_n - I_nK_{n+1} = (1/z) \cos n\pi$ for $n = 0$, we obtain finally $Q = 2A\kappa\pi$, and thus for the temperature at any point the expression

$$\theta = (Q/2\pi\kappa) e^{nr} K_0(nr), \quad (15)$$

which is H. A. WILSON'S⁽⁹⁾ solution for a line source.

Let $u(\xi) d\xi$ represent the total flux of heat from an elementary portion $d\xi$ of the x -axis between $x = 0$ and $x = \beta_0$. The contribution of this element to the temperature at any point is given by (15) in the form

$$d\theta = (1/2\pi\kappa) u(\xi) d\xi e^{n(x-\xi)} K_0[n\sqrt{y^2 + (x-\xi)^2}].$$

Since equation (5) is linear the temperature due to a distribution of line-sources along the x -axis between $x = 0$ and $x = \beta_0$ is obtained by integration,

$$\theta(x, y) = (1/2\pi\kappa) \int_0^{\beta_0} u(\xi) e^{n(x-\xi)} K_0[n\sqrt{y^2 + (x-\xi)^2}] d\xi. \quad (16)$$

The boundary condition over $y = 0$ is expressed by the relation

$$2\pi\kappa\theta(x) = \int_0^{\beta_0} u(\xi) e^{n(x-\xi)} K_0[n(x-\xi)] d\xi. \quad (17)$$

If the temperature is prescribed over the boundary, equation (17) constitutes an *integral equation* for the determination of $u(\xi)$; if the flux of heat $u(\xi)$ is prescribed over the boundary from $x = 0$ to $x = \beta_0$ the same equation gives the temperature of the stream in contact with the boundary. In either case the total heat-loss of the cylinder per unit length is given by

$$H = \int_0^{\beta_0} u(\xi) d\xi, \quad (18)$$

and the temperature at any point by (16).

Section 4. Calculation of Heat-Loss under the Hypothesis of Constant Flux over the Boundary.

The solution of the problem in hand which gives results in best agreement with experiment for the case of convection of heat from small cylinders is that obtained by

⁽⁸⁾ GRAY and MATTHEWS, *loc. cit.*, p. 68.

⁽⁹⁾ H. A. WILSON, "On Convection of Heat," 'Proc. Camb. Phil. Soc.', 12, p. 413, 1904.

assuming the flux $u(\xi)$ to be constant over the boundary. From (18) we have, writing $u(\xi) = u_0$,

$$H = u_0 \beta_0. \quad (19)$$

As a result of the high heat-conductivity of the cylinder in the experiments carried out the temperature θ_0 of the cylinder may be considered constant over its entire boundary, an assumption justified by a calculation carried out in Section 13. There will therefore be a discontinuity in the temperature over the boundary; we assume that the temperature of the stream in contact with the cylinder becomes finally equal to that of the cylinder at the point β_0 where it leaves the boundary. The integral in (17) must be divided into two parts in order to make the argument $n(x-\xi)$ in the function $K_0[n(x-\xi)]$ positive; we then obtain

$$2\pi\kappa\theta(x) = u_0 \left[\int_0^x e^{n(x-\xi)} K_0\{n(x-\xi)\} d\xi + \int_x^{\beta_0} e^{n(\xi-x)} K_0\{n(\xi-x)\} d\xi \right].$$

Making the substitution $u = n(x-\xi)$ in the first integral and $v = n(\xi-x)$ in the second, we may write the above equation in the form

$$2\pi\kappa\theta(x) = (u_0/n) \left[\int_0^{nx} e^u K_0(u) du + \int_0^{n(\beta_0-x)} e^{-v} K_0(v) dv \right]. \quad (20)$$

Writing $\theta = \theta_0$ when $x = \beta_0$ in the above equation and making use of (19) we find for the heat-loss of the cylinder per unit length the expression

$$H = 2\pi\kappa\theta_0 n \beta_0 / \left[\int_0^{n\beta_0} e^u K_0(u) du \right], \quad (21)$$

where θ_0 denotes the temperature of the cylinder above that of the stream at a great distance. The discontinuity of temperature occurring over the boundary may easily be calculated at any point from (20); the maximum discontinuity occurring at $x = 0$ is easily seen to be given by

$$\theta_1/\theta_0 = \left[\int_0^{n\beta_0} e^{-u} K_0(u) du \right] / \left[\int_0^{n\beta_0} e^u K_0(u) du \right]. \quad (22)$$

Section 5. Numerical Evaluation of the Functions Employed in the Preceding Section.

Before we can proceed to evaluate the functions occurring in the preceding section we proceed to derive the convergent and asymptotic expansions of the function $F(x) = \int_0^x e^u K_0(u) du$. Expanding e^u in powers of u , and making use of the expansion (10), we obtain on integrating term by term the convergent series applicable for small values of the variable x ,

$$(1/x) \int_0^x e^u K_0(u) du = (1 + \frac{1}{4}x + \frac{1}{6}x^2 + \frac{1}{168}x^3 + \dots) - (\gamma + \log \frac{1}{2}x) (1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{5}{48}x^3 + \dots). \quad (23)$$

Integrating term by term the asymptotic expansion (9), we obtain

$$\int_0^x e^u K_0(u) du + 1 = \sqrt{(2\pi x)} \left[1 + \frac{1}{8x} - \frac{1^2 \cdot 3}{(8x)^2 \cdot 2!} + \frac{1^2 \cdot 3^2 \cdot 5}{(8x)^3 \cdot 3!} - \dots \right]. \quad (24)$$

The term 1 on the left-hand side represents a term of integration arising from the lower limit of the integral, which is proved by actual numerical calculation to be unity.

In order to evaluate $F(x)$ numerically over the interval where neither (23) or (24) are convergent, use is made of EULER'S formula for the quadrature of the function $y = f(x)$ ⁽¹⁰⁾

$$\int_{x_0}^{x_n} y dx = h \left(\frac{1}{2}y_0 + y_1 + y_2 + \dots + \frac{1}{2}y_n \right) - \frac{1}{12}h^2 [f'(x_n) - f'(x_0)] \\ + \frac{1}{720}h^4 [f'''(x_n) - f'''(x_0)] - \dots, \quad (25)$$

where h is the interval between the successive values of x , that is, $h = (x_n - x_0)/n$. From a table ⁽¹¹⁾ of the BESSEL'S functions $K_0(x)$ and $K_1(x)$, the values of $e^x K_0(x)$ and $e^x K_1(x)$ were tabulated over various ranges of equidistant intervals from $x = 0.1$ to $x = 6.0$. Numerical values of $F(x)$ for values of $x < 0.1$ were easily calculated from the convergent formula (23). Beyond this point the integral between the limits $x = 0.1$ and $x = x_n$ was calculated by the use of EULER'S formula (25), making use of a calculating machine for the purpose in such a way that the various entries were recorded as successive values of a single series of operations. Beyond $x = 6.0$ the function was evaluated from (24), the constant of integration proving to be unity. The functions $e^x K_0(x)$, $e^x K_1(x)$, $\int_0^x e^x K_0(x) dx$ and $x / \left[\int_0^x e^x K_0(x) dx \right]$ are tabulated in Table I. together with a more detailed description of the method of computation; a graph of the last function is also given.

Section 6. Approximate Formulæ for the Heat-Loss.

When the variable x is small, equation (23) enables us to write

$$y = x / \left[\int_0^x e^x K_0(x) dx \right] \quad \text{in the form} \quad y = 1 / [(1 - \gamma) - \log \frac{1}{2}x]. \quad (26)$$

Hence when the variable $n\beta_0$ is so small that it may be neglected in comparison with $\log(1/n\beta_0)$ the expression (21) for the heat-loss may be written in the form

$$H = 2\pi\kappa\theta_0 / [\log(4b/\beta_0)], \quad (27)$$

where

$$c = s\sigma \quad \text{and} \quad b = e^{1-\gamma} / (2n) = \kappa e^{1-\gamma} / (s\sigma V). \quad (28)$$

⁽¹⁰⁾ EULER, 'Comm. Acad. Sci. Imp. Petrop.', vi. (1732-33).

⁽¹¹⁾ JAHNKE and EMDE, 'Funktionentafeln' (Teubner's, 1909), p. 135. The functions $(\pi i/2) H_0^1(ix)$ and $-(\pi/2) H_1^1(ix)$ of the above tables are here denoted by $K_0(x)$ and $K_1(x)$ respectively, and their computation is due to W. S. ALDIS ('Roy. Soc. Proc.', vol. 64, p. 219, 1898).

When the variable x is large, equation (24) enables us to write approximately

$$y = 1/(2\pi) + \sqrt{x/(2\pi)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Thus when the term $n\beta_0$ is sufficiently large and the term $1/(8n\beta_0)$ is small in comparison with unity, equation (21) takes the form

$$H = \kappa\theta_0 + \kappa\theta_0\sqrt{2\pi n\beta_0} = \kappa\theta_0 + \sqrt{\pi\kappa s\sigma\beta_0} V^{\frac{1}{2}}\theta_0. \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Expressions (27) and (30) hold for cylinders of any shape for which the constant β_0 can be calculated from the hydrodynamical problem. It is easily proved in the case of an elliptic cylinder of semi-axes (a, b) that

$$\beta_0 = 2(a+b). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

This result holds good independently of the direction of the axes of the cross-section with that of the stream and may be utilized in the special cases of calculating the heat-loss from a circular cylinder or a strip of breadth $2a$. The case of a circular cylinder of radius a represents the conditions of the experiments described in Part II. of the present paper; writing $\beta_0 = 4a$ we obtain finally the approximate formulæ⁽¹²⁾ for the heat-loss

$$\text{Small velocities} \quad . \quad . \quad . \quad H = 2\pi\kappa\theta_0/[\log(b/a)], \quad . \quad . \quad . \quad . \quad (32)$$

$$\text{Large velocities} \quad . \quad . \quad . \quad H = \kappa\theta_0 + 2\sqrt{\pi\kappa s\sigma a} V^{\frac{1}{2}}\theta_0. \quad . \quad . \quad . \quad . \quad (33)$$

The limits within which these approximate formulæ represent the values of the heat-loss given by the exact expression (21) is examined in the description of Diagram I.

⁽¹²⁾ It is interesting to compare (32) with LANGMUIR's formula for free convection,

$$H = 2\pi(\phi_2 - \phi_1)/[\log b/a],$$

where ϕ denotes a function of the thermal conductivity and the temperature given by the relation

$\phi = \int_0^\theta \kappa d\theta$. This result has been shown by LANGMUIR to represent with fair accuracy the results of his experiments on the free convection from small platinum wires ('Phys. Rev.,' 34, p. 401, 1912). Interpreted in the light of equation (32) the term b of LANGMUIR's equation represents a term depending on the "effective" velocity $\bar{V}$ of the free convection current set up by the heated wire.

Equation (33) may be compared with that derived by KENNELLY ('Trans. A.I.E.E.,' 26, p. 969, 1907, and 'Trans. A.I.E.E.,' 28, p. 368, 1909) as a result of his experiments on the forced convection of heat from small copper wires. In the notation of the present paper KENNELLY's formula may be written in the form

$$H = (C + Bd) \theta_0 \sqrt{V + v_0},$$

where C and B are constants given by $C = 300 \times 10^{-7}$, $B = 5.8 \times 10^{-8}$, the heat-loss H being measured in watts per unit length, the temperature difference θ_0 being expressed in degrees C. and the velocity V in cm./sec. v_0 is a constant whose value is $v_0 = 25$ cm./sec., and d is the diameter of the wire in cm. It must be noticed that in deriving the above formula no correction was made in the experiments for the "swirl" of the rotating arm in the determination of the velocity.

It is shown that when the cooling stream is air at ordinary pressure, there exists a value of the product Vd (V expressed in cm./sec. and the diameter d in cm.) given by $Vd = 0.0187$ which discriminates between the two formulæ: when $Vd < 0.0187$ (32) is appropriate to the problem, while for $Vd > 0.0187$ (33) must be employed. In practically all cases except for very small wires and extremely low velocities the condition $Vd > 0.0187$ is satisfied and equation (33) expresses with sufficient accuracy nearly all applications of the formula.

Section 7. Note on the Interpretation and Application of the Preceding Theory.

That the expression for the heat-loss from a cylinder cooled by a stream of fluid derived in the preceding sections and leading to equation (21) may be identified with reality involves many delicate considerations as to the nature of the boundary condition over the surface of the cylinder. A comparison with the results of experiment described in Section 13 gives strong support to the validity of this formula and seems to justify the boundary condition of constant flux by means of which it was derived. Looking at the matter from the point of view of the kinetic theory of gases, temperature conditions cannot strictly be defined in the immediate neighbourhood of the heated cylinder, and it is only at a distance of several free paths when equipartition is nearly complete that we may define temperature and that normal thermal conduction takes place. The boundary condition of constant flux is one which must look for its explanation in the light of the kinetic theory, making use of the precise knowledge which is now coming to hand regarding the nature of molecular impacts on solid boundaries from recent investigations on the properties of ultra-rarefied gases.⁽¹³⁾ This aspect of the question must, however, be left over for future discussion. That there should exist a discontinuity of temperature between the stream and the cylinder at ordinary pressures is not surprising; this discontinuity was considered possible by POISSON and is now known to exist in the case of rarefied gases.⁽¹⁴⁾

The solution of the present problem on the conduction of heat in moving media is only a particular case which may be applied to many other problems for which the same formal expression of the physical conditions holds; for instance, the results apply *mutatis mutandis* to the problem of diffusion or evaporation from liquid surfaces into streams of gases flowing over them. It must be kept in mind, however, that in each type of problem to which this analysis may be applied, the nature of the boundary condition must in each case be referred to a comparison with the results of experiment.

⁽¹³⁾ A summary of recent work on the subject is given by KNUDSEN ('La Théorie du Rayonnement et les Quanta,' Gauthier-Villars, Paris, 1912, p. 133), and more recently by DUNOYER ('Les Idées Modernes sur la Constitution de la Matière,' Gauthier-Villars, Paris, 1913, p. 215 *et seq.*).

⁽¹⁴⁾ KUNDT and WARBURG, 'Pogg. Ann.,' vol. 156, 1875, p. 177; also SMOLUCHOWSKI, 'Wied. Ann.,' 64, 1898, p. 101.

PART II.

EXPERIMENTAL DETERMINATION OF THE CONVECTION CONSTANTS OF SMALL PLATINUM WIRES.

Section 8. Introduction.

The experimental study of thermal losses from heated bodies under various conditions dates back to the classical researches of DULONG and PETIT⁽¹⁵⁾ in 1817. Since that date numerous experiments have been carried out on the radiation and convection of heat, most of which fall under two categories:—

(i.) The study of the total heat losses from a heated body to an enclosure maintained at constant temperature and containing different gases under various conditions of pressure. In so far as these experiments refer to the heat losses from wires, we may cite the classical work of AYRTON and KILGOUR⁽¹⁶⁾ and that of PETAVEL.⁽¹⁷⁾ More recently we may quote the experiments of KENNELLY⁽¹⁸⁾ on the forced convection of heat from small copper wires, and to a series of detailed papers recently published by LANGMUIR⁽¹⁹⁾, where exhaustive references to experiments on convection problems are to be found.

(ii.) The study of radiation losses from heated solids to enclosures maintained at constant temperature, in which connection the earliest measurements carried out by an electrical method appear to have been due to BOTTOMLEY.⁽²⁰⁾ In high vacua this subject has in recent years been made the field of much research, especially in connection with the development of the metallic filament lamp. In the present work the heat-loss by radiation plays a very subordinate part and was not made the subject of special investigation.

The measurement of thermal losses in stagnant media, while simpler to carry out experimentally, and perhaps more important in practical applications, has thus far defied mathematical investigation on a rational physical basis. On the other hand, the mathematical interpretation of heat convection as a problem of heat conduction in moving media admits of fewer restrictions and leads to the results of BOUSSINESQ⁽²¹⁾

⁽¹⁵⁾ DULONG et PETIT, 'Ann. de Chimie et de Physique,' t. 7, 1817.

⁽¹⁶⁾ AYRTON and KILGOUR, "The Thermal Emissivity of Thin Wires in Air," 'Phil. Trans.,' vol. 183, Part I., p. 371, 1892.

⁽¹⁷⁾ PETAVEL, 'Phil. Trans.,' vol. 191, p. 501, 1898; also 'Phil. Trans.,' vol. 197, p. 229, 1901.

⁽¹⁸⁾ KENNELLY, WRIGHT, and VAN BYLEVELT, 'Trans. A.I.E.E.,' vol. 26, p. 969, 1907; also 'Trans. A.I.E.E.,' vol. 28, pp. 363-396, 1909.

⁽¹⁹⁾ LANGMUIR, 'Phys. Rev.,' vol. 34, p. 401, 1912; also 'Proc. A.I.E.E.,' June, 1912, p. 1011; 'Proc. A.I.E.E.,' April, 1913.

⁽²⁰⁾ BOTTOMLEY, 'Phil. Trans.,' A, vol. 178, p. 429, 1888; 'Roy. Soc. Proc.,' vol. 66, p. 269, 1900.

⁽²¹⁾ BOUSSINESQ, *loc. cit.*, p. 297.

and to the more general formulæ developed in the preceding sections. It is not difficult to understand from a physical point of view that the results of experiments on forced convection should be simpler in their interpretation in that the disturbance due to the free convection set up by the heated wire may be neglected in comparison with the impressed velocity provided the latter be sufficiently large. The first experimenter to have shown that the convection loss in a current of air is proportional to the temperature difference and to the square root of the velocity seems to have been SER⁽²²⁾, whose measurements on the variation of heat-loss with extent of surface indicate approximations to the theoretical formulæ. KENNELLY'S⁽²³⁾ observations led to an empirical formula closely resembling that finally derived in Section 6.

The experiments of COMPAN⁽²⁴⁾ on the cooling of spheres in air currents have verified BOUSSINESQ'S approximate equation between comparatively narrow limits of temperature and air-velocity. While the present investigation was in progress, the work of MORRIS⁽²⁵⁾ was published, verifying the application of a formula of the type obtained by BOUSSINESQ to the cooling of fine wires heated by an electric current to temperatures of about 70° C. above the surrounding air and for air-velocities as high as 40 miles an hour.

The present investigation was undertaken with two purposes in view:—

(i.) To study the laws of convection of heat from small platinum wires heated by an electric current over as wide a range as possible of temperature, air-velocity, and diameter in the light of the formulæ developed in the preceding sections and to obtain in absolute measure the convection constants of such wires.

(ii.) To make use of the constants thus obtained in the design of accurate and portable wind-measuring apparatus to form the basis of a standard system of anemometry, as well as to serve for use in a great variety of engineering and aerotechnical problems.

Section 9. Experimental Arrangements.

(i.) General.

The general arrangement of apparatus necessary to carry out the requisite measurements of heat losses from a series of platinum wires, of diameters 1 to 6 mils, consisted of a rotating arm capable of adjustment to any speed as calculated from a

(²²) SER, 'Traité de Physique industrielle,' t. I., pp. 142–162, 1888. The paper is briefly abstracted by BOUSSINESQ, *loc. cit.*, p. 290.

(²³) KENNELLY, 'Trans. A.I.E.E.,' 28, pp. 363–397, June, 1909. The formula referred to is (28), p. 388.

(²⁴) COMPAN, 'Ann. de Chimie et de Physique,' 26, p. 488, 1902.

(²⁵) MORRIS, "The Electrical Measurement of Wind Velocity," 'Electrician,' October 4, 1912, p. 1056. Paper read at the British Association Meeting, Dundee, September, 1912. See also 'Electrician,' October 4, 1912, p. 1056; 'Engineer,' September 27, 1912.

chronograph record. At various lengths along this arm could be clamped a light fork designed to hold the specimens of wire under test. The latter formed part of a Kelvin double bridge, electrical connection being obtained through a central mercury connecting switch and overhead wires to the remainder of the bridge. By means of a rheostat it was possible at each speed to adjust a measured current through the wire so as to bring its resistance to a value corresponding to a predetermined temperature. In this way it was possible in the case of each wire to vary in any chosen way the various factors of temperature, air-velocity and heat-loss. The general arrangement of apparatus and details are drawn in Diagram II. and shown photographically in Plate 8.

(ii.) *Details of Resistance Bridge.*

In order to measure the temperature of a length of platinum wire heated by an electric current under given conditions of wind-velocity, it is necessary to design a form of resistance-bridge suitable for an accurate determination under these conditions. The well-known connections of the Kelvin double bridge at once recommend themselves for the purpose and are shown diagrammatically in fig. 2.

A represents the platinum wire whose resistance it is required to measure. B is a ten-metre bridge wire of No. 23 S.W.G. manganin wire. Connecting A and B is a resistance δ including that of the ammeter and leads to the fork on the rotating arm. The ratio-resistances a , b , α , β were in the neighbourhood of 10^5 ohms, so that the current in the wire

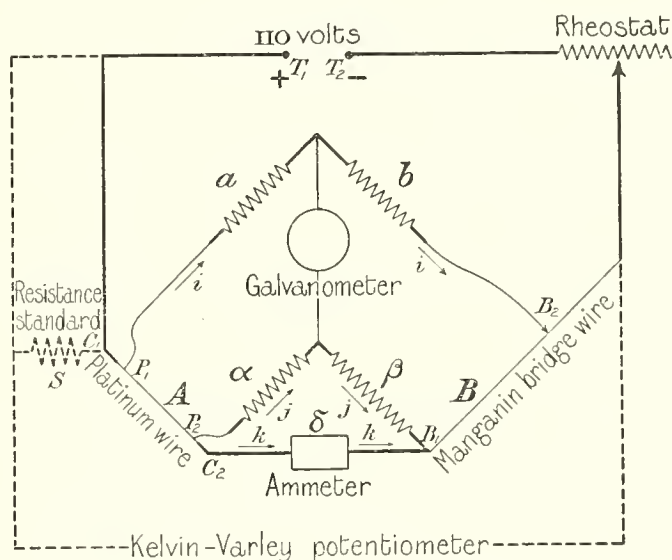


Fig. 2. Kelvin bridge connections.

A is to a very close approximation that indicated by the ammeter. A and B were connected through an adjustable rheostat to the 110-volt mains. The following formulæ are given for future reference. When the galvanometer is balanced it is not difficult to prove that

$$\frac{A}{B} = \frac{a}{b} + \frac{\beta}{B} \frac{\delta}{\alpha + \beta + \delta} \left(\frac{a}{b} - \frac{\alpha}{\beta} \right). \quad (34)$$

Also if I be the current in the platinum wire, k that in the ammeter, i that through the coils (a, b), and j that through (α, β), we have

$$I = k [1 + \delta/(\alpha + \beta)], \quad j = k \cdot \delta/(\alpha + \beta), \quad i = I [A/a + \alpha/a \cdot \delta/(\alpha + \beta + \delta)]. \quad (35)$$

We notice from (34) that when the ratio coils are so adjusted that $a/b = \alpha/\beta$, then,

$A/B = a/b = \alpha/\beta$ independently of any connecting- or contact-resistance in the bridge. It is this characteristic property of the Kelvin bridge which makes it especially applicable to the present series of measurements.

In the actual apparatus the wire A to be tested is mounted in a specially constructed fork which is rotated at the extremity of a long revolving arm. The main circuit and the potential terminals P_1P_2 are carried through four mercury-contact slip-rings to the overhead wires and thence to the bridge.

In the actual apparatus the ratio coils had nearly equal resistances of the value 10^5 ohms; when compared by means of a Kelvin-Varley slide they were found to have the following ratios:—

α .	β .	a .	b .
24,984	24,986	25,020	25,010

making the correction factor $(a/b - \alpha/\beta) = 3/2500$.

The connecting resistance δ was found to be about 0.55 ohm; the resistance B was never less than 0.50 ohm, so that the correction in equation (34) due to small departures from equality in the ratio-coils is given by

$$A/B = a/b + \delta/B \cdot 3/2500. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

It will be seen that $A=B$ to less than one-tenth of 1 per cent., which, as will be seen later, is well within the possible accuracy of the present experiment. We also notice from (35) that I does not differ from the ammeter current k by an appreciable amount.

The galvanometer employed was a Broca instrument of resistance 97 ohms and adjusted to a sensitivity such that 8.5×10^{-10} amperes gave a deflection of 1 mm. on a scale at a metre distance—a shunt was used with the galvanometer so that the sensitivity could be reduced to any required fraction of this amount.

The ammeter was a direct-reading Weston instrument, consisting of milli-voltmeter and shunt, by means of which ranges 0.2, 2.0, 20 amperes were available; these ranges could be further subdivided by the insertion of a suitable resistance in series in the milli-voltmeter circuit. The instrument was calibrated against a Weston laboratory standard and was found to read correctly to within $\frac{1}{4}$ of 1 per cent., which represents the order of accuracy aimed at through the experiment.

The procedure carried out in order to obtain a set of readings was as follows: the wire under test was placed in the fork and potential terminals of fine platinum wire (about 1 mil diameter) fused on the wire at a measured distance apart. A 2-volt cell was inserted at T_1T_2 and a Kelvin-Varley slide placed in parallel with the circuit $SP_1P_2B_1B_2$, S being a standard resistance connected in series with the circuit A, B so that the resistance of the wire in ohms might be obtained. By using a very small measuring current (about 0.05 amperes) and the galvanometer at full sensitivity the resistance of the wire between potential terminals at standard room temperature

(17° C.) was obtained, care being taken to test for the absence of any appreciable heating effect of the measuring current since this may become quite noticeable in the case of the very fine wires. Points were then determined on the bridge-wire corresponding to a series of predetermined ratios R/P_{17} , each of these representing when balanced against the heated platinum wire a certain temperature as calculated from the constants of the wire. The series of temperatures employed was as far as possible kept the same for all the wires. In order to measure the heat-loss, the standard resistance and the Kelvin-Varley slide were cut out of the circuit, and the terminals T_1T_2 connected to the 110-volt mains through rheostats allowing of the continuous adjustment of current from zero to about 5 amperes. The rotating arm was then adjusted to a fixed speed, and the current set to such a value that a balance was obtained on the galvanometer (shunted to $\frac{1}{1000}$ of its sensitivity) when the adjustable contact B_2 was set on each of the points of the bridge-wire previously determined to represent certain temperatures. Readings were also taken with the apparatus at a standstill, and the wire in three positions, horizontal, vertical, and inclined at an angle of 45 degrees to the vertical. At each point the current was read off on the ammeter, and from the resistance of the wire the heat-loss in watts per unit length could be calculated. This series of readings was repeated for various velocities as high as 25 miles an hour. In order to eliminate the end correction the wires tested were for the most part of considerable length (about 23 cm.); it was found impossible to go to higher speeds owing to the sagging and vibration of the wires under wind-pressure and centrifugal force with consequent risk of breaking at high temperatures and the loss of a set of observations.

(iii.) *On the Measurement of Wind-velocity.*

The simplest laboratory method of obtaining a stream-line wind-velocity, whose value is known directly without reference to the calibration of Pitot-tubes or anemometers, is realized by the use of a whirling table to which is attached a light arm at the extremity of which the object to be experimented upon is attached. The advantage of this method is, however, more apparent than real. It is well-known that in such a disposition of apparatus a vortex is created in the neighbourhood of the rotating arm, so that the velocity of a point at any radius relative to the room does not represent its velocity relative to the air. In addition to the difficulty mentioned results obtained from air-velocities measured from motion in a circle cannot in some classes of work be applied with safety to linear motion. In fact so serious have these objections proved themselves to be that the method of obtaining velocity by means of a rotating arm has been abandoned in meteorological work and in aeronautical problems.⁽²⁶⁾

⁽²⁶⁾ These difficulties are described in detail by FRY and TYNDALL in a paper "On the Value of the Pitot Constant," 'Phil. Mag.,' 21, p. 352, 1911; also in the 'Report of the Advisory Committee for Aeronautics,' 1909-10, p. 15; 1910-11, 'Report,' No. 34, p. 50, by Messrs. BRAMWELL and SILLICK.

For most purposes a specially constructed wind-tunnel is employed in the production of wind-velocity. In order to eliminate the effect of the walls and to produce a uniform velocity over a considerable cross-section, a large size of tunnel is necessary, requiring at high velocities a very large delivery of air obtainable only from massive and expensive equipment. Moreover, it is necessary for the actual measurement of wind-velocity to depend on a Pitot-tube or other form of anemometer; in addition it is difficult to secure stream-line motion at high velocities.

(iv.) *On the Determination of the Correction Factor in the Measurement of Velocity.*

For the purposes of the present experiment the measurement of velocity by means of a rotating arm recommends itself as the simplest and the most direct. With the wire held parallel to the axis of rotation, the objection to circular motion does not hold, while the true velocity at any point on the arm relative to the air can be determined as follows:—

We denote by V_r the velocity of a point at radius r on the rotating arm relative to the room (apparent velocity), by V the true velocity relative to the air, and by v the velocity of the vortex at radius r relative to the room. Observation then shows that at the same radius the true velocity is proportional to the apparent velocity consistently with an expression of the form

$$V = V_r - v = (1-s) V_r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

The constant s may conveniently be called the *swirl* and expressed in percentages of the apparent velocity. The quantity $v = sV_r$ represents the velocity at radius r of the vortex set up by the rotating arm. In order to measure this velocity one of the wires is placed in the fork of the rotating arm, and a series of currents required to heat up the wire to a determinate resistance for various velocities measured. The relation between the current and the apparent velocity is found to be of the form

$$i^2 = i_0^2 + \kappa_r V_r^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

and the constants i_0^2 and κ_r were determined by observation. The *same* wire was then mounted in another fork fixed relatively to the room and placed in such a position that the rotating fork passed within a centimetre or less from the fixed wire. The whirling table was then set into motion and the current required to bring the resistance to the same value for each of a series of velocities measured. In this way the apparent velocity of the vortex v_r was calculated from (38). It was found that the apparent velocity of the arm V_r was connected with v_r by a linear relation $v_r = s_r V_r$, s_r now denoting the *apparent swirl*. Making use of (37) we have the relations $v_r = v/(1-s) = sV_r/(1-s) = s_r V_r$ giving $s_r = s/(1-s)$ and finally $1-s = 1/(1+s_r)$ so that

$$V = V_r/(1+s_r), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

expressing the true velocity in terms of the apparent velocity and the apparent swirl, both easily measurable quantities. Details of measurements by means of which the swirl was obtained for the three positions on the rotating arm employed in the present experiment are given briefly under the description of Diagram II. We note in passing that the value of the swirl will depend on the construction and disposition of the apparatus as well as on the dimensions and shape of the room in which the rotating arm is set up; in the present case the swirl amounted to about 5 per cent. and could be determined to within 5 per cent., so that the true velocity of the wires relatively to the air of the room could be obtained to an accuracy well within $\frac{1}{4}$ of a per cent. In this manner the simplicity and ease of working of the rotating arm can be employed without loss of accuracy.⁽²⁷⁾

(v.) *Note on the Construction of a Constant Low Resistance Set of Mercury Contact Slip-rings.*

In order that the resistance of the wire between potential terminals may be measured with accuracy at the extremity of the rotating arm, it is necessary to provide some form of slip-ring; since the contacts enter into the connections of the bridge, it is important that the contact resistances be as small as possible and remain constant while the apparatus is in rotation. A specially designed set of four slip-rings which fulfils these requirements is shown in (b) Diagram II., while the specifications are given under the accompanying description. The insertion of this set of slip-rings in the circuit $P_1P_2B_1B_2$ of fig. 2, increased the resistance by the amount 0.0077 ohm. only and was found to remain extremely constant. No change in the balance of the bridge when connected to measure the resistance of the wire at room temperature could be detected when the rotating arm was set into motion at various speeds.

(vi.) *Note on the Fork Carrying the Wire to be Tested and on the Whirling Table.*

The rotating arm consisted of a light strip of pine (6.3×1.3 cm.) about 2.6 m. long fastened to a whirling table in the manner shown in Diagram II. and illustrated by Plate 8. At three determinate positions on this arm could be clamped the fork for carrying the wires. This fork was designed to disturb as little as possible the flow of air past the wire, and is illustrated in detail in (c) Diagram II. In order to eliminate the cooling effect of the terminals and their disturbance on the flow of air past the ends of the wires, potential leads of extremely fine platinum wire (1 mil diameter) were fastened at the points P_1P_2 at distances of about 2 cm. from the ends C_1C_2 . These potential leads were most easily fixed in position by heating the wire to

⁽²⁷⁾ The results here described and the method of correction for the swirl agree with the observations of BRAMWELL and SILICK (footnote ⁽¹²⁾), who found nearly the same value for the ratio $s = v/V_r$. The correction is now regularly made in tests carried out on a rotating arm.

a bright red when the fine platinum wires could be easily fused in their proper places ; this preliminary heating also served to anneal the wire before its resistance was determined. A small weight, attached by means of a silk thread to the lower current terminal C_2 , served to keep the wire under slight tension while the distance between the potential terminals was accurately measured.

The whirling table was operated through suitable reducing gear by means of a $\frac{1}{4}$ h.p., 1200 r.p.m., 110-volt D.C. motor, with speed varied by means of an adjustable resistance in series with the armature, controlled by a rheostat conveniently situated near the bridge. The field of the motor was separately excited : in order to obtain constant speed it was found necessary to excite the field some time before commencing a series of observations in order that a steady temperature and therefore steady resistance in the field-coils shall have been attained. Another advantage is the elimination of disturbing effects on the galvanometer as the motor is stopped and started. A timing device attached to the driving shaft of the whirling table caused a contact to be made every tenth revolution of the rotating arm and to be registered on a chronograph simultaneously with the beats of a seconds pendulum. An examination of the chronograph sheet showed that with the above precautions the speed remained constant to about one-tenth of 1 per cent., well within the range of accuracy of the present observations.

Section 10.

(i.) On the Constants of the Platinum Wires under Test.

The most difficult measurement in the present study of heat convection and that which imposes a limit to the accuracy attainable is the calculation of the temperature of the wire from the change of resistance. In fact this point confines the present experiment to the use of platinum wires whose constants in the measurement of high temperatures are well known. It must be kept in mind, however, that the constants of a platinum wire which has been kept for some time at a high temperature for a considerable time as in the present experiments are liable to change. This source of uncertainty is aggravated in the case of very fine wires where the additional difficulty of a sensible change of resistance due to "*evaporation*" from the wire is to be met with.⁽²⁸⁾ Evidence on both these points will be noticed from an inspection of Table II., and the limit of accuracy due to these and other difficulties may be roughly set at $\frac{1}{4}$ of 1 per cent.

The wires employed in the present experiment were drawn through diamond dies⁽²⁹⁾ from a length of 6 mil pure platinum wire whose constants when used in platinum

⁽²⁸⁾ See BURGESS and LE CHATELIER, 'The Measurement of High Temperatures' (Wiley and Sons, New York, 1912), p. 232 ; also p. 196.

⁽²⁹⁾ The writer is indebted to the Imperial Wire and Cable Co., of Montreal, for the use of diamond dies employed in drawing down the wires.

thermometry were given (in the usual notation) as $\alpha = 0.00388$, $\delta = 1.54$. It was hoped that these constants would remain unchanged by drawing down the wire, so that by keeping the series of ratios in the bridge adjustment the same for all the specimens, the corresponding temperatures would only have to be calculated once. On testing some of the wires after they had been heated to high temperatures, it was found that the temperature coefficients varied appreciably with the size of the wires and that they differed materially from the rated value. The temperature coefficient was therefore separately determined for each wire by comparing its resistance at the ice- and steam-points with that of a manganin standard, making use of a potentiometer method for which the Kelvin-Varley slide was employed. In this way the resistance between potential terminals could be expressed in ohms, and the resistance at any temperature calculated. The results are given in Table II. and are explained in further detail in the accompanying description. It will be shown later that a considerable change in the value of δ does not affect the calculation of temperature beyond the limit of other experimental errors; the value of δ was therefore not re-determined, and the value given above was taken. A length of 3 mil wire drawn down from the original specimen was sent to a firm to be drawn down to 2.0, 1.5, and 1.0 mil. It will be noticed from Table II. that the temperature-coefficients of these extremely fine wires are so small that it is improbable that they are drawings from the specimen sent out. These fine wires were found extremely difficult to work with and could only be tested in comparatively short lengths of 3 to 4 cm.; they often broke while being heated to a dull redness for annealing so that it was not possible to measure the convection losses over as great a range of temperature as in the case of the larger wires. The friability of the wires under heating was probably due to their being drawn from a specimen of impure platinum.

(ii.) *On the Measurement of Diameters and Lengths.*

The measurement of the diameters of the wires to a fraction of a per cent. requires an estimation of length to about 10^{-5} cm. The direct measurement of the diameter by means of a high-power microscope was ultimately found to be the most satisfactory method.⁽³⁰⁾ The wires were mounted on a microscope slide and the diameters measured at centimetre intervals by means of a micrometer screw in the eye-piece. The readings of the micrometer screw were afterwards calibrated in centimetres by means of a Bausch and Lomb stage micrometer, care being taken to calibrate the eye-piece micrometer screw over the various portions of the field employed in measuring the

⁽³⁰⁾ The microscope employed was a Zeiss 1E, with an 8 mm. apochromatic objective fitted with a screw micrometer and $\times 6$ compensating ocular; the total magnifying power was 187. The writer was indebted to Prof. J. C. SIMPSON, of the Department of Histology of McGill University, for the loan of this microscope.

wire in order to avoid errors due to optical distortion. It was found possible to reset the cross-hairs consistently to each division of the micrometer head, *i.e.*, to about 10^{-5} cm. The results are tabulated in Table II. and further details are given in the accompanying description.

The measurement of the lengths of wire between potential terminals offered no difficulties. In order to calculate the heat-loss per unit length at any temperature it was necessary to correct for expansion with temperature by means of the formula $l_{\theta} = l_0 (1 + \alpha\theta + \beta\theta^2)$, where l_{θ} is the length at θ° C., l_0 that at 0° C., and α and β the coefficients of linear expansion given by HOLBORN and DAY⁽³¹⁾ as $\alpha = 0.08868 \times 10^{-4}$, $\beta = 0.001324 \times 10^{-6}$.

Section 11. On the Calculation of Temperatures.

From the mean temperature-coefficient of resistance c of platinum between 0° C. and 100° C. given by $c = (R_{100} - R_0)/100R_0$, the temperature of the wires were calculated from the formula $\theta_p = (R/R_0 - 1)/c$, R being the resistance at the temperature θ_p on the platinum scale and R_0 that at 0° C. In the present instance the resistances of the platinum wires were referred to the temperature $t = 17^{\circ}$ C., so that the ratio R/R_{17} are those from which the temperatures are to be calculated. We may take very approximately $R_t = R_0 (1 + ct)$ for $t = 17^{\circ}$ C., in which case it is not difficult to prove that $\theta_p - t = (R/R_t - 1) (1/c + t)$ by means of which temperatures on the platinum scale were calculated. The temperatures of the wires were then obtained in terms of the true thermodynamic scale by means of CALLENDAR's formula,

$$\theta^{\circ} \text{ C.} = \theta_p + \delta (t/100 - 1). \quad t/100 \text{ with } \delta = 1.54.$$

Actual calculation of temperatures was avoided by the use of the conversion tables given by BURGESS and LE CHATELIER⁽³²⁾ calculated for $\delta = 1.50$. From an auxiliary table the temperature correction for a value of δ other than this value is given; it is seen that the correction $\Delta\theta$ for $\Delta\delta = 0.01$ is $\Delta\theta = 0.9^{\circ}$ C. at $\theta = 1000^{\circ}$ C., showing that the value of δ can vary considerably without altering the value of θ to an extent greater than the limit of accuracy of the other measurements.

No attempt was made to work with the wires at a higher temperature than 1200° C. owing to the fact that the use of CALLENDAR's formula beyond this point is subject to uncertainty⁽³³⁾; also at high temperatures the wires soften and are liable to break when driven through the air at the high velocities.

⁽³¹⁾ 'Smithsonian Physical Tables,' 5th edition, 1910, p. 222.

⁽³²⁾ BURGESS and LE CHATELIER, 'The Measurement of High Temperature,' Table V., p. 493; also Table VII., p. 495.

⁽³³⁾ See LANGMUIR, 'Journal Amer. Chem. Soc.,' vol. 28, p. 1357, 1906.

Section 12. On Possible Sources of Error in the Calculation of Temperatures.

(i.) *Calculation of the Radial Temperature Gradient.*

The question arises as to whether under conditions of rapid rate of cooling there may not exist a temperature gradient in the wire of sufficient magnitude to cause the temperature as calculated from the apparent resistance of the wire to differ appreciably from the surface-temperature.⁽³⁴⁾ This point can be settled by the following analysis :—

Let ρ be the specific resistance of the wire at the temperature under consideration, e the voltage drop per unit length, constant at all points of the cross-section. The heat generated per unit volume is e^2/ρ and the equation for the radial conduction of heat is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(K r \frac{\partial \theta}{\partial r} \right) = \frac{e^2}{\rho} + c \frac{\partial \theta}{\partial t}, \dots \dots \dots (40)$$

where K is the heat-conductivity and c the specific heat per unit volume. If H is the heat-loss per unit length at the boundary $r = a$, we have

[illegible]

Also, if R is the apparent resistance of the wire at temperature θ we have $1/R = \int_0^a (2\pi r/\rho) \cdot dr$, giving from (40) and (41) when $\partial\theta/\partial t = 0$, the result $H = -e^2/R$ which holds independently of radial temperature gradients or variations of specific resistance and heat-conductivity with temperature. The radial temperature gradient may be calculated to a first approximation by assuming that the gradient is so small that K and ρ may be considered constant throughout the wire. We then have $1/R = \pi a^2/\rho$ and therefore $H = \pi a^2 \cdot e^2/\rho$. The integral of (40) consistent with (41) is $K(\partial\theta/\partial r) = -Hr/2\pi a^2$, which integrates further to

$$\theta - \theta_a = H/4\pi K \cdot (1 - r^2/c^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

θ_a being the surface temperature at $r = a$. The maximum temperature difference is that between the centre of the wire and the surface and is given by $\theta - \theta_a = H/4\pi K$. For platinum the heat-conductivity is $K = 0.1664$ calories per cm.² per sec., or $K = 0.698$ watts cm.² sec. at 0°C. ; in the case of the largest wire H does not exceed 5 watts per cm. at 1000°C. , so that we have $\theta - \theta_a < 5/(4\pi \times 0.698)$ or $\theta - \theta_a < 0.6^\circ \text{C.}$ We note that this small temperature gradient justifies the simplifying assumption regarding the constancy of K and ρ over the cross-section of the wire, and conclude, finally, that the error in the calculation of the surface temperature introduced by a radial temperature gradient is within the limits of accuracy of the other measurements.

(³⁴) A similar problem is dealt with by KELVIN, 'Roy. Soc. Proc.,' June 10, 1875; also 'Collected Works,' vol. 3, p. 245.

(ii.) *On the Error Introduced by the Cooling Effect of the Leads and Potential Terminals.*

It is of importance in the practical design of anemometers to determine the limits of error introduced by the leads and potential terminals. Practically all cases are

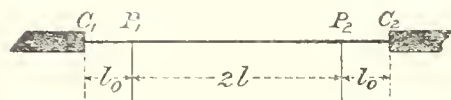


Fig. 3. Cooling effect of leads and potential terminals.

covered by that illustrated in fig. 3, where the end-leads C_1C_2 may be considered so heavy that the heated wire is maintained at its extremities very nearly at air-temperature; in addition it is necessary to allow for the heat conducted away by the potential leads fused to the anemometer-wire at P_1P_2 . The problem

presents itself in the following way:—A wire whose constants have been determined from a very long specimen in which end-corrections are negligible is placed between two heavy terminals C_1C_2 at a distance $2(l+l_0)$ apart; a current i is passed through the wire until the resistance of length $2l$ between potential terminals P_1P_2 is $2Rl$; it is required to determine to what an extent the determination of air-velocity from the measurement of current and resistance is affected by the cooling effect of the leads and potential terminals. Let ω represent the cross-section of the anemometer-wire, ω' that of the potential leads, and let K be the heat-conductivity of platinum; further, let W and W' represent the heat-losses per unit length of the two wires respectively when the air-velocity is V . The temperature θ at any point of the anemometer-wire is determined from the equation

$$\omega \frac{\partial}{\partial x} \left(K \frac{\partial \theta}{\partial x} \right) + W = i^2 R, \quad \dots \quad (43)$$

where W and R are known functions of the temperature θ .

The complete expression for W is shown by the present experiment to be of the form

$$W = [\beta V^{\frac{1}{2}} + \gamma_0 \{1 + c(\theta - \theta_0)\}] (\theta - \theta_0). \quad \dots \quad (44)$$

In the case of platinum the variation of resistance with temperature can be represented by a formula of the type

$$R = R_0 [1 + a(\theta - \theta_0) + b(\theta - \theta_0)^2]. \quad \dots \quad (45)$$

It will be noticed that, according to the experimental results expressed in (44) and (45), a solution of (43) can be completely determined in terms of elliptic integrals if we neglect the variation of the heat conductivity K with the temperature.⁽³⁵⁾ For

⁽³⁵⁾ Little seems to be known with regard to the variation of the heat conductivity of metals at high temperatures. The existence of definite experimental relations of the form (44) and (45) seems to indicate the possibility through an exact solution of (43) of carrying out an experiment for the determination of K at high temperatures along the lines of STRANEO'S method (CARSLAW, 'Fourier Series and Integrals,' Macmillan and Co., 1906, p. 282).

the purpose of determining the small end-corrections it is sufficient to employ approximate expressions of the form

$$W = A(\theta - \theta_0) \quad \text{and} \quad R = R_0[1 + \bar{\alpha}(\theta - \theta_0)], \quad (46)$$

where $A = \beta V^{\frac{1}{2}} + \bar{\gamma}$, and $\bar{\alpha}$ and $\bar{\gamma}$ are average values of the coefficients over the range of temperature considered.

Under these conditions (43) becomes

$$K\omega \frac{d^2\theta}{dx^2} - A(\theta - \theta_0) = -i^2 R_0[1 + \bar{\alpha}(\theta - \theta_0)],$$

or, writing $p^2 = (A - i^2 R_0 \bar{\alpha})/K\omega$, the above equation takes the form

$$\frac{d^2\theta}{dx^2} - p^2(\theta - \theta_0) = i^2 R_0/K\omega. \quad (47)$$

Since we may neglect the current in the potential terminals (see equation (35)), the temperature θ' at a distance x' from the junction is determined from the equation

$$K\omega' \frac{d^2\theta'}{dx'^2} - A'(\theta' - \theta_0) = 0, \quad (48)$$

where the accented symbols refer to the potential wire. The solution of (48), which makes $\theta' = \theta_0$ at $x' = \infty$ and $\theta' = \theta'_0$ at the junction $x' = 0$, may be written

$$\theta'_0 - \theta' = (\theta'_0 - \theta_0) e^{-x' \sqrt{(A'/K\omega')}}. \quad (49)$$

The quantity of heat carried away by the potential lead from the point of junction is given by

$$H' = K\omega' d\theta'/dx' = (\theta'_0 - \theta_0) \sqrt{(K\omega' A')} \quad \text{at} \quad x' = 0. \quad (50)$$

Taking the origin at the centre of the anemometer-wire, the solution of (47), which makes $\theta = \theta'_0$ at $x = l$ is given by

$$\theta - \theta_0 = i^2 R_0/K\omega p^2 - [i^2 R_0/K\omega p^2 - H'/\sqrt{(K\omega' A')}] \cdot \cosh px / \cosh pl. \quad (51)$$

The flow of heat H_1 towards the junction at $x = l$ is given by

$$H_1 = -K\omega d\theta/dx = K\omega p [i^2 R_0/K\omega p^2 - H'/\sqrt{(K\omega' A')}] \tanh pl. \quad (52)$$

For the determination of the temperature in the portion of the anemometer-wire between the potential terminal P_1 and the terminal C_1 , take the origin at the junction. The solution of (47), which makes $\theta = \theta'_0$ at $x = 0$ and $\theta = \theta_0$ at $x = l_0$, is given by

$$\theta - \theta_0 = \frac{i^2 R_0}{K\omega p^2} \left\{ 1 - \frac{\sinh px}{\sinh pl_0} \right\} + \left\{ \frac{H'}{\sqrt{(K\omega' A')}} - \frac{i^2 R_0}{K\omega p^2} \right\} \frac{\sinh p(l_0 - x)}{\sinh pl_0}. \quad (53)$$

The flow of heat away from the junction $x = 0$ is given by

$$H_2 = -K\omega d\theta/dx = (i^2 R_0/p) \cdot \text{cosech } pl_0 + [H'/\sqrt{(K\omega' A')} - i^2 R_0/K\omega p^2] \cdot K\omega p \coth pl_0. \quad (54)$$

various diameters in cases where the end corrections are liable to be large, *i.e.*, $\theta - \theta_0 = 1165^\circ \text{C.}$, $V = 81 \text{ cm./sec.}$ for $R_0/\bar{R} = 1/4$. It will be noticed that p is always fairly large, and since in all the experimental arrangements we had $pl > 10$ and $pl_0 > 2$, the expression (57) may be written approximately in the form

$$\begin{aligned} Q \sinh pl &= (1 + P \sinh pl_0)/(e^{pl_0} + P \sinh pl_0) \\ &= (P + \operatorname{cosech} pl_0)/(P + 1 + \coth pl_0). \end{aligned}$$

We have, finally, from (59) the convenient formula

$$R = R(1 - \epsilon),$$

where

$$\epsilon = (1 - R_0/\bar{R})(P + 2e^{-pl_0})/\{pl(P + 2)\} \dots \dots \dots (62)$$

The values of ϵ are tabulated under the description of Diagram IV., from which it will be seen that the calculation of temperature from the resistance between the potential leads may be taken to refer to an infinitely long wire to an accuracy represented by these values of ϵ ; in the case of the larger wires the experiment was carried out so that $l > 10 \text{ cm.}$ and $l_0 > 2 \text{ cm.}$, while the potential leads were of 1-mil wire. In this case the error introduced by the terminal conditions is well within the limits of accuracy imposed by the other measurements. In the case of the finest wires it was necessary to employ short specimens so that the possible error in the calculation of temperature due to terminal conditions is of the order of 2 or 3 per cent.

The preceding calculations are important in the design of hot-wire anemometers when the constant of the instrument is determined from the convection constants of platinum wire as determined in the present paper. Terminal errors do not need to be calculated if the anemometer is directly calibrated as long as the conditions of air velocity in the neighbourhood of the terminals are the same during actual service as during calibration.

(iii.) *On the Error due to the Vibration of the Anemometer Wire.*

A wire placed in a strong current of air tends to vibrate at right angles to the direction of the stream with a frequency depending both on the tension of the wire and the velocity of the air current.⁽³⁶⁾ Although the amplitude of vibration of the wire may be small, the high frequency may lead to an appreciable source of error the magnitude of which we now propose to investigate. We suppose the wire to perform simple harmonic vibrations of amplitude a and period T at right angles to the stream of velocity V . Representing the displacement from the equilibrium position by $x = a \sin \omega t$, the velocity at right angles to the stream is $v = a\omega \cos \omega t$. The velocity

⁽³⁶⁾ See RAYLEIGH, 'Sound,' vol. II., p. 412 (1896).

of the wire relative to the stream at any instant is given by $U = \sqrt{(V^2 + v^2)}$. The heat-loss at any instant is given by the experimental relation $W = \beta(\theta - \theta_0)\sqrt{U} + \gamma$, so that the average heat-loss, $\overline{W} = R\dot{\theta}^2$, is given by

$$\overline{W} = 1/T \int_0^T W dt = \beta(\theta - \theta_0) T^{-1} \int_0^T (V^2 + v^2)^{\frac{1}{2}} dt + \gamma.$$

Writing $w = \omega a = 2\pi a/T$ and $\phi = 2\pi t/T$, we find

$$\overline{W} = \beta(\theta - \theta_0) V^{\frac{1}{2}} 2\pi^{-1} \int_0^{\pi/2} [1 + (w^2/V^2) \cos^2 \phi]^{\frac{1}{2}} d\phi + \gamma. \quad (63)$$

If $\overline{V}$ be the apparent velocity corresponding to the observed heat-loss $\overline{W}$ we have, writing $\omega = 0$,

$$\overline{W} = \beta(\theta - \theta_0) \sqrt{\overline{V}} + \gamma. \quad (64)$$

Comparing (63) and (64) we obtain finally

$$(V/\overline{V})^{\frac{1}{2}} = 1 / \left\{ 2\pi^{-1} \int_0^{\pi/2} [1 + (w^2/V^2) \cos^2 \phi]^{\frac{1}{2}} d\phi \right\} = \pi \sqrt{k'} / \left\{ 2 \int_0^K \text{dn}^{\frac{3}{2}}(u, k) du \right\}, \quad (65)$$

where K is the complete elliptic integral of the first kind with modulus $k = w^2/(w^2 + V^2)$. When w/V is small we have approximately,

$$(V/\overline{V})^{\frac{1}{2}} = 1 - \frac{1}{8} w^2/V^2 + \dots \quad \text{or} \quad V/\overline{V} = 1 - \frac{1}{4} w^2/\overline{V}^2. \quad (66)$$

As an illustration, take the amplitude $a = 0.01$ cm., $T = 1/100$ sec., so that $w = 6.2$ cm./sec. The error due to vibration will remain less than $\frac{1}{4}$ of a per cent. as long as V is less than 60 cm./sec. An increase of amplitude to $a = 0.1$ cm. will introduce an appreciable error for velocities less than 600 cm./sec. It is thus important in the design of practical anemometers to construct the apparatus in such a way as to eliminate vibration or reduce it to so small an amplitude that the error contributed is negligible. In the present series of experiments the method adopted of keeping the wire under tension by means of a small weight in the manner shown in Diagram II. (c) proved to be effective in preventing the vibrations over the range of velocities employed. At very high velocities vibrations of considerable amplitude were at times set up in spite of the frictional constraint at the lower end of the wire.

Section 13. On the Reduction and Interpretation of the Observations.

Sample series of observations are given in Tables III. and IV. showing the method of entering the observations in the case of each wire. For each velocity (corrected for *swirl*) the current required to heat the wire to a predetermined series of resistances (from which the temperatures were calculated) were measured. The corresponding heat-loss in watts per unit length was then calculated and entered as the second entry

in each compartment of the table. The theory of Part I. suggests that these results be examined in the light of the formula

[illegible]

where B and C are functions of the temperature and of the dimensions of the wire. If W is plotted against $\sqrt{V}$ for each temperature, we should, according to (67), obtain a family of straight lines, and figs. 4 and 5 show this conclusion to be justified within the limits of experimental error. By determining the line of closest fit through the observed points, we are enabled to calculate the constants B and C . The position of this line can be found by calculating for each system of observed points the position of the major axis of inertia of the corresponding system of material points of equal weight.⁽³⁷⁾ It was, however, found to be extremely tedious to make use of this method and it was deemed sufficiently accurate to adopt the simpler method of dividing the observations into two groups, finding the centre of gravity of each and taking the position of the line joining these two points as an approximation to the line of closest fit. The constants B and C were found in this way directly from the tabulated values of W and $\sqrt{V}$. The accuracy of the observations was tested by plotting for each wire the family of approximately straight lines corresponding to the various temperatures, and in this way a check was obtained on the determination of the constants B and C .

The constants B and C obtained in this manner are set out with the temperature and diameter of the wire in Tables V. and VI., and will now be discussed separately.

(i.) Analysis of the Convection Constant B.

The theoretical investigations of Sections 4, 5, and 6 suggest that the constant B be examined in the light of the formula $B = \beta(\theta - \theta_0)$. The third entry in each compartment of Table V. gives the value of $\beta = B/(\theta - \theta_0)$ for $\theta_0 = 17^\circ \text{ C.}$, from which it is seen in fact that β is very nearly independent of the temperature; the slight increase in its value with increasing temperature may be represented by a small temperature coefficient b such that $\beta = \beta_0 [1 + b(\theta - \theta_0)]$ and which may be interpreted as due to the combined variation with temperature of the diameter of the wire, and the specific heat, density, and heat conductivity of air. The variation of β with temperature is very little greater than that due to the errors of experiment, and the coefficient b was roughly determined by plotting β against the temperature, drawing in the line of closest fit and hence determining β_0 and b , giving a mean value $b = 0.000080$.

Theory further requires that β_0 be given by the expression $\beta_0 = 2 \sqrt{\pi s_0 \kappa_0 \sigma_0 a_0}$, where the suffix $_0$ refers to the temperature 17°C . That the ratio $\beta_0/\sqrt{a_0}$ is constant is

(³⁷) Formulæ for the determination of this straight line are given by KARL PEARSON ('Phil. Mag.,' 2, November, 1901, p. 559), and also by SNOW, E. C. ('Phil. Mag.,' 21, March, 1911, p. 367).

and is found to agree with the determinations of LANGMUIR⁽⁴¹⁾ obtained by a somewhat different calculation. The value of the radiation loss $E = 2\pi ae$ per unit length is plotted against the temperature, and from this curve the radiation loss for any wire may be easily determined at any temperature. The values of the radiation losses thus obtained are set down in Table VI. under the constants C and the true convection loss $C_0 = C - E$ obtained by subtraction. For each wire the ratio $C_0/(\theta - \theta_0)$ is determined over the range of temperatures and is plotted against $\theta - \theta_0$. The result shows very approximately a straight line whose constants are determined from the position of the line of closest fit in the manner already described. In this way the constants in the formula

$$C_0 = \gamma_0 (\theta - \theta_0) [1 + c (\theta - \theta_0)]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

were obtained for each wire. The constant γ_0 is very nearly independent of the diameter of the wire as we should expect from the theoretical discussion of Part I. By plotting its value against the radius we find approximately

$$\gamma_0 = 2.50 \times 10^{-4} (1 + 70a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

expressed in watts. According to the theoretical development we should expect the relation $C_0 = \kappa_0 (\theta - \theta_0)$. Taking $\kappa_0 = 5.66 \times 10^{-5}$ calories = 2.37×10^{-4} watts, this result is in surprisingly good agreement with the value of γ_0 given in (73), when the uncertainty of the exact correction for radiation is taken into account.⁽⁴²⁾ The slight dependence of γ_0 on the radius of the wire is probably due to the effect of viscosity. The coefficient c is seen to vary remarkably little from the mean value $c = 0.00114$, and may be considered to represent in large measure the variation of the heat-conductivity with the temperature.⁽⁴³⁾

⁽⁴¹⁾ LANGMUIR, "Convection and Conduction of Heat in Gases," 'Phys. Rev.,' 34, p. 415, 1912.

⁽⁴²⁾ The success of the theoretical formula (33),

$$H = \kappa\theta_0 + 2\sqrt{\pi\kappa\sigma sa} V^{\frac{1}{2}}\theta_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

in representing experimental results leads to a possibility worth future investigation as to whether the convection method may not be used in the determination of the heat-conductivity and specific heat of gases as a method of continuous flow. By making the temperature difference θ in (i.) small and introducing refinements in the methods of measurement, the procedure of the present experiment is especially well adapted to the study of the variation of the conductivity and specific heat of gases over an extremely wide range of temperature and pressure. A slight modification of the disposition of the apparatus would enable the viscosity of the same specimen of gas to be measured under the same conditions, an experimental procedure of the greatest importance in the interpretation of these results according to the kinetic theory of gases (see CHAPMAN, "The Kinetic Theory of a Gas Constituted of Spherically Symmetrical Molecules," 'Phil. Trans.,' A 482, vol. 211, pp. 462-476).

⁽⁴³⁾ It is interesting to note that WINKLEMANN gives a temperature coefficient 0.00190 for the variation of thermal conductivity with temperature (see the 'Smithsonian Physical Tables,' 1910, p. 200).

It follows from (32) that the heat-loss by convection can be obtained from a formula of the form

$$H = 2\pi\kappa_0 (\theta - \theta_0) [1 + c (\theta - \theta_0)] / [\log b/a], \quad . \quad . \quad . \quad . \quad . \quad (75)$$

to which must be added the radiation loss given by

$$E = 2\pi a \times 0.514 (\theta/1000)^{5.2}, \quad . \quad . \quad . \quad . \quad . \quad (76)$$

E being expressed in watts per cm.² Formula (75) is tested in the light of LANGMUIR's observations on free convection⁽⁴⁴⁾ which were carried out over a much wider range of temperature than those of the writer. The analysis of these observations is discussed under Table VIII., from which it appears that to a fair degree of accuracy the denominator $\log b/a$ is independent of the temperature, but depends on the radius of the wire in the manner shown in fig. 9, the use of which affords a simple method of calculating the actual convection loss in any actual case.

It is interesting from the relation (28), $b = \kappa e^{1-\gamma}/(s\sigma\bar{V})$ to determine the effective velocity $\bar{V}$ of the convection current by means of which the wire is cooled. The variation of the term $\sqrt{\kappa s\sigma}$ with the temperature is small, and from the results of Section 13 may be expressed by means of the relation

$$\sqrt{\kappa s\sigma} = \sqrt{\kappa_0 s_0 \sigma_0} [1 + b (\theta - \theta_0)], \quad . \quad . \quad . \quad . \quad . \quad (77)$$

where $b = 0.000080$. Thus making use of (74) and writing

$$\bar{V}_0 = \kappa_0 e^{1-\gamma}/(s_0 \sigma_0 b), \quad . \quad . \quad . \quad . \quad . \quad (78)$$

we have

$$\bar{V} = \bar{V}_0 [1 + c (\theta - \theta_0)]^2 / [1 + b (\theta - \theta_0)]^2. \quad . \quad . \quad . \quad . \quad . \quad (79)$$

Values of the effective velocities $\bar{V}_0$ and $\bar{V}$ are given in Table VIII. It is probable that the values of $\bar{V}$ present velocities of the same order as the actual average velocity of the convection current generated by the heated wire, estimates of which are useful in determining the limits of temperature and velocity within which a hot-wire anemometer can be employed with accuracy.

(⁴⁴) LANGMUIR, 'Phys. Rev.,' 34, p. 415, 1912.

PART III.

ON THE DESIGN OF PORTABLE HOT-WIRE ANEMOMETERS.

Section 16. Introduction.

That the measurement of the current required to keep a wire at a given temperature (measured by its resistance) might be employed as a method of measuring air-velocity seems to have first been suggested by KENNELLY⁽⁴⁵⁾ in 1909. This method was developed independently both by BORDINI⁽⁴⁶⁾ and by MORRIS⁽⁴⁷⁾ about the same time, and actual measurements were carried out by these investigators. In the latter case a platinum wire was inserted in one arm of a Wheatstone bridge, the remaining resistances being constructed of manganin and unaffected by variation in temperature due to the heating of the measuring current; under these circumstances it was found that the square of the current required to keep a wire at a constant temperature of about 70° C. above the surrounding air was very nearly proportional to the square root of the air velocity for as high values as 40 miles per hour; it was found necessary to make a small correction corresponding to the term C of the formula $W = B\sqrt{V+C}$.

A form of integrating anemometer, suitable for measuring the total flow across a given cross-section, has recently been described by GERDIEN.⁽⁴⁸⁾ All of the instruments described are more especially suited to the measurement of average velocities over a considerable area.⁽⁴⁹⁾

The aim of the experiments described in Part II. of the present paper was to determine the convection constants of platinum wire in absolute measure, in order that it might be possible to construct from the easily determined electrical constants

⁽⁴⁵⁾ See footnote ⁽²³⁾.

⁽⁴⁶⁾ BORDINI, U., "Un procedimento per la misura della velocità dei gas," read before the Società Italiana per il Progresso delle Scienze, October 13, 1911; published in the 'Nuovo Cimento,' Series VI., vol. III., pp. 241-283, April, 1912; see also the 'Electrician,' 70, p. 278, November 22, 1912.

⁽⁴⁷⁾ MORRIS, J. T., "The Electrical Measurement of Wind Velocity," read at the British Association, Dundee, September, 1912; published in the 'Engineer,' September 27, 1912; the 'Electrician,' October 4, 1912, p. 1056; see also the 'Electrician,' 70, p. 278, November 22, 1912.

⁽⁴⁸⁾ GERDIEN, H., "Der Luftgeschwindigkeitsmesser der Siemens und Halske," 'Ber. der Deutschen Phys. Gell.,' Heft 20, 1913.

⁽⁴⁹⁾ An interesting application of electric heating to the measurement of gas flow is to be found in the Thomas Gas Meter (THOMAS, C. C., 'Jour. of the Franklin Inst.,' November, 1911, pp. 411-460; also 'Proc. of the American Gas Inst.,' 7, p. 339, October, 1912; and 'Trans. Amer. Soc. Mech. Eng.,' 31, p. 655, 1909). In this form of instrument, however, the entire cross-section of gas is heated by means of a special heating grid, and the electric power required to maintain a constant difference of temperature, as measured by means of a pair of differential electric thermometers on each side of the heater, is recorded.

of the wire an anemometer capable of giving an accurate measure of air-velocity without reference to a calibration in terms of some other form of wind-measuring instrument. The problem is analogous to the determination of the constants of platinum wire for the purposes of thermometry; in fact standard thermometer wire is the most suitable for the purpose of hot-wire anemometry in that its electrical constants are usually determined with great accuracy.

The specifications described in the next section were carried out independently of any previous work, and as they offer several distinct advantages with regard to the construction of a practical standard of anemometry, it has been thought by the writer worth while to describe them in some detail. The special type of instrument developed may be called a *linear anemometer* consisting only of a single wire, and is especially suited for the study of turbulent flow and the analysis of sharp gradients of velocity such as are to be found in the neighbourhood of obstacles in streams, in jets, &c. The advantage to be derived from the use of a single wire lies in the fact that it is possible to measure with considerable accuracy the velocity in the neighbourhood of the wire with no appreciable disturbance of the flow at a distance of a few diameters away. This statement is easily verified from the classical solution of the flow of a stream past a cylindrical obstacle. In the case of a circular cylinder of radius a in a stream of a perfect fluid of undisturbed velocity V , the velocity potential of the point (r, θ) is given by $\phi = V(r + a^2/r) \cos \theta$, θ being measured from the down-stream direction of the stream. It is easily proved that the maximum velocity at a distance r , ($r > a$), occurs for $\theta = \pi/2$ and is given by $q = V(1 + a^2/r^2)$. Thus at a distance $r = 10a$, the velocity of the stream is disturbed by only 1 per cent. of its value; hence, in the case of a 3 mil anemometer wire, the theoretical resolving power in measuring a sharp gradient may roughly be set at 15 mils (about 0.04 cm.), the accuracy of velocity measurement being set at 1 per cent.

Section 17. General Considerations on the Design of Hot-Wire Anemometers.

The Kelvin bridge connections employed in the determination of the convection constants, as described in Part II., offered so many advantages that this system of connections was at once taken as the starting point in the design of an instrument for the measurement of wind-velocity.

The experiments showed that over the entire range of diameters employed, the power required to maintain the wire at the same resistance, corresponding to a temperature $\theta^\circ \text{C.}$, is given by the formula

$$W = B\sqrt{V} + C, \quad \text{where } B = \beta(\theta - \theta_0), \quad \beta = \beta_0[1 + 0.00008(\theta - \theta_0)],$$

and $\beta_0 = 1.432 \times 10^{-3} \sqrt{a}$; furthermore, $C = C_0 + E$ where the radiation is given by $E = 2\pi a \times 0.514 (\theta/1000)^{5.2}$, $C_0 = \gamma_0(\theta - \theta_0)[1 + 0.00114(\theta - \theta_0)]$, and finally

$\gamma_0 = 2.50 \times 10^{-4}(1+70a)$. From this series of formulæ and constants it is possible to calibrate in absolute measure an anemometer wire provided its thermometric constants and its diameter are accurately known. It is simpler, however, to make use of a wire whose constants have been directly determined by calibration either by reference to a standard instrument or preferably in absolute measure by means of a rotating arm. The advantage of the Kelvin bridge connections enables a number of wires with potential terminals fused in place to be independently calibrated and inserted at will in a suitable anemometer fork, the readings of the instrument when balanced being independent of the connecting or contact resistances. It will be noticed from the above formulæ that by maintaining the anemometer wire at a sufficiently high temperature, the variations of the factor $\theta - \theta_0$ with changes of room temperature can be made small enough to be neglected. This result is important from the point of view of a practical instrument in that the calculation of wind velocity from the current required to balance the resistance of the anemometer wire against a fixed resistance is independent of the room temperature for such variations as may be met with under ordinary conditions of use; in any case, a small correction for the variation of room temperature may be applied if necessary. The most suitable temperature was found to be in the neighbourhood of 1000°C . The most convenient method of obtaining this adjustment is to adjust the manganin resistance B (fig. 2) so that its resistance is four times that of the platinum wire between potential terminals at room temperature. At this temperature variations of as much as 5°C . in room temperature only affect the velocity determination by about 1 per cent.—about the limit of accuracy in most measurements of turbulent flow. At the high temperatures employed the wires were found to remain in a very constant state, and no variations of electrical constants sufficient to affect the calibration constants for velocity measurements could be detected after extended use. It was found of great convenience in practice to be able to make rough adjustments of current necessary to balance the resistances by judging the degree of brightness of the anemometer wire which is dull red at a temperature for which the resistance is four times the resistance at room temperature. It will be noticed from Table VIII. that the effective velocity of the free convection current increases comparatively slowly with increased temperature to 1000°C ., and in most applications does not interfere seriously with the measurement of velocity.

Although it is desirable to operate the instrument with as small an expenditure of power as possible by making use of a very fine anemometer wire, it was found by experience to be impracticable to employ wires of diameter smaller than 3 mils; for a wire of this diameter, maintained in the neighbourhood of 1000°C ., velocities from zero to 25 miles per hour are represented by currents between 1 and 2 amperes. For wires smaller than 3 mils, the electrical constants are subject to change on drawing down the wire, and errors due to the cooling effect of the potential leads become relatively larger, although by special calibration an instrument with a wire as small as 1 mil, or less, could be successfully constructed to meet special requirements.

For the same wire the current i required to bring the wire to the same temperature is given by a relation of the form

[illegible]

where i_0^2 and k are the constants of the instrument determined either by calculation or by direct calibration ; in practice there is some advantage in making use of a dynamometer form of ammeter in which the scale reading is proportional to i^2 , provided it is sufficiently accurate ; the range of readings can be much increased by arranging the instrument so that the scale has a suppressed zero.

Another point connected with hot-wire anemometry may be mentioned here. The constants of the instrument or its calibration-curve determined by means of a rotating arm represent determinations of velocity appropriate to stream-line motion; when employed to measure turbulent flow the velocity measured by the hot-wire instrument may be referred to as the *effective velocity*. The question then arises as to whether this measure is a consistent one and is independent of the diameter of the wire. This point was settled in the course of experiments carried out by Prof. A. M. GRAY⁽⁵⁰⁾ and the writer on the distribution of air-velocities in the neighbourhood of rapidly rotating cylinders. The velocity gradient in the neighbourhood of a rapidly revolving wooden pulley was measured by the use of an anemometer fitted with a 3-mil wire and also under the same conditions with a 5-mil wire. It was found that the velocity gradients agreed within the limits of experimental error. It would thus seem possible to obtain a consistent measure of turbulent flow under conditions where other methods of measurement would be inadequate.

Section 18. *Note on the Applications of the Hot-Wire Anemometer.*

The advantages of the anemometer designs and connections described under Diagram III. in the establishment of a standard system of anemometry may be briefly stated as follows :—

- (i.) The use of the Kelvin bridge connections makes it possible to standardize and calibrate anemometer wires at a central laboratory independently of the remainder of the apparatus with which the wire is to be employed.
- (ii.) These connections also enable the wire to be heated to a high temperature and so makes the determinations of velocity practically independent of ordinary variations of room temperatures.
- (iii.) The use of the linear anemometer makes it possible to establish a consistent measure of turbulent flow.
- (iv.) The linear anemometer makes it possible to analyse sharp gradients of velocities without disturbing the flow at the point of measurement.

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(v.) The properties of the Kelvin bridge connections make it possible to connect an anemometer through low resistance slip-rings and provide a means of attacking such problems as the analysis of velocities in the neighbourhood of rapidly revolving areoplane propeller blades, or between the blades of centrifugal fans.

In addition to the applications already mentioned incidentally, it will be noticed that the use of the linear anemometer makes it easily possible to analyse the distribution of air-velocities in the neighbourhood of obstacles of various dimensions and shapes. An important application of this method is to be found in aerotechnical problems, such as the analysis of propeller wakes, distribution of velocities over planes of various dimensions and camber, &c. A compilation of such data would prove to be of the greatest service in developing a rational theory of turbulent flow. An instrument of the type under consideration could easily be constructed to give a continuous graphical record, thereby greatly extending its usefulness in analyses of complicated velocity distributions.

The application of the theoretical part of the present paper to the measurement of the thermal conductivity of gases has already been mentioned; in the case of liquids the chief application of the method lies in the investigation of a possible instrument for the convenient and rapid measurement of thermal conductivity. Experiments by ROGOVSKIJ⁽⁵¹⁾ seem to indicate the existence of a law of heat-loss of the form already studied.

In a somewhat different field of application, the laws of heat-loss already studied probably determine the temperature which a small solid object is able to acquire in a current of highly heated gas such as a Bunsen or blow-pipe flame. In fact the hot-wire method seems capable of affording a convenient means of analysing the velocities and true temperatures of flames or of heated gases in boiler-flues, &c.

It is hoped at a latter date to undertake in detail the study of some of the problems suggested in the present section.

Before closing the writer wishes to express his sincere thanks to Prof. H. T. BARNES, F.R.S., Director of the Macdonald Physics Laboratory, for the kind way in which he has facilitated the present work by every means in his power.

⁽⁵¹⁾ ROGOVSKIJ, E. A., 'Comptes Rendus Acad. Sci., Paris,' vol. 136, pp. 1391-1393, 1903; also vol. 141, pp. 622-624, 1905.

NUMERICAL TABLES AND DIAGRAMS: COMPARISON OF THEORY WITH OBSERVATION.

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Table of the Functions $e^x K_0(x)$, $e^x K_1(x)$, $\int_0^x e^x K_0(x) dx$, $x / \left[\int_0^x e^x K_0(x) dx \right]$.

The functions tabulated in the first two columns of the table opposite are based on JAHNKE and EMDÉ's tables⁽¹⁾ of the functions $K_0(x)$ and $K_1(x)$ for values of x between $x = 0.1$ and $x = 12.0$. The process of multiplication by the factor e^x was easily carried out by means of a "Brunsviga" calculating machine.⁽²⁾ For values of the argument $x < 0.1$ the functions were calculated directly from the convergent expansions

$$K_0(x) = -I_0(x) [\gamma + \log \frac{1}{2}x] + \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} (1 + \frac{1}{2}) + \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} (1 + \frac{1}{2} + \frac{1}{3}) + \dots \quad (\text{i.})$$

$$-K_1(x) = I_1(x) [\gamma + \log \frac{1}{2}x] + \frac{1}{2} \cdot \frac{x}{2} \left(\frac{2}{x} \right)^2 - \frac{1}{2} \cdot \frac{x}{2} \left[\frac{1}{1!} + \frac{1+1+\frac{1}{2}}{1! \cdot 2!} \left(\frac{x}{2} \right)^2 + \frac{1+\frac{1}{2}+1+\frac{1}{2}+\frac{1}{3}}{2! \cdot 3!} \left(\frac{x}{2} \right)^2 + \dots \right] \quad (\text{ii.})$$

For large values of the argument numerical values were obtained from the asymptotic expansions

$$e^x K_0(x) = \left(\frac{\pi}{2x} \right)^{1/2} \left[1 - \frac{1}{8x} + \frac{1^2 \cdot 3^2}{2! (8x)^2} - \dots \right] \quad (\text{iii.})$$

$$-e^x K_1(x) = \left(\frac{\pi}{2x} \right)^{1/2} \left[1 + \frac{3}{8x} - \frac{3 \cdot 5}{2! (8x)^2} + \frac{3 \cdot 5 \cdot 7}{3! (8x)^3} - \dots \right] \quad (\text{iv.})$$

For values of $x < 0.1$, the function $\int_0^x e^x K_0(x) dx$ is most easily calculated from the convergent series obtained by integrating (i.) term by term after multiplying by the factor e^x :

$$x^{-1} \int_0^x e^x K_0(x) dx = 1 + \frac{1}{4}x + \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots - (\gamma + \log \frac{1}{2}x) (1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{5}{48}x^3 + \dots) \quad (\text{v.})$$

From the point $x = 0.1$ onwards it is convenient to make use of EULER'S⁽³⁾ formula for quadratures in the form

$$2h^{-1} \int_{x_0}^{x_n} y dx = [(y_0 + y_1) + (y_1 + y_2) + \dots + (y_{n-1} + y_n)] - \frac{1}{6}h [f'(x_n) - f'(x_0)] + \frac{1}{360}h^3 [f'''(x_n) - f'''(x_0)] - \dots \quad (\text{vi.})$$

In the case under consideration we have

$$f'(x) = e^x [K_0(x) + K_1(x)], \quad f'''(x) = (4 - 3x^{-1} + 2x^{-2}) e^x K_1(x) + (4 - x^{-1}) e^x K_0(x). \quad (\text{vii.})$$

The first term on the right-hand side of (vi.) in square brackets [] is especially well suited for computation on an adding machine; the totals $(y_0 + y_1)$, $(y_0 + y_1) + (y_1 + y_2)$, &c., are obtained as successive results of a single series of operations and are thus easily tabulated. Opposite these entries are written down the values of $\frac{1}{6}hf'(x_n)$, obtained by using equation (vii.) from the first two columns of Table I. The third order correction term in (vi.) can be easily shown to be too small to affect the value of the series to 1 part in 1000, as seen from the numerical example given below. Starting with $\int_0^{0.1} y dx = 0.3572$ calculated from the convergent series, and taking $h = 0.1$, the work is tabulated as follows:—

x .	y .	$\frac{1}{6}hf'(x)$.	$\frac{1}{360}h^3f'''(x)$.	$2h^{-1} \int_0^{0.1} y dx + S_n$.	C'_n .	C'''_n .	$\int_0^{x_n} y dx$.
0.1	2.6824	-0.1368	-0.0052	—	—	—	0.3572
0.2	2.1407	-0.0615	-0.0006	11.9681	0.0753	0.0046	0.5946

(1) See footnote (11).

(2) For a description of this instrument see D'OCAGNE, 'Le Calcul Simplifié' (Gauthier Villars, Paris, 1905), pp. 62-65.

(3) See footnote (10).

where

$$S_n = (y_0 + y_1) + (y_1 + y_2) + (y_2 + y_3) + \dots + (y_{n-1} + y_n),$$

$$C'_n = \frac{1}{8}h[f'(x_n) - f'(x_0)], \quad \text{and} \quad C'''_n = \frac{1}{360}h^3[f'''(x_n) - f'''(x_0)].$$

By carrying out this process the function $\int_0^x e^x K_0(x) dx$ was tabulated as far as $x = 6.0$, beyond which point the calculation by means of the asymptotic series involves less labour. We obtained, in equation (24), an expansion of the form

$$\int_0^x e^x K_0(x) dx + C = \sqrt{(2\pi x)} \left[1 + \frac{1}{8x} - \frac{1^2 \cdot 3}{2!(8x)^2} + \frac{1^2 \cdot 3^2 \cdot 5}{3!(8x)^3} - \dots \right] \dots \dots \dots \quad (\text{viii.})$$

That the constant C is unity is very probably true although an analytical proof of the result would be very laborious; for the purposes of the present work the case is sufficiently justified by the following numerical calculations of the sum of the series on the right-hand side of (viii.). These calculations, at the same time, furnish a check on the results obtained by quadratures:—

	$x = 1.$	$x = 2.$	$x = 3.$	$x = 6.$
Series (v.)	1.779	—	—	—
Series (viii.)	[2.778] ⁽⁴⁾	3.7519	4.5165	6.2643
Quadratures	1.7804	2.7499	3.5127	5.2640
C	<u>[1.002]</u>	<u>1.0020</u>	<u>1.0038</u>	<u>1.0003</u>

The values of the function $x / \left[\int_0^x e^x K_0(x) dx \right]$ are easily obtained from the functions previously tabulated. This function when plotted against $\sqrt{x}$, as shown in Diagram I., gives very approximately a straight-line relation over the range dealt with in the present experiments. Simple approximate formulæ for the function in question are derived and tested in the description of Diagram I.

(⁴) The series (viii.) fails to give accurate values for so small a value of the argument as $x = 1$. The sum of four terms is 2.798, and of five 2.758, the value given above being the mean of these two determinations.

TABLE I.

x .	$e^x K_0(x)$.	$-e^x K_1(x)$.	$\int_0^x e^x K_0(x) dx$.	$x / \left[\int_0^x e^x K_0(x) dx \right]$.	$\sqrt{x}$.
·0001	9·3272	10001·000	·001033	·09683	·01
·0004	7·9412	2500·998	·003576	·1119	·02
·0009	7·1355	1112·108	·007319	·1230	·03
·0016	6·5642	625·994	·012095	·1323	·04
·0025	6·1227	400·992	·017788	·1405	·05
·0036	5·7630	278·767	·024313	·1481	·06
·0049	5·4611	205·068	·031601	·1551	·07
·0064	5·2005	157·233	·039578	·1617	·08
·0081	4·9718	124·435	·048224	·1680	·09
·0100	4·7638	100·974	·057472	·1740	·10
·0200	4·1139	50·966	·1015	·1971	·1414
·0300	3·7338	34·282	·1406	·2134	·1732
·0400	3·4726	25·940	·1765	·2266	·2000
·0500	3·2949	20·931	·2102	·2378	·2236
·0600	3·1142	17·588	·2421	·2478	·2449
·0700	2·9814	15·199	·2726	·2568	·2646
·0800	2·8680	13·406	·3018	·2650	·2828
·0900	2·7693	12·009	·3300	·2727	·3000
·1000	2·6823	10·891	·3572	·2799	·3162
·200	2·1407	5·8334	·5946	·3363	·4472
·300	1·8527	4·1253	·7936	·3780	·5477
·400	1·6626	3·2587	·9683	·4131	·6325
·500	1·5241	2·7309	1·1273	·4435	·7071
·600	1·4167	2·3738	1·2742	·4709	·7746
·700	1·3301	2·1151	1·4064	·4977	·8367
·800	1·2581	1·9179	1·5407	·5192	·8944
·900	1·1971	1·7623	1·6634	·5411	·9487
1·000	1·1444	1·6361	1·7804	·5616	1·0000
1·10	1·0983	1·5315	1·8925	·5812	1·049
1·20	1·0574	1·4429	2·0002	·6000	1·095
1·30	1·0208	1·3668	2·1041	·6178	1·140
1·40	·9883	1·3009	2·2045	·6351	1·183
1·50	·9582	1·2432	2·3018	·6516	1·225
1·60	·9311	1·1917	2·3963	·6677	1·265
1·70	·9059	1·1462	2·4881	·6832	1·304
1·80	·8826	1·1046	2·5775	·6983	1·342
1·90	·8611	1·0677	2·6646	·7121	1·378
2·00	·8416	1·0337	2·7498	·7273	1·414
2·1	·8232	1·0020	2·8330	·7412	1·449
2·2	·8057	·9738	2·9145	·7548	1·483
2·3	·7893	·9475	2·9942	·7681	1·517
2·4	·7740	·9229	3·0724	·7811	1·549
2·5	·7596	·9002	3·1490	·7938	1·581
2·6	·7459	·8789	3·2243	·8064	1·612
2·7	·7328	·8592	3·2982	·8186	1·643
2·8	·7206	·8405	3·3709	·8306	1·673
2·9	·7090	·8231	3·4424	·8424	1·703
3·0	·6978	·8066	3·5127	·8540	1·732

TABLE I. (continued).

x .	$e^x K_0(x)$.	$-e^x K_1(x)$.	$\int_0^x e^x K_0(x) dx$.	$x / \left[\int_0^x e^x K_0(x) dx \right]$.	$\sqrt{x}$.
3.2	.6768	.7762	3.6499	.8768	1.789
3.4	.6580	.7491	3.7836	.8986	1.844
3.6	.6405	.7243	3.9135	.9200	1.897
3.8	.6245	.7023	4.0399	.9406	1.949
4.0	.6093	.6814	4.1632	.9607	2.000
4.2	.5953	.6627	4.2837	.9804	2.049
4.4	.5823	.6453	4.4014	.9997	2.098
4.6	.5700	.6292	4.5169	1.0184	2.145
4.8	.5586	.6142	4.6300	1.0367	2.191
5.0	.5478	.6003	4.7406	1.0547	2.236
5.2	.5377	.5871	4.8491	1.0724	2.280
5.4	.5281	.5750	4.9557	1.0897	2.324
5.6	.5187	.5633	5.0604	1.1066	2.366
5.8	.5100	.5526	5.1633	1.1233	2.408
6.0	.5019	.5422	5.2640	1.1398	2.449
6.5	.4828	.5187	5.5114	1.1794	2.550
7.0	.4658	.4981	5.7482	1.2178	2.646
7.5	.4506	.4797	5.9775	1.2547	2.739
8.0	.4367	.4632	6.1972	1.2909	2.828
8.5	.4240	.4482	6.4120	1.3255	2.915
9.0	.4123	.4347	6.6221	1.3591	3.000
9.5	.4016	.4222	6.8328	1.3903	3.082
10.0	.3916	.4108	7.0234	1.4239	3.162
11.0	.3738	.3904	7.4076	1.4850	3.317
12.0	.3582	.3729	7.7723	1.5431	3.464
13.0	.3442	.3574	8.1248	1.6000	3.606
14.0	.3321	.3439	8.4583	1.6551	3.742
15.0	.3210	.3317	8.7887	1.7067	3.873
16.0	.3110	.3207	9.1036	1.7575	4.000
17.0	.3016	.3105	9.4102	1.8066	4.123
18.0	.2934	.3014	9.7090	1.8540	4.243
19.0	.2857	.2932	9.9974	1.9005	4.359
20.0	.2787	.2856	10.2790	1.9457	4.472
25.0	.2492	.2544	11.596	2.1559	5.000
36.0	.2082	.2111	14.090	2.5550	6.000
49.0	.1786	.1804	16.591	2.9535	7.000
64.0	.1564	.1576	19.092	3.3522	8.000
81.0	.1390	.1399	21.594	3.7509	9.000
100.0	.1252	.1258	24.097	4.1500	10.000

Table of the Functions $e^x K_0(x)$, $e^x K_1(x)$, $\int_0^x e^x K_0(x) dx$, $x / \left[\int_0^x e^x K_0(x) dx \right]$, and $\sqrt{x}$.

TABLE II.

Constants of Wires Used in the Determination of Convection Constants.

Specimen.	r_{17} .	r_{10} .	r'_{10} .	α .	d cm.	$\rho_0 \times 10^{-6}$.
1	·0582	·0547	·0546 ⁽¹⁾	·00375 ₅ ⁽⁴⁾	·01530	10·02
2	·0706	[·0662]	—	[·00375]	[·01387] ⁽⁷⁾	[10·00]
3	·0881	·0828	·0834 ⁽²⁾	·00374 ₅ ⁽⁵⁾	·01230	9·98
4	·1030	·0968	·0953	·00374 ₅	·01165	10·19
5	·1340	·1262	·1225	·00371 ₆	·01013	9·91
6	·1672	·1577	·1534	·00370 ₂	·00930	10·38
7	·2220	·2092	·2192 ⁽³⁾	·00370 ₀ ⁽⁶⁾	·00775	10·38
8	1·570	1·519	1·590	·00188 ₅	·00383	18·3
9	1·599	1·550	1·606	·00183 ₉	·00390	19·15
10	1·852	1·795	1·847	·00332 ₅	·00283	11·60
10 ₁	1·865	1·765	—	·00328	·00278	10·70
10' ₁	1·771	1·692	—	·00341	·00278	10·25

⁽¹⁾ Mean of two determinations, 0·0545 and 0·0547.⁽²⁾ Mean of two determinations each, 0·0834.⁽³⁾ Mean of two determinations, 0·2193 and 0·2191.⁽⁴⁾ Mean of three determinations, 0·003761, 0·003752, and 0·003752.⁽⁵⁾ Mean of 0·003746 and 0·003745.⁽⁶⁾ Mean of 0·003703 and 0·003715.⁽⁷⁾ Wire No. 2 burned out during the experiment and the constants given in square brackets are the means of those of wires No. 1 and No. 2.

Description of Table II.—Constants of Wires Used in the Determination of Convection Constants.

The resistance per unit length at 17°C. , denoted in Table II. by r_{17} , was obtained by the method described in Section 9 with the wire placed in the fork and annealed for several minutes before the convection measurements were made; care was taken to use a very small measuring current to avoid heating. These values of resistance were reduced to 0°C. , making use of the temperature coefficients given in the fifth column. After the test had been carried out the temperature coefficient was determined for each wire by coiling it loosely in the grooves of a double screw thread cut in an ebonite cylinder. Potential terminals were fused to the wire at a measured distance apart and these were with the current terminals brought out to the bridge connections by means of fine copper leads from the glass test-tube in which the ebonite cylinder was inserted. The resistance between potential terminals was compared with that of a manganin standard at the ice and steam points. In this way both the resistance per unit length r'_0 and the temperature coefficient α were determined. It will be noticed that in the case of the large wires the prolonged annealing is more effective in diminishing the resistance than the reduction of diameter by "evaporation" in increasing it, while the contrary may be noticed in the finer wires. Wires No. 1 to No. 7 show a marked diminution in the temperature coefficient with diameter, a fact well worth a special investigation. Wires No. 8 to No. 10 were probably drawn from a specimen of impure platinum.

As described in Section 10, the diameters of the wires were measured directly by means of a microscope.⁽⁸⁾ It was found that the diameters of various portions of each wire varied about 1 per cent. The mean diameter was found from ten to fifteen observations. A typical set of readings for the finest wire (1 mil) is given below.

Wire No. 10. Readings of micrometer head at intervals of 3 mm. :—

0.405 0.395 0.412 0.413 0.395 0.395 0.400 0.400 0.395 0.041 Mean 0.401

Calibration of micrometer readings; micrometer readings corresponding to 0.030 cm. on stage micrometer situated in the same part of the field :—

0.432 0.434 0.431 Mean 0.432

Mean diameter of wire No. 10 $d = 0.00278\text{ cm.}$

(⁸) An attempt was made to measure the diameter of the wire by rolling it between two narrow plates of smooth ground glass and counting the number of revolutions of the wire for a given displacement of the plates as measured by a micrometer screw—according to the method described by HORTON ('Phil. Trans.,' A, vol. 204, p. 407, 1905) in the measurement of the diameter of quartz fibres. In the present instance the platinum wire proved to be too soft for the employment of this method; the surface of the wire proved on microscopic examination to be badly roughened and the diameter sensibly changed by the action of the ground glass. The use of smooth glass plates was found to be unsuitable owing to the danger of slip.

TABLE III.

*Specimen Observation Sheet, September 15, 1912.*Air temperature, $16^{\circ}\cdot 8$ C. Relative humidity, 62 per cent.

Wire No. 1. Diameter, 0.01530 cm. Length between potential terminals, 23.2 cm.

Total length, 27.2 cm. Resistance per unit length at 17° C., 0.0582 ohms.

R/R_{17} Temperature, degrees C. r/r_{17} r ohms	1.727 227° 1.730 .1008	2.37 426° 2.40 .1397	2.965 627° 2.98 .1733	3.51 817° 3.54 .2025	4.00 1004° 4.05 .2360
T = 17.63 sec. $V_r = 19.11$ $2\pi L = 337$ cm. $V = 17.4$ (1-s) = .912 $\sqrt{V} = 4.17$	1.40 .198	1.76 .435	2.02 .703	2.22 1.000	2.44 1.405
T = 6.84 sec. $V_r = 49.3$ $2\pi L = 337$ cm. $V = 45.0$ (1-s) = .912 $\sqrt{V} = 6.70$	1.615 .263	2.02 .572	2.29 .908	2.52 1.285	2.72 1.745
T = 9.77 sec. $V_r = 86.4$ $2\pi L = 844$ cm. $V = 82.4$ (1-s) = .956 $\sqrt{V} = 9.07$	1.81 .331	2.27 .721	2.50 1.081	2.74 1.521	2.92 2.015
T = 9.50 sec. $V_r = 174.8$ $2\pi L = 1660$ cm. $V = 165.0$ (1-s) = .947 $\sqrt{V} = 12.83$	2.05 .424	2.54 .905	2.82 1.375	3.08 1.925	3.28 2.54
T = 5.55 sec. $V_r = 299$ $2\pi L = 1660$ cm. $V = 283$ (1-s) = .947 $\sqrt{V} = 16.8$	2.33 .549	2.84 1.127	3.17 1.740	3.42 2.370	3.64 3.140
T = 3.425 sec. $V_r = 485$ $2\pi L = 1660$ cm. $V = 459$ (1-s) = .947 $\sqrt{V} = 21.4$	2.57 .667	3.08 1.328	3.43 2.030	3.70 2.77	3.90 3.60
T = 2.60 sec. $V_r = 638$ $2\pi L = 1660$ cm. $V = 603$ (1-s) = .947 $\sqrt{V} = 24.55$	2.66 .712	3.25 1.480	3.61 2.260	3.89 3.07	4.10 3.97
T = 1.954 sec. $V_r = 850$ $2\pi L = 1660$ cm. $V = 803$ (1-s) = .947 $\sqrt{V} = 28.3$	2.86 .825	3.44 1.660	3.81 2.525	4.10 3.42	4.32 4.41
Constants B and C in the formula $W = B\sqrt{V} + C$ determined from the line of closest fit.	B = .0264 C = .089	.0508 .241	.0772 .388	.1012 .602	.1272 .884

Description of Tables III. and IV.—Specimen Observation Sheets.

In Tables III. and IV. are set out the observations and calculations of the heat-losses in the case of the specimens Nos. 1 and 10 of the wires tested at different velocities. Although observations of relative humidity and of barometric pressures were made at the time of the experiment, no effect due to the variation of these factors could be detected in the final results. The ratio R/R_{17} refers to the entire length of wire between the potential terminals, while the ratio r/r_{17} refers to unit length at the temperature under consideration and is corrected for thermal expansion by the formula of Section 10. The time of a single revolution of the rotating arm, denoted by T , was obtained from the measurement of a chronograph sheet. The fork containing the wire was clamped in one of three positions so that the radius L was one of the values 53.7 cm., 134.0 cm., and 264.3 cm.; the apparent velocity of the wire through the air was given by $V_r = 2\pi L/T$. Owing to the "swirl" set up by the rotating arm, the true velocity of the wire through the air was obtained from the formula $V = (1-s) V_r$, the correction factor $(1-s)$ having been determined for each value of the radius in the manner described under Diagram II. The values printed in italics in the Tables III. and IV. are the heat-losses expressed in watts per unit length, as calculated from the current required to increase the resistance of the wire to the value R , and entered in the same compartment. These values when plotted against the square root of the velocity give rise to a family of straight lines, the temperature being the variable parameter. These are illustrated in figs. 4 and 5, and indicate how closely these curves correspond to the theoretical curve shown in Diagram I. It was found impossible to work at very low velocities owing to the disturbing effect of the free convection current set up by the heated wire. It was thus impossible to follow out the experimental curves to small velocities with a view to comparison with theory. It will be seen from the graph of the theoretical curve that the straight-line asymptote lies extremely close to the curve over the interval covered by experiment. In the reduction of the observations the line of closest fit through the experimentally determined points was taken to represent the equation of the asymptote given by formula (33).

TABLE IV.

*Specimen Observation Sheet, October 28, 1912.*Air temperature, $15^{\circ} \cdot 3$ C. Relative humidity, 55 per cent.

Wire No. 10. Diameter, 0.00283 cm. Length between potential terminals, 3.64 cm.

Total length, 4.6 cm. Resistance per unit length at 17° C., 1.852 ohms.

R/R_{17} Temperature, degrees C. r/r_{17} r ohms	1.398 162° 1.400 2.59	1.727 272° 1.730 3.20	2.00 366° 2.010 3.72	2.37 499° 2.39 4.43	2.60 584° 2.63 4.87	2.965 724° 2.98 5.52
T = 18.00 sec. $V_r = 18.72$ $2\pi L = 337$ cm. $V = 17.1$ (1-s) = .912 $\sqrt{V} = 4.13$	—	.200	.226	.260	.280	.296
	—	.128	.190	.300	.382	.486
T = 6.64 sec. $V_r = 50.8$ $2\pi L = 337$ cm. $V = 46.3$ (1-s) = .912 $\sqrt{V} = 6.80$	—	.228	.256	.288	.306	.328
	—	.166	.244	.394	.456	.597
T = 9.097 sec. $V_r = 182.7$ $2\pi L = 1660$ cm. $V = 173.0$ (1-s) = .947 $\sqrt{V} = 13.13$	.171	.284	.314	.349	.370	.392
	.0751	.258	.367	.540	.668	.851
T = 4.83 sec. $V_r = 342$ $2\pi L = 1660$ cm. $V = 324$ (1-s) = .947 $\sqrt{V} = 18.0$	.190	.314	.348	.388	.410	.432
	.0939	.316	.451	.668	.819	1.035
T = 3.77 sec. $V_r = 441$ $2\pi L = 1660$ cm. $V = 417$ (1-s) = .947 $\sqrt{V} = 20.4$	.201	.328	.362	.408	.427	.451
	.1051	.345	.488	.740	.890	1.126
T = 2.41 sec. $V_r = 688$ $2\pi L = 1660$ cm. $V = 650$ (1-s) = .947 $\sqrt{V} = 25.47$	.222	.358	.396	.440	.460	.485
	.128	.411	.583	.859	1.030	1.305
T = 1.80 sec. $V_r = 922$ $2\pi L = 1660$ cm. $V = 871$ (1-s) = .947 $\sqrt{V} = 29.5$	(.234) ⁽¹⁾	.380	.420	.462	.489	.515
	(.1422)	.463	.657	.944	1.163	1.470
Constants B and C in the formula $W = B\sqrt{V} + C$ determined from the line of closest fit.	B = .00425 C = .0182	.01292 .081	.0181 .123	.0256 .205	.0308 .257	.0384 .338

⁽¹⁾ Owing to an interruption in the series of readings the velocity corresponding to this observation was $V = 856$ cm./sec.

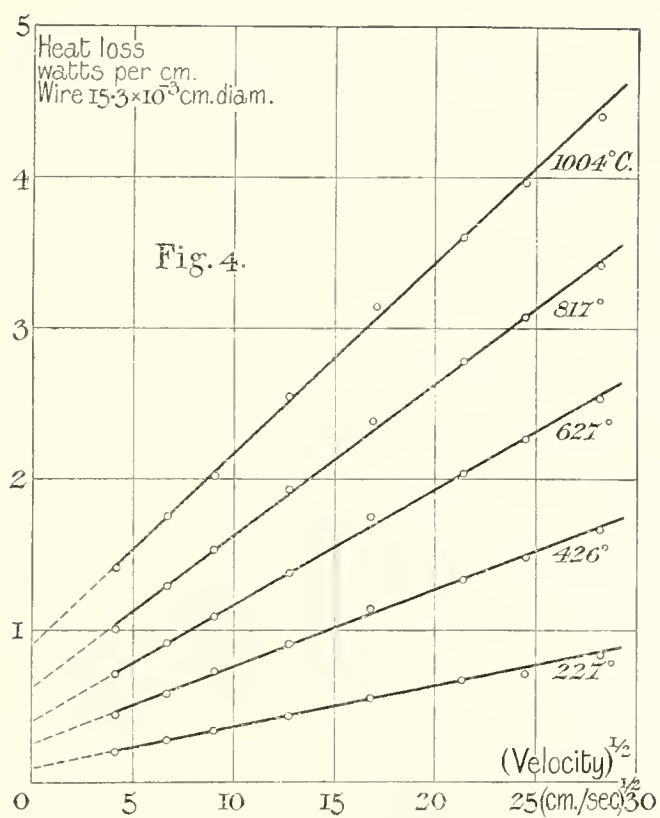


Fig. 4. Heat-Loss Velocity Curves.
Wire No. 1.

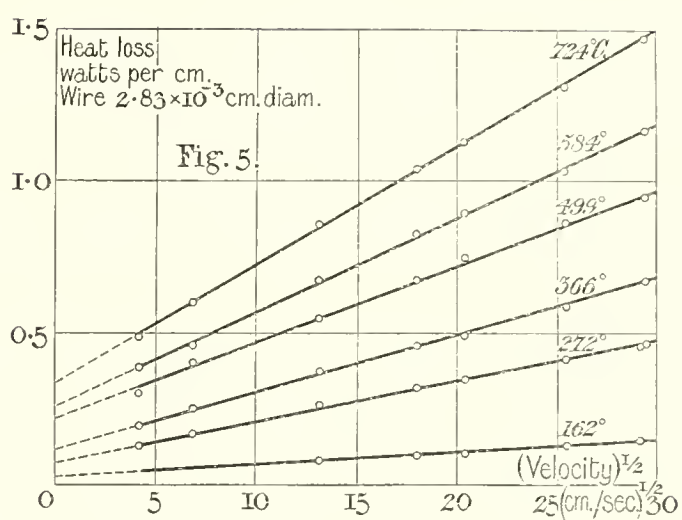


Fig. 5. Heat-Loss Velocity Curves.
Wire No. 10.

TABLE V.

Wire No. 1	$\theta - \theta_{17}$	210°	409°	610°	800°	987°		$\sqrt{a} = .0875$ $\beta_0 = 1.257 \times 10^{-4}$ $\beta_0 / \sqrt{a} = 1.437 \times 10^{-3}$ $b = 4.3 \times 10^{-5}$
$d = .01530$	$B =$	.0264	.0508	.0772	.1012	.1272		
$a = .00765$	$B/(\theta - \theta_{17})$	1.255	1.241	1.265	1.265	1.290×10^{-4}		
Wire No. 2	$\theta - \theta_{17}$	210°	410°	609°	801°	990°	1161°	$\sqrt{a} = .0832$ $\beta_0 = 1.207 \times 10^{-4}$ $\beta_0 / \sqrt{a} = 1.448 \times 10^{-3}$ $b = 3.7 \times 10^{-5}$
$d = .01387$	$B =$	.0255	.0515	.0728	.0985	.1233	.1481	
$a = .00693$	$B/(\theta - \theta_{17})$	1.230	1.255	1.192	1.230	1.248	1.275×10^{-4}	
Wire No. 3	$\theta - \theta_{17}$	210°	411°	608°	803°	993°	1165°	$\sqrt{a} = .0784$ $\beta_0 = 1.130 \times 10^{-4}$ $\beta_0 / \sqrt{a} = 1.441 \times 10^{-3}$ $b = 8.2 \times 10^{-5}$
$d = .01230$	$B =$	.0238	.0468	.0718	.0961	.1190	.1422	
$a = .00615$	$B/(\theta - \theta_{17})$	1.133	1.142	1.180	1.196	1.198	1.222×10^{-4}	
Wire No. 4	$\theta - \theta_{17}$	210°	411°	608°	803°	993°	1165°	$\sqrt{a} = .0763$ $\beta_0 = 1.078 \times 10^{-4}$ $\beta_0 / \sqrt{a} = 1.412 \times 10^{-3}$ $b = 11.4 \times 10^{-5}$
$d = .01165$	$B =$	.0224	.0458	.0696	.0932	.1168	.1410	
$a = .00582$	$B/(\theta - \theta_{17})$	1.067	1.116	1.145	1.161	1.175	1.208×10^{-4}	
Wire No. 5	$\theta - \theta_{17}$	213°	413°	613°	807°	1001°		$\sqrt{a} = .0710$ $\beta_0 = 1.020 \times 10^{-4}$ $\beta_0 / \sqrt{a} = 1.438 \times 10^{-3}$ $b = 8.1 \times 10^{-5}$
$d = .01013$	$B =$	.0228	.0422	.0655	.0880	.1102		
$a = .00506$	$B/(\theta - \theta_{17})$	1.070	1.023	1.070	1.090	1.102×10^{-4}		
Wire No. 6	$\theta - \theta_{17}$	213°	416°	616°	812°	1001°	1179°	$\sqrt{a} = .0682$ $\beta_0 = .988 \times 10^{-4}$ $\beta_0 / \sqrt{a} = 1.448 \times 10^{-3}$ $b = 8.3 \times 10^{-5}$
$d = .00930$	$B =$	.0221	.0417	.0632	.0834	.1094	.1291	
$a = .00465$	$B/(\theta - \theta_{17})$	1.038	1.002	1.027	1.027	1.092	1.092×10^{-4}	
Wire No. 7	$\theta - \theta_{17}$	213°	413°	612°	805°	997°		$\sqrt{a} = .0622$ $\beta_0 = .872 \times 10^{-4}$ $\beta_0 / \sqrt{a} = 1.405 \times 10^{-3}$ $b = 12.0 \times 10^{-5}$
$d = .00775$	$B =$	.0193	.0377	.0577	.0771	.0984		
$a = .00387$	$B/(\theta - \theta_{17})$	.907	.913	.927	.958	$.985 \times 10^{-4}$		
Wire No. 8	$\theta - \theta_{17}$	230°	434°	613°	877°			$\sqrt{a} = .0438$ $\beta_0 = .640 \times 10^{-4}$ $\beta_0 / \sqrt{a} = 1.460 \times 10^{-3}$
$d = .00383$	$B =$	.0140	.0272	.0372	.0540			
$a = .00192$	$B/(\theta - \theta_{17})$	.657	.652	.625	$.628 \times 10^{-4}$			
Wire No. 9	$\theta - \theta_{17}$		439°		885°			$\sqrt{a} = .0441$ $\beta_0 = .630 \times 10^{-4}$ $\beta_0 / \sqrt{a} = 1.430 \times 10^{-3}$
$d = .00390$	$B =$		.0261		.0542			
$a = .00195$	$B/(\theta - \theta_{17})$		.634		$.625 \times 10^{-4}$			
Wire No. 10	$\theta - \theta_{17}$	145°	255°	349°	482°	567°	707°	$\sqrt{a} = .0380$ $\beta_0 = .531 \times 10^{-4}$ $\beta_0 / \sqrt{a} = 1.396 \times 10^{-3}$ $b = 8.1 \times 10^{-5}$
$d = .00283$	$B =$	.00425	.01292	.0181	.0256	.0308	.0384	
$a = .00144$	$B/(\theta - \theta_{17})$	.332	.544	.545	.551	.561	$.557 \times 10^{-4}$	
$W = B\sqrt{V} + C, \quad B = \beta(\theta - \theta_{17})$ $\beta = \beta_0[1 + b(\theta - \theta_{17})]$								Mean values { $\beta_0 / \sqrt{a} = 1.432 \times 10^{-3}$ $b = 8.0 \times 10^{-5}$

Description of Tables V. and VI.—Analysis of the Convection Constants B and C.

In Tables V. and VI. are summarised the results of the entire series of observations on the ten specimens of platinum wire. The measurement of the diameters and the calculation of the temperature of the wires have already been described in Sections 10 and 11. In the case of each wire the constants B and C of the formula $W = B\sqrt{V} + C$ were determined from the position of the lines of closest fit to the observed points illustrated in figs. 4 and 5.

In Table V. the constant B is printed in heavy-faced type and beneath it in italics the ratio $\beta = B/(\theta - \theta_0)$. The variation of B with temperature is illustrated for each wire by the graphs of fig. 6. The constant β increases slightly with the temperature in the manner indicated by the formula $\beta = \beta_0 [1 + b(\theta - \theta_0)]$. The constants β_0 and b were determined graphically by plotting β against $(\theta - \theta_0)$. Finally the ratio $\beta_0/\sqrt{a}$ was found to be very nearly constant in the case of each wire as shown in the graph of fig. 7, and its mean value is seen to be in fair agreement with that calculated from the theoretical investigation of Part I.

Table VI. contains the analysis of the constant C. The part of this term due to radiation is calculated in the manner described in Section 13, making use of the observations of LUMMER and KURLBAUM. In heavy-faced type are given the values of $C_0 = C - E$ and beneath it, in italics, the ratios $\gamma = C_0/(\theta - \theta_0)$, which it will be seen vary comparatively slowly with temperature and radius. For each wire the constants γ_0 and c of the formula $\gamma = \gamma_0 [1 + c(\theta - \theta_0)]$ were determined graphically. The constant γ_0 varies slightly with the radius and its value is not far removed from that required by the theory of Section 6. The value of γ_0 for Wire No. 2 shows a discrepancy which was explained, on microscopic observation, as due to the fact that owing to the use of an imperfect die the wire was badly scored along its length.

TABLE VI.

Wire No. 1	$\theta - \theta_{17}$	210°	409°	610°	800°	987°		
	C	.089	.241	.388	.602	.884		$\gamma_0 = 3.85 \times 10^{-4}$
$a = .00765$	E	.002	.005	.014	.040	.087		
	$C_0 = C - E =$	.087	.237	.374	.562	.797		
	$C_0/(\theta - \theta_{17})$	4.15	5.79	6.13	7.03	8.07×10^{-4}		$c = .001075$
Wire No. 2	$\theta - \theta_{17}$	210°	410°	609°	801°	990°	1161°	
	C	.085	.183	.392	.601	.850	1.220	$\gamma_0 = 3.06 \times 10^{-4}$
$a = .00693$	E	.001	.004	.012	.036	.079	.154	
	$C_0 = C - E =$	.084	.179	.380	.565	.771	1.066	
	$C_0/(\theta - \theta_{17})$	4.00	4.37	6.23	7.05	7.78	9.17×10^{-4}	$(c = .001649)$
Wire No. 3	$\theta - \theta_{17}$	210°	411°	608°	803°	993°	1165°	
	C	.092	.228	.357	.574	.814	1.106	$\gamma_0 = 3.62 \times 10^{-4}$
$a = .00615$	E	.001	.003	.010	.030	.069	.129	
	$C_0 = C - E =$	.091	.225	.347	.544	.745	.977	
	$C_0/(\theta - \theta_{17})$	4.34	5.48	5.62	6.65	7.51	8.38×10^{-4}	$c = .001045$
Wire No. 4	$\theta - \theta_{17}$	210°	411°	608°	803°	993°	1165°	
	C	.096	.217	.382	.567	.823	1.085	$\gamma_0 = 3.49 \times 10^{-4}$
$a = .00582$	E	.001	.004	.011	.032	.071	.138	
	$C_0 = C - E =$	.095	.213	.371	.535	.752	.947	
	$C_0/(\theta - \theta_{17})$	4.53	5.18	6.11	6.66	7.58	8.13×10^{-4}	$c = .001159$
Wire No. 5	$\theta - \theta_{17}$	213°	413°	613°	807°	1001°		
	C	.070	.209	.369	.561	.800		$\gamma_0 = 3.39 \times 10^{-4}$
$a = .00506$	E	.001	.003	.009	.026	.063		
	$C_0 = C - E =$	.069	.206	.360	.535	.737		
	$C_0/(\theta - \theta_{17})$	3.24	5.00	5.87	6.63	7.36×10^{-4}		$c = .001181$
Wire No. 6	$\theta - \theta_{17}$	213°	416°	616°	812°	1001°	1179°	
	C	.088	.211	.354	.536	.764	1.057	$\gamma_0 = 3.31 \times 10^{-4}$
$a = .00465$	E	.001	.003	.008	.025	.056	.110	
	$C_0 = C - E =$	.087	.208	.346	.511	.708	.947	
	$C_0/(\theta - \theta_{17})$	4.08	5.00	5.62	6.28	7.07	8.03×10^{-4}	$c = .001155$
Wire No. 7	$\theta - \theta_{17}$	213°	413°	612°	805°	997°		
	C	.081	.213	.358	.546	.728		$\gamma_0 = 3.29 \times 10^{-4}$
$a = .00387$	E	.001	.002	.007	.020	.046		
	$C_0 = C - E =$	.080	.211	.351	.526	.682		
	$C_0/(\theta - \theta_{17})$	3.76	5.10	5.74	6.53	6.83×10^{-4}		$c = .001164$
Wire No. 8	$\theta - \theta_{17}$	230°	434°	613°	877°			
	C	.096	.204	.349	.584			$\gamma_0 = 3.21 \times 10^{-4}$
$a = .00192$	E	.000	.001	.004	.009			
	$C_0 = C - E =$	.096	.203	.345	.575			
	$C_0/(\theta - \theta_{17})$	4.17	4.68	5.64	6.56×10^{-4}			$c = .001191$
Wire No. 9	$\theta - \theta_{17}$		439°		885°			
			—		—			
$a = .00195$			—		—			
Wire No. 10	$\theta - \theta_{17}$	145°	255°	349°	482°	567°	707°	
	C	.0182	.081	.123	.205	.257	.338	$\gamma_0 = 2.26 \times 10^{-4}$
$a = .00144$	E	.000	.000	.001	.001	.002	.005	
	$C_0 = C - E =$	.0182	.081	.122	.204	.255	.333	
	$C_0/(\theta - \theta_{17})$	—	3.18	3.50	4.23	4.50	4.72×10^{-4}	$(c = .001681)$

$$W = B \sqrt{V + C},$$

$$C_0 = \gamma_0 (\theta - \theta_{17}) [1 + c (\theta - \theta_{17})],$$

$$C_0 = C - E,$$

$$E = 2\pi a \times .514 (\theta/1000)^{5.2},$$

$$\gamma_0 = 2.50 \times 10^{-4} (1 + 70a),$$

$$\text{Mean value of } c$$

$$c = .00114$$

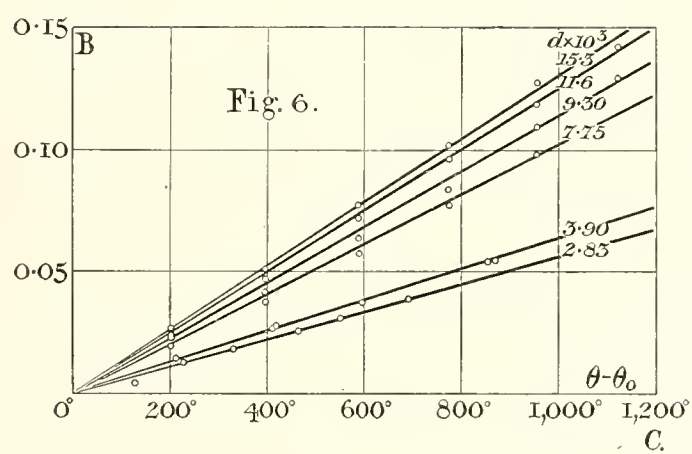


Fig. 6. Variation of B with the Temperature.

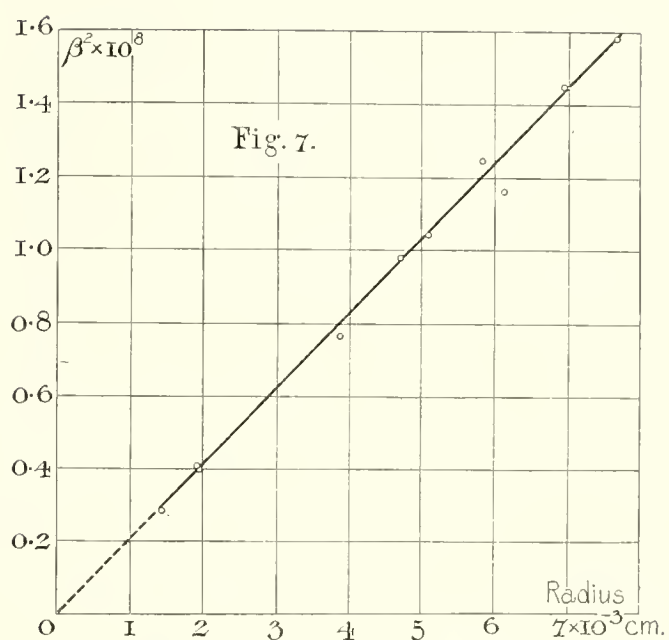


Fig. 7. Variation of β_0 with the Radius of the Wires.

TABLE VII.

Wire No. 1. Diameter, 0.0153 cm. Inclined at angle ϕ with the vertical: direction of stream horizontal.

$\theta - \theta_{17}$	210°	409°	610°	800°	987°	
$\phi = 0^\circ$ C =	.089	.241	.388	.602	.884	$b = 1.5 \times 10^{-5}$
B =	.0264	.0508	.0772	.1012	.1272	
$B/(\theta - \theta_{17})$	1.255	1.241	1.265	1.265	1.290×10^{-4}	$\beta_0 = 1.26 \times 10^{-4}$
$\phi = 31^\circ$ C =	.109	.259	.362	.520	.860	$b = 8.0 \times 10^{-5}$
B =	.0227	.0460	.0725	.0985	.1213	
$B/(\theta - \theta_{17})$	1.081	1.122	1.188	1.231	1.230×10^{-4}	$\beta_0 = 1.15 \times 10^{-4}$
$\phi = 45^\circ$ C =	.084	.206	.413	.639	1.000	$b = 5.0 \times 10^{-5}$
B =	.0216	.0452	.0644	.0843	.1034	
$B/(\theta - \theta_{17})$	1.030	1.103	1.056	1.054	1.049×10^{-4}	$\beta_0 = 1.02 \times 10^{-4}$
$\phi = 63^\circ$ C =	.090	.224	.396	.562	.846	$b = 9.0 \times 10^{-5}$
B =	.0172	.0346	.0522	.0704	.0885	
$B/(\theta - \theta_{17})$	.818	.845	.856	.880	$.897 \times 10^{-4}$	$\beta_0 = .81 \times 10^{-4}$
$\phi = 90^\circ$ C =	.077	.177	.318	.551	.823	$b = 4.0 \times 10^{-5}$
B =	.0138	.0285	.0430	.0549	.0694	
$B/(\theta - \theta_{17})$	.658	.696	.704	.687	$.703 \times 10^{-4}$	$\beta_0 = .67 \times 10^{-4}$

Effect of Inclination on the Convection Constants B and C.

TABLE VIII.

$\theta - \theta_0$	200°	400°	600°	800°	1000°	1200°	1400°	1600°	$c = .00114.$
M	246	582	1000	1530	2140	2840	3640	4520	$b = .000080.$
N	1.46	1.98	2.56	3.23	3.93	4.67	5.43	6.30	$\kappa_0 = 2.37 \times 10^{-4}$ watts.
Wire No. I.	.11	.24	.40	.60	.81	1.09	1.44	1.96	$d = .00404$ cm.
$\log b/a$	3.33	3.60	3.73	3.80	3.93	3.88	3.77	3.43	
$\bar{V}_0$	6.9	5.3	4.6	4.3	3.8	4.0	4.5	6.2	$\text{Mean } \log b/a = 3.68$
$\bar{V}$ cm./sec.	10.0	10.5	11.8	13.9	15.0	18.7	24.4	39.0	
Wire No. II.	.12	.29	.47	.70	.95	1.24	1.62	2.19	$d = .00691$ cm.
$\log b/a$	3.05	2.98	3.17	3.26	3.35	3.41	3.35	3.07	
$\bar{V}_0$	5.3	5.7	4.7	4.3	3.9	3.7	4.0	5.2	$\text{Mean } \log b/a = 3.20.$
$\bar{V}$ cm./sec.	7.7	11.3	12.0	13.9	15.3	17.2	21.7	32.8	
Wire No. III.	.13	.31	.52	.75	1.03	1.29	1.64	2.18	$d = .01262$ cm.
$\log b/a$	2.81	2.79	2.87	3.04	3.09	3.28	3.31	3.08	
$\bar{V}_0$	3.7	3.8	3.5	3.0	2.8	2.3	2.3	2.8	$\text{Mean } \log b/a = 3.03.$
$\bar{V}$ cm./sec.	5.4	7.5	9.0	9.7	11.0	10.7	12.5	17.6	
Wire No. IV.	.17	.38	.66	.95	1.29	1.67	2.06	2.49	$d = .02508$ cm.
$\log b/a$	2.15	2.28	2.26	2.40	2.46	2.53	2.63	2.70	
$\bar{V}_0$	3.6	3.2	3.2	2.8	2.7	2.5	2.2	2.1	$\text{Mean } \log b/a = 2.43.$
$\bar{V}$ cm./sec.	5.2	6.3	8.2	9.0	10.6	11.7	11.9	13.2	
Wire No. V.	.22	.51	.85	1.28	1.71	2.22	2.85	3.50	$d = .0510$ cm.
$\log b/a$	1.66	1.70	1.75	1.78	1.86	1.91	1.91	1.92	
$\bar{V}_0$	2.8	2.8	2.6	2.6	2.4	2.3	2.3	2.2	$\text{Mean } \log b/a = 1.81.$
$\bar{V}$ cm./sec.	4.1	5.5	6.7	8.4	9.4	10.7	12.5	13.9	

$$H = 2\pi\kappa_0(\theta - \theta_0)[1 + c(\theta - \theta_0)]/[\log b/a], \quad M = (\theta - \theta_0)[1 + c(\theta - \theta_0)].$$

$$N = [1 + c(\theta - \theta_0)]^2/[1 + b(\theta - \theta_0)]^2, \quad V = N\bar{V}_0, \quad \bar{V}_0 = \kappa_0 e^{1-\gamma}/(s_0\sigma_0 b).$$

Analysis of LANGMUIR'S Observations on Free Convection from Small Platinum Wires.

Description of Table VII.—Effect of Inclination on the Convection Constants of Small Wires.

In Table VII. is set out the analysis of observations on the effect of inclination on the convection constants of one of the wires. The wire was mounted in the fork in such a manner that the end connections offered no obstruction to the flow of air, illustrated in (d) Diagram II. As described in Section 14, the previously determined laws of convection were found to hold good; the values of the constants b and β_0 were determined in the manner already described, and the variation of the latter with the inclination is shown graphically in fig. 8.

Description of Table VIII.—Analysis of Observations on Free Convection.

In the accompanying table are set out LANGMUIR'S⁽¹⁾ observations on the free convection of heat from small platinum wires, discussed in terms of the theory of Part I. of the present paper. For the small velocities of the free convection current set up by the heated wire this theory affords a rational interpretation of the empirical formula obtained by LANGMUIR. In heavy-faced type are given the values of the heat-losses corrected for radiation; the denominator $\log(b/a)$ of formula (75) is set out in italics in the following line and is seen to be nearly independent of the temperature, especially for the larger wires; the variation of this term with the radius is shown graphically in fig. 9. The use of this diagram, combined with formulæ (75) and (76), offers a convenient method of roughly estimating the free convection losses from small platinum wires. The velocity of the air-flow, which would give rise to the observed heat-loss (called the "effective" free convection current), is calculated and is set out in italic characters in the last line of the accompanying table.

(¹) LANGMUIR, 'Phys. Rev.', 34, p. 415, Table VIII., 1912.

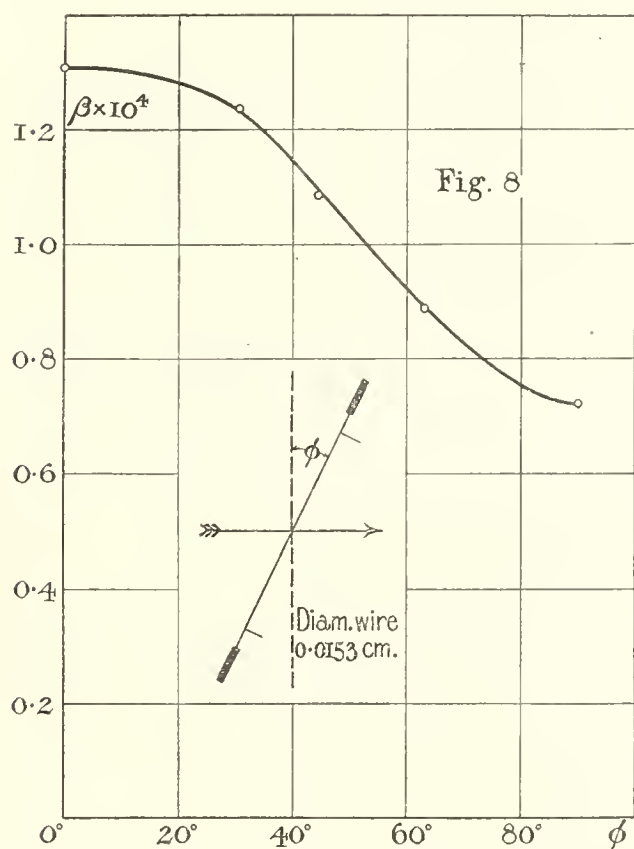


Fig. 8. Effect of inclination on the convection constant β_0 .

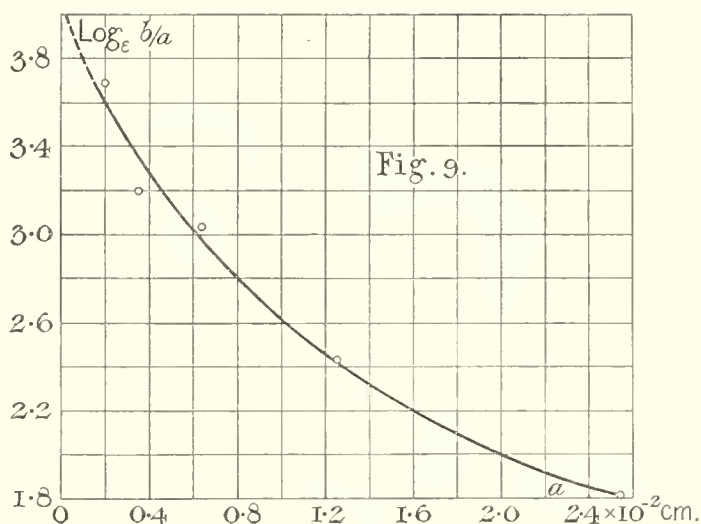


Fig. 9. Variation of $\log b/a$ with diameter of the wire.

Diagram I.—Graph of the Function $x / \left[\int_0^x e^x K_0(x) dx \right]$.

The values of the function $y = x / \left[\int_0^x e^x K_0(x) dx \right]$ are required for the purpose of making a comparison between theory and experiment in the convection losses of a cylindrical wire cooled by a stream of air. These values may be easily obtained from Table I. or from the graph of the function given in Diagram I. We notice that, for small values of the variable x , equation (23) enables us to write approximately

$$y = 1/[(1 - \gamma) - \log(x/2)] = 1/[1.11593 + \log(1/x)]. \quad \dots \dots \dots (i.)$$

(Error < 1 per cent. if $x < 0.02$.)

When x is sufficiently large (24) gives approximately

$$y = 1/(2\pi) + \sqrt{x/2\pi} = 0.15915 + 0.39894 \sqrt{x} \quad \dots \dots \dots (ii.)$$

(Error < 1 per cent. if $x > 0.3$.)

It is of importance in the application of these approximate equations to know for what values of x they may be employed with sufficient accuracy. The following table sets forward this comparison:—

x .	$\sqrt{x}$.	y .	Formula (i.)	Per cent. error.	Formula (ii.)	Per cent. error.	x .	$\sqrt{x}$.	y .	Formula (ii.)	Per cent. error.
·01	·1	·1740	·1748	+0·5	·1990	+15·0	4	2	·9607	·9571	−0·4
·04	·2	·2266	·2307	+2·0	·2389	+5·0	9	3	1·3591	1·3561	−0·2
·09	·3	·2727	·2809	+3·0	·2788	+2·2	16	4	1·7575	1·7551	−0·1
1·0	1·0	·5616	—	—	·5581	−0·6	100	10	4·1500	4·1490	−0·02

From this table we notice that to an accuracy of 1 per cent. we must have $x < 0.02$ in (i.) and $x > 0.3$ in (ii.). The values given by formulæ (i.) and (ii.) become equal at about $x = 0.08$, at which point the error is somewhat less than 2.5 per cent. This point may in practice enable us to define a limiting air-velocity, affording a criterion as to which of the approximate formulæ (32) or (33) should be used. Remembering that in the case of a circular cylinder of radius $d/2$ we have $x = s\sigma V d/x$, and inserting the values $s = 0.171$, $\sigma = 1.3 \times 10^{-3}$, $\kappa = 5.2 \times 10^{-5}$, we find the limiting value of Vd given by

$$Vd = 1.87 \times 10^{-2} \dots \dots \dots (iii.)$$

In the case of a 3-mil wire this limiting velocity is about 2.4 cm./sec. representing an accuracy of 2.5 per cent. To an accuracy of 1 per cent. the measurement of velocity by a 3-mil wire can be obtained for velocities greater than 9 cm./sec., provided the free convection current due to the heating current is less than this amount.

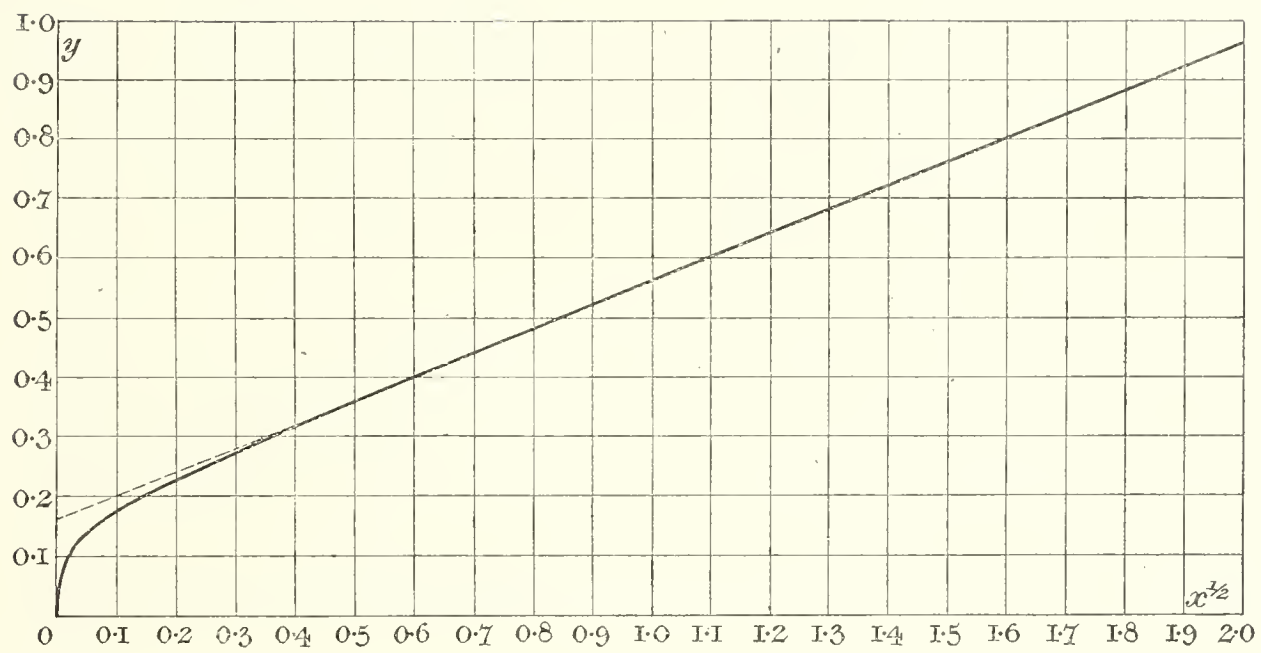


Diagram I. Graph of the Function $x / \left[\int_0^x e^x K_0(x) dx \right]$.

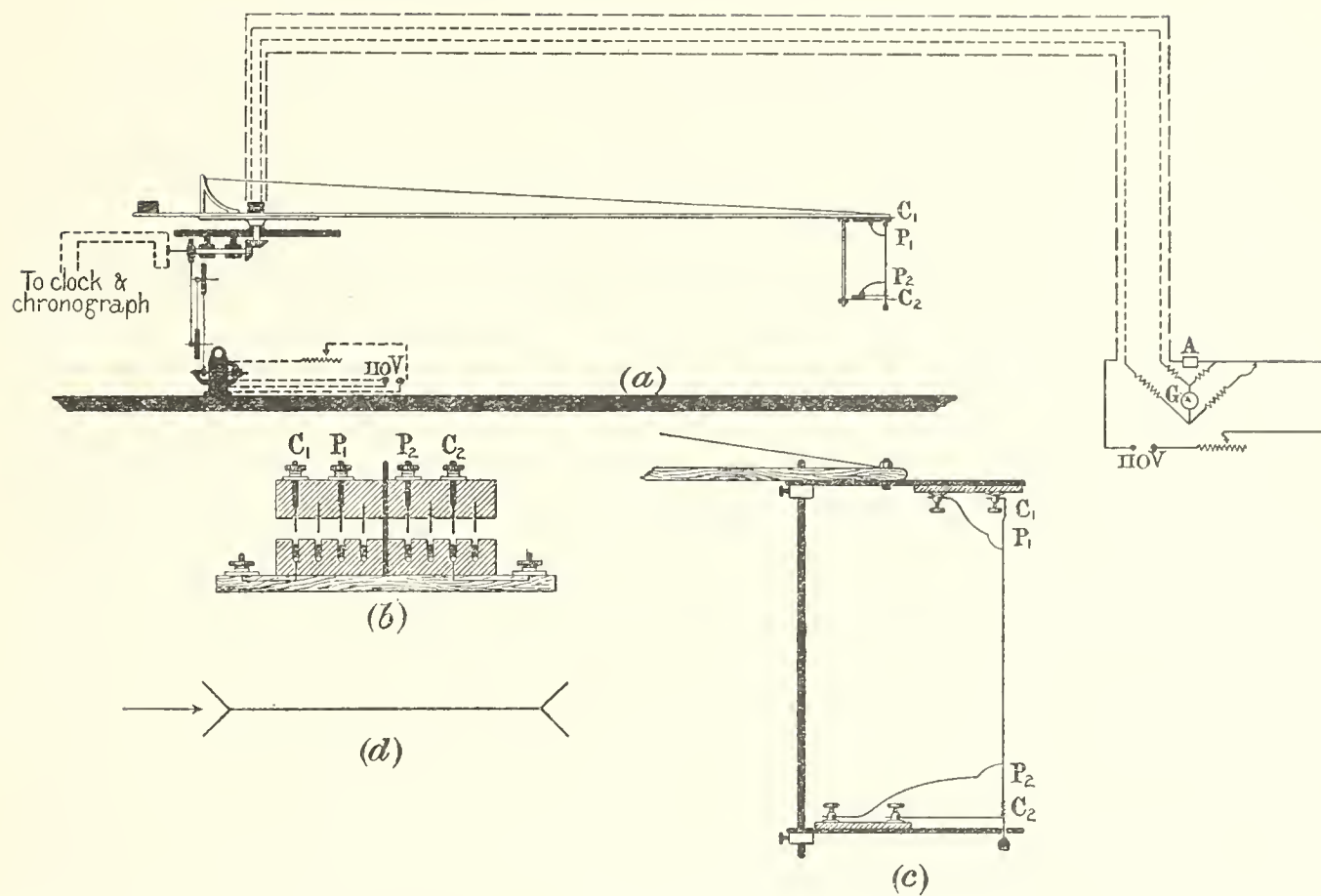


Diagram H. Rotating arm and diagram of connections.

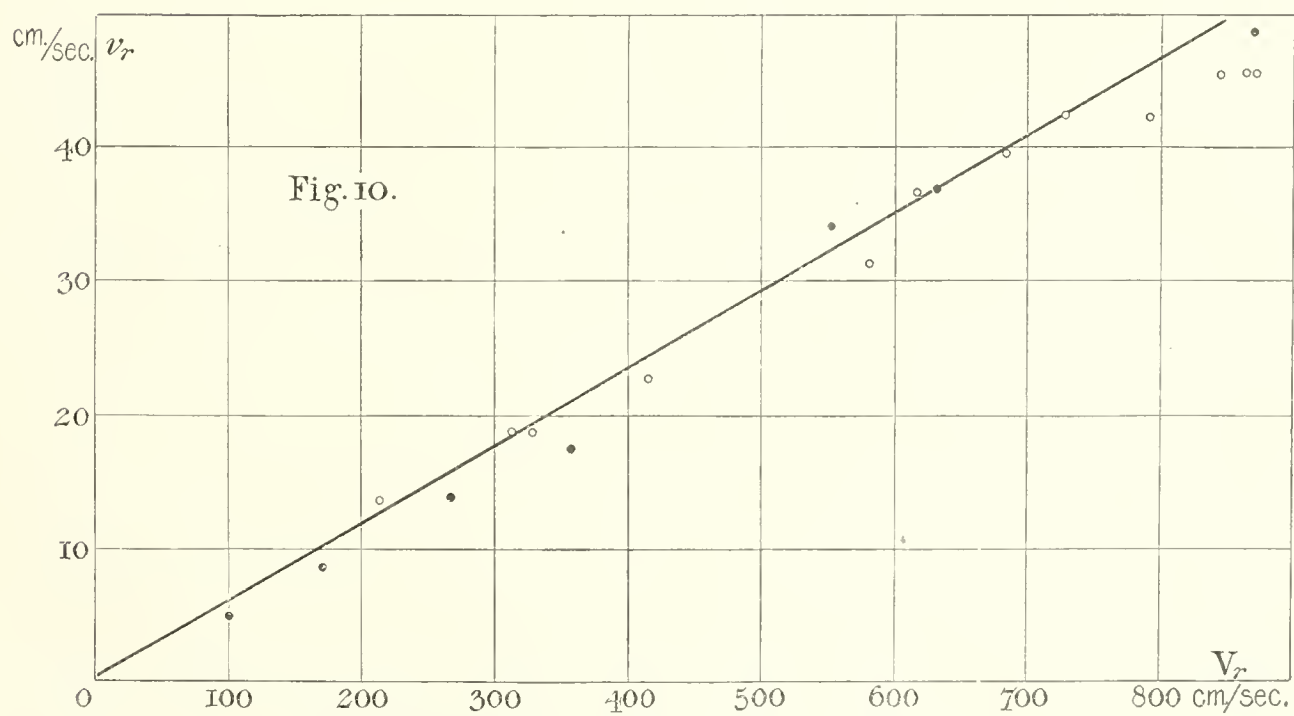


Fig. 10. Velocity of "swirl" set up by rotating arm.

Description of Diagram III.—Details and Specifications in the Construction of Hot-Wire Anemometers.

The most suitable wire for use in hot-wire anemometry was found to be platinum thermometer wire drawn down to a diameter of about 3 mils. It is advisable to age the wire before mounting by heating it to a red heat for some hours by means of an electrical current. The wire should then be carefully examined under a high-power microscope, and a portion of sufficient length selected free from flaws and pittings and of as uniform a diameter as possible. The wire may at once be mounted in position in the fork described below, heated to a bright red heat, and the potential terminals of 1-mil wire fused in place. Care should be taken to avoid allowing the heated wire to come into contact with easily fusible metals or organic matter. No connections should be fused in a blow-pipe flame, and no solder should be employed except in those connections which remain cool. The Kelvin-bridge connections obtain the condition that the reading of the instrument is independent of the contact resistances at C_1 and C_2 , which need only be tolerably good. The potential leads may be fused to the anemometer at any convenient distance apart, and the instrument will give correct readings of air-velocity if the constants are determined experimentally in such a manner that the potential terminals are in the same position relatively to the flow of air as in the distribution of flow to be measured. If, however, the calibration curve of the instrument is to be determined from the dimensions and electrical constants of the wire, making use of the convection constants obtained in Part II. of the present paper, the position of the potential terminals is subject to the limitations imposed by keeping within narrow limits the errors introduced by the cooling effect of the end-connections and potential wires. The maximum values of these errors are investigated in Section 12 (ii.); the values of P and p occurring in formulæ (60) and (61) are calculated for anemometer-wires of 6, 3, and 1 mils diameter, fitted with potential terminals of 1-mil wire. The thermal conductivity of platinum is taken to be $K = 0.7$ watts per sq. cm. per sec. per degree C. The temperature of the wire is that corresponding to the ratio $\bar{R}/R_0 = 4$, adopted as the most suitable for hot-wire anemometry and representing a value $\theta - \theta_0 = 1165^\circ \text{C}$. The heat-loss W employed in the table given below is that obtained from experiment at the lowest velocity $V = 81 \text{ cm./sec.}$:—

TABLE (i.).—*Values of p and P .*

Wire.	6-mil.	3-mil.	1-mil.
Diameter	·0153 cm.	·00775 cm.	·00283 cm.
Cross-section	$1.84 \times 10^{-4} \text{ sq. cm.}$	$472 \times 10^{-4} \text{ sq. cm.}$	$·0628 \times 10^{-4} \text{ sq. cm.}$
$K\omega(\theta - \theta_0)$	·150	·0385	·00513
Heat-loss W watts at 1165°C. , for $V = 81 \text{ cm./sec.}$ }	2.00	1.60	(1.10) (calculated)
$p = \sqrt{(R_0/\bar{R})} \cdot \sqrt{[W/K\omega(\theta - \theta_0)]}$	1.83	3.22	7.31
$P = \sqrt{(\bar{R}/R_0)} \cdot (a/a')^{5/4}$	·213	·51	2.0

We now take the case of an anemometer-wire of length $2(l + l_0)$. The resistance between potential terminals at a temperature $\theta^\circ \text{C}$. is denoted by $2Rl$, l being the length between potential terminals. If calculated from the resistance per unit length for the same temperature the result would be $2\bar{R}l$. It is shown that, owing to the cooling effect of the leads and potential terminals, these differ by a small correction-factor given by $R = \bar{R}(1 - \epsilon)$, ϵ being given by formula (62). In designing an anemometer for which the constants are to be determined by calculation the length and diameter of the wire and the position of the potential terminals must be so disposed that the quantity ϵ remain small. In the following table, based on the preceding Table (i.), values of ϵ are given for various diameters and positions of the potential terminals and represent overestimates of this correction :—

TABLE (ii.).—*Values of the Correction Factor ϵ .*

Wire.	6-mil.	3-mil.	1-mil.
cm.			
$l = 10.0$	.0050	.0047	.0051
$l_0 = 2.0$			
$l = 2.0$	.092	.041	.026
$l_0 = 0.5$			

A convenient form of fork suitable for holding in position the anemometer wires, and offering a minimum of disturbance to the flow of air in its neighbourhood, is illustrated in fig. (a) of Diagram III. Fastened to a block of ebonite are the two arms of the fork, consisting of steel strips about 5 mm. in width. At the end of each is soldered a small brass block, drilled to receive two fine needles fastened about 1 cm. apart. Threaded through the eyes of these two needles is a 3-mil platinum wire, having its extremities firmly elamped in the brass block just mentioned. The anemometer-wire is held by these two loops as indicated in the figure and by this means is kept under tension by the elastic support, and is protected from accidental damage by the two needles on either side of each end. The tension is adjusted by a fine thread carried down from each of the brass blocks to an adjustable screw in the centre of the ebonite block; this thread is also effective in preventing lateral vibrations of the fork. Carried up from each end of the ebonite block are two thin steel strips crossing each other to the opposite arm of the fork, insulated from each other and also from the fork by means of thin mica strip. These strips serve to brace the fork and at the same time serve as potential leads. At each extremity is soldered a small brass block drilled to hold a fine needle, at the extremity of which is soldered a thin copper wire. The 1-mil platinum potential terminals fused to the anemometer-wire are carried to these copper wires to which they are easily soldered. The complete apparatus is illustrated photographically in Plate 8 (c), mounted on a micrometer screw for measuring rapid gradients of turbulent flow. In the experiments previously referred to in Section 17 it was found possible to resolve a gradient in which the velocity changed by 5 cm./sec. over a distance of 1/10 mm.

In fig. (b) of Diagram III. are drawn the connections which were found convenient in practice. The resistances a and b were made equal and about 500 ohms while α and β were adjusted to equality at about 250 ohms. In order to protect the anemometer-wire from accidentally burning out, a key K_1 was inserted by means of which it was automatically short-circuited; a double-contact key K_2 was inserted in the galvanometer circuit in such a way that contact was first made through a high resistance for preliminary adjustments; it was also found convenient to connect the galvanometer to an adjustable shunt. The resistance B was constructed of No. 23 B. and S. manganin wire wound non-inductively on an asbestos frame in such a manner as to dissipate a maximum amount of heat; its resistance measured between potential terminals soldered to the wire was adjusted to four times that of the anemometer-wire at room temperature. By means of a fine-adjustment rheostat R the current in the anemometer-wire could be adjusted until a balance was obtained on the galvanometer. It is important that the rheostat be always re-adjusted to the position of minimum current to avoid over-heating the wire should the velocity of the air-flow suddenly diminish; this may be easily accomplished by means of a spring control.

In taking a measurement of velocity, the key K_1 is pressed down and the current as read by the ammeter slowly increased until on pressing down the key K_2 a balance is obtained on the galvanometer. From the reading of the current the velocity may readily be obtained from a calibration curve corresponding to the formula (80), $i^2 = i_0^2 + k \sqrt{V}$. It has already been mentioned that a dynamometer form of instrument will, if at the same time sufficiently accurate, prove to be more suitable for the measurement of current in that the scale-readings will be more open at high velocities; the same result could also be achieved by suitably shaping the pole-pieces of the permanent magnet of a direct-current

instrument and in the event of being able to employ standardized anemometer-wires, the scale of the ammeter could be graduated to read velocities directly.

The galvanometer employed was a Weston portable instrument with jewel bearings and capable of detecting a current of about 10^{-6} amperes. This degree of sensitivity is more than necessary; in fact a millivoltmeter was found to be sufficiently sensitive for most purposes. The constants of damping are very important in determining the rapidity with which observations can be made, and it was found that equally sensitive galvanometers varied within wide limits in this respect. By employing an alternating current and a telephone receiver instead of a galvanometer it was found that the same calibration curve was obtained as in the case of direct current measurements.

It was found possible to test for any suspected change in the constants of the anemometer-wire by measuring the current required to bring the wire to the standard temperature in a stagnant atmosphere, care being taken to protect the instrument from draughts.

In analysing a complicated distribution of air-flow, the direction of the current of air may be determined from the effect discussed in Section 14; if the anemometer is rotated about an axis perpendicular to the wire, it will be at right-angles to the stream when the current required to bring it to the standard

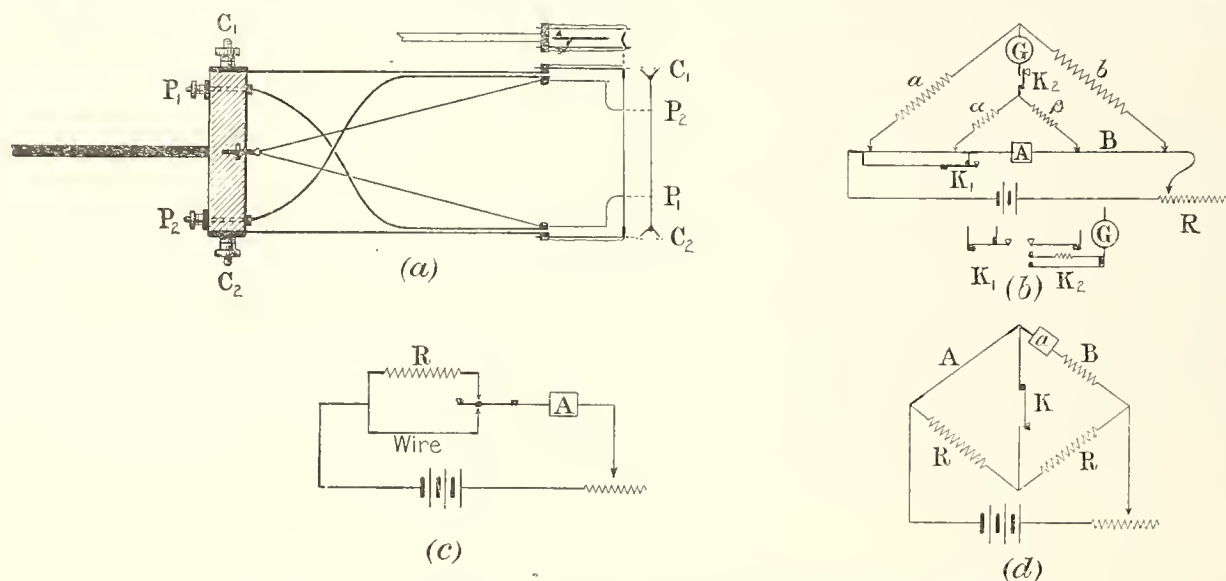
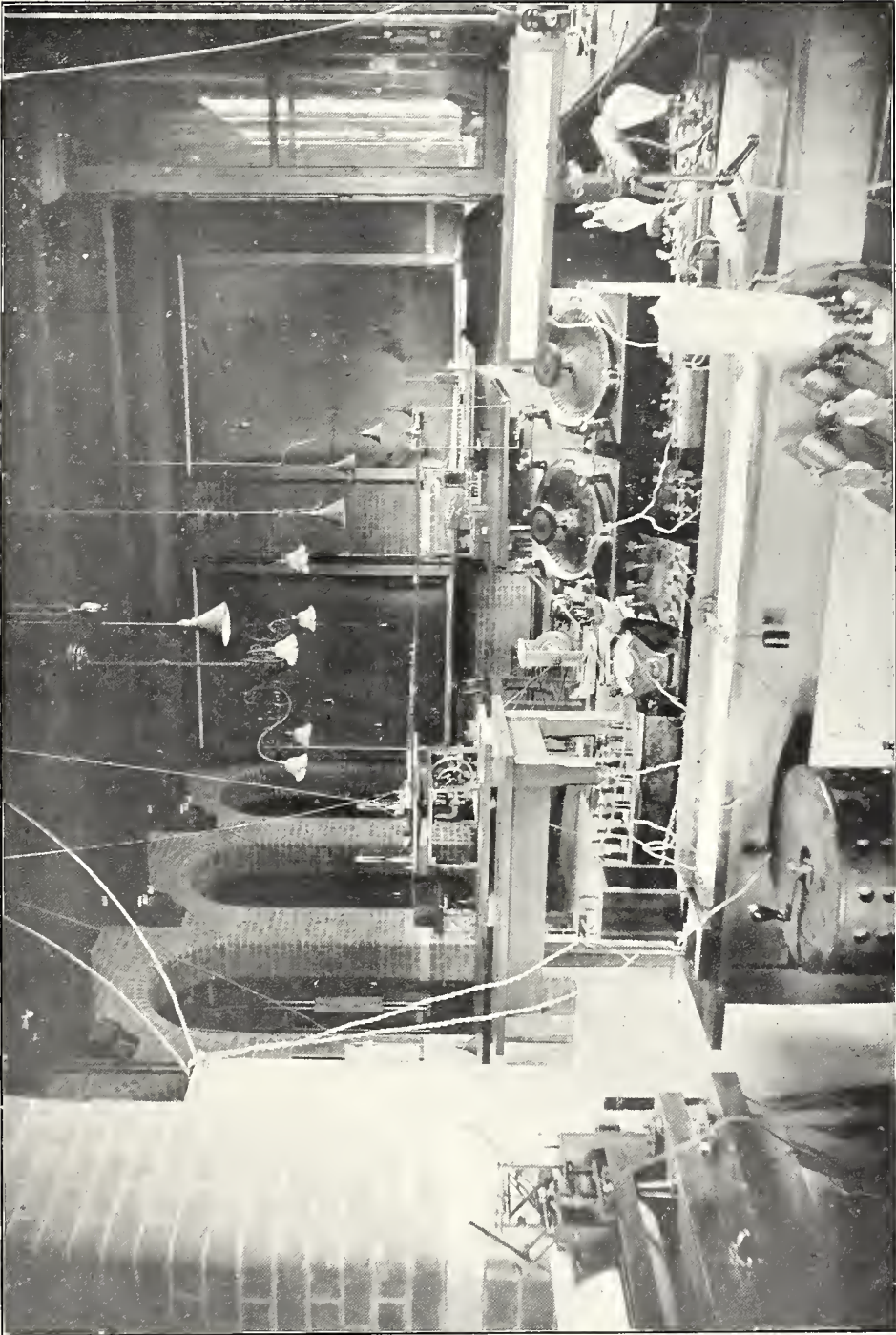


Diagram III. Details of hot-wire anemometers and connections.

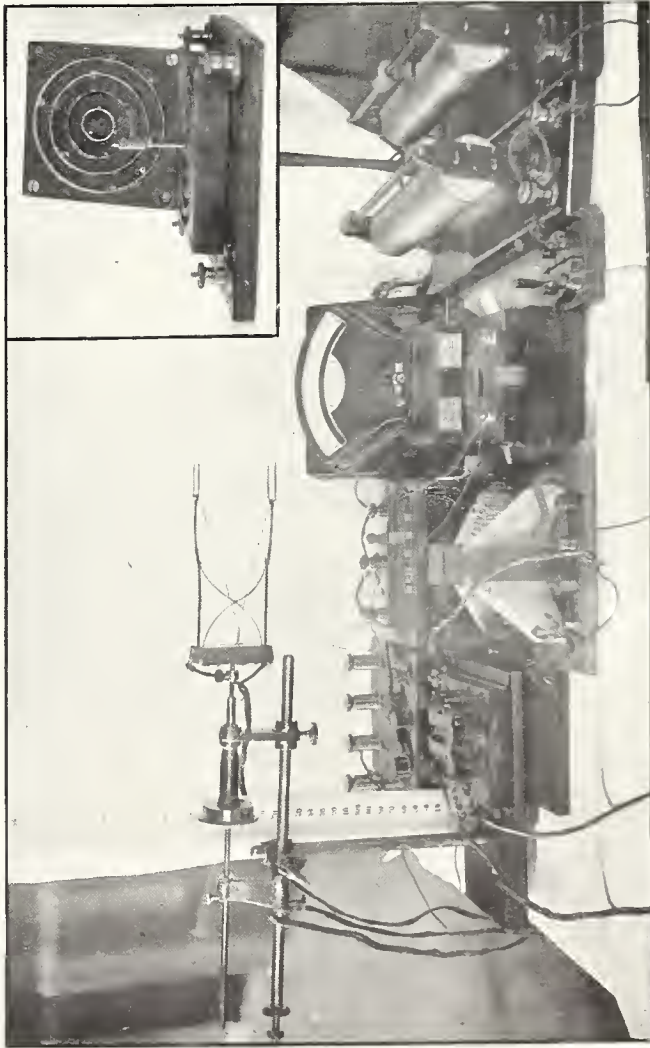
temperature is a maximum. This effect is sufficiently marked to enable the direction of a stream to be fixed with a fair degree of accuracy.

In figs. (c) and (d) of Diagram III. are illustrated simple methods of making the connections to an anemometer-wire so as to dispense with a galvanometer. In fig. (c) a low resistance storage-cell is connected by means of a three-way key in such a way that the anemometer-wire and a manganin resistance of four times the resistance may be successively included in the circuit; if the current be adjusted so that the ammeter reading is unchanged on successively including these two resistances, they must have the same resistance. Fig. (d) illustrates an adaptation of KELVIN'S method for measuring a galvanometer resistance. The ammeter a is in one of the arms in series with a manganin resistance so that the total resistance is four times that of the anemometer-wire in the adjacent arm. The ratio resistances R are adjusted to equality and have approximately the same value as that of the remaining arms. A contact key K replaces the galvanometer. If the current in the bridge be adjusted so that on depressing the key there is no change in the ammeter reading, it is easily seen that the condition of balance of the Wheatstone-bridge is satisfied and the resistances A and B are equal.

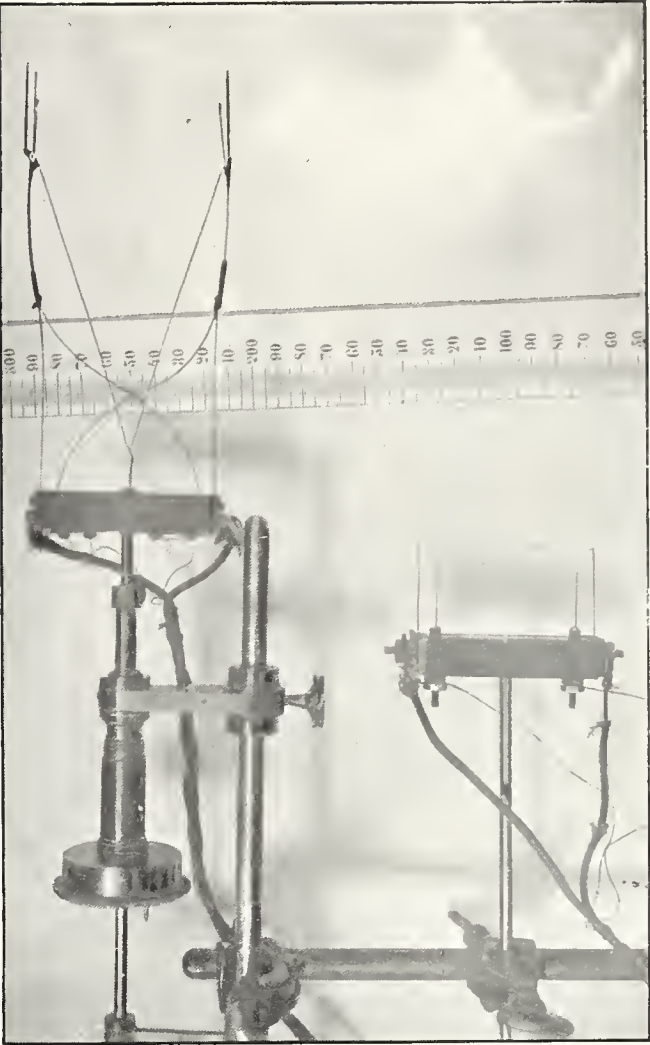
The two methods just described suffer the disadvantage that the contact resistances of the anemometer-wire are included in the resistance of the wire itself, and is likely to be uncertain if the wire is to be employed at high temperatures. The potential terminals and the Kelvin-bridge connections are recommended for use as the most satisfactory and should be employed if possible.



(a) General disposition of apparatus.



(b) Portable hot-wire anemometer. Details of mercury slip-rings.



(c) Details of linear hot-wire anemometers.



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[PLATES 9-10.]

ON ACQUIRED RADIO-ACTIVITY.

BY

SIR WILLIAM CROOKES, O.M., LL.D., D.Sc., PRES.R.S.



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XIII. *On Acquired Radio-Activity.*

By SIR WILLIAM CROOKES, O.M., LL.D., D.Sc., Pres.R.S.

[PLATES 9-10.]

Received October 26, 1914.

Experiments with Cathode Rays.

1. ABOUT forty years ago I sealed in a vacuum tube a yellow diamond cut as a brilliant. This diamond was chosen because it phosphoresced in the dark after exposure to bright sunshine,—it also phosphoresced slightly under the influence of ultra-violet light. In the vacuum tube, as the anti-cathode, it emitted a brilliant yellowish white light giving almost as much light as a candle. It was often exhibited to illustrate the glow of a diamond under the influence of cathode rays *in vacuo*; scarcely a week passed without the vacuum tube containing the diamond being exhibited to friends. It was by far the most attractive tube in my collection. After forty years of hard work the diamond has become much discoloured. It was of interest to see if the repeated bombardment, as the anti-cathode, in the vacuum tube had conferred radio-activity on the diamond. Accordingly I opened the tube, quickly removed the diamond, and in the dark put it on a sensitive film, a thin sheet of black paper intervening. Over the diamond was placed a pad of cotton-wool and a weight, to prevent the stone from shifting its position. After nine days' contact the film was developed. An exceedingly slight action could, with difficulty, be detected, showing that the off and on action for forty years had conferred practically no radio-activity on the diamond.

2. A sensitive electroscope was now made, with sulphur insulation. The fall of the leaf was observed with a small telescope having a photographed scale. The normal fall due to natural leakage was 5° in 165 seconds. A piece of a thoria gas mantle caused a fall of 5° in two seconds, and radio-active diamonds caused a fall in from two or three seconds to a fraction of a second. Tested in this instrument I found this special diamond to be entirely devoid of action.

Action of Cathode Rays on Diamond.

3. On one occasion when M. MOISSAN was in my laboratory I darkened some diamonds by means of cathode rays. One of these he took away, and subsequently reported the result of his experiments to the French Academy.* He heated

* 'Comptes Rendus,' vol. cxxiv., No. 13, p. 653, March, 1897.

the diamond to 60° C. in an oxidising mixture of potassium chlorate and fuming nitric acid, prepared from monohydrated sulphuric acid and potassium chlorate fused and free from moisture (9, 12, 32). The action on the dark layer is very slow, requiring at least eight or ten days for its complete removal. There is at first produced graphitic oxide which at an increased temperature yields pyrographitic acid easily destroyed by nitric acid. Hence the variety of carbon which coated the diamond is graphite. The transformation of diamond into graphite requires the temperature of the electric arc, and the higher the temperature at which it is formed the greater is its resistance to oxidation. M. MOISSAN came to the conclusion that the temperature reached by the surface of the diamond blackened in my radiant matter tube was probably about 3600° C.

4. Six diamonds—having different degrees of phosphorescence under cathode rays—were tested in a Becquerel phosphoroscope to see if the order of intensity of the residual phosphorescence due to illumination with an arc light was the same as the order under cathode rays. All had a slight residual phosphorescence, but the order of intensity differed in the two cases.

Action of Cathode Rays on various Mineral Substances.

5. For many years I have experimented on the changes produced in ordinary and quartz glass and various crystals by exposure to radium emanations and rays.

Considering the identity of the cathode discharge in a high vacuum tube with the β -rays from radium, experiments were started to ascertain if the cathode rays would confer radio-activity in various solid bodies submitted to its influence. A vacuum tube was prepared with a removable window at one end to allow the contents to be exchanged for other bodies. In the tube were placed pieces of uranium glass, a crystal of ruby, a crystal of garnet, a piece of quartz, a plate of platinum and one of gold. The tube was exhausted to a high vacuum just short of the non-conducting point. On excitation the whole tube was filled with the green glow of phosphorescent glass, the ruby and uranium glass became very phosphorescent, the other bodies remained quite dark. After an hour's exposure to the cathode discharge the tube was opened, the objects quickly removed in the dark and placed on a sensitive film; the upper sides that had received most bombardment being placed downwards on the film. They were kept thus for eighteen hours and the film then developed. There was absolutely no impression of any of the bodies on the film. Thus it appears that the cathode stream will not confer radio-activity on the above-named bodies.

Action of Cathode Rays on Phosphorescent Bodies.

6. Experiments were next tried to see if earths and compounds which became strongly phosphorescent under the influence of the cathode discharge would thereby

become radio-active. Some ignited yttrium sulphate, in the condition most sensitive to the cathode rays, and giving under their action a phosphorescent glow with a discontinuous spectrum, was pressed tightly into a shallow aluminium tray and exposed for an hour to the discharge in the vacuum tube. It was then removed and immediately covered with a sensitive film pressed down with a slight weight. After forty-eight hours in total darkness the film was developed. Only a very slight image of the yttria was visible. This result does not prove that the cathode stream had rendered the yttria radio-active, for there is always a residual glow in the yttria in these circumstances; it is not unlikely that the light of this glow, acting for the first hour or so of the forty-eight, might have been strong enough to impress the film (7).

7. Three shallow trays were filled—one with phosphorescent calcium sulphide, another with zinc sulphide, and a third with platinocyanide of barium. After subjecting them to the cathode stream in the above manner, they were removed and covered with a sensitive film, and kept in darkness for twenty-four hours. On development only an impression of the zinc sulphide was seen. As this sulphide also has a certain amount of residual glow after exposure to the cathode rays it is probable that the photographic action is only the result of this glow (6).

Action of Radium on Diamond.

In June, 1904,* I read a paper before the Royal Society on the action of Radium on Diamond.

8. A few years later I repeated the original experiment, exposing two New South Wales diamonds to the action of the radium rays and emanations for a longer period. The diamonds selected were of an identical pale yellow, devoid of radio-activity. A quartz tube, containing 15 mgrms. of pure radium bromide was well exhausted and sealed before the oxy-hydrogen blowpipe. One of the diamonds (fig. 3*a*) was put close to the tube of radium and kept in its place with a piece of gummed paper. The other diamond (fig. 3*b*) was put away in a cabinet and kept far from any radium compound. The two diamonds were thus left for a little more than six months. At the end of this time they were examined. No appreciable difference could be detected in the colour of the two diamonds—the one (fig. 3*a*) that had been close to the tube of radium bromide not being darker than the one (fig. 3*b*) which had been away from radium the whole time.

Coloration of Diamond by α -Ray.

9. The diamond, 3*a*, was now enclosed for seventy-eight days in a tube containing radium bromide; at the end of the time it had acquired a bluish green colour. It was

* 'Roy. Soc. Proc.,' vol. lxxiv., p. 47.

then heated for ten days in a mixture of fuming nitric acid and potassium chlorate so as to dissolve off any outer skin of graphite which might have contributed to the colour (3, 12, 32). This treatment brightened its appearance but did not alter the colour. The diamond was next put on a sensitive photographic film and kept there for twenty-four hours. On developing, a strong impression was seen.

β - and γ -Rays produce Phosphorescence.

10. This experiment shows that the alteration of the colour is not due to the phosphorescent state of excitement to which the diamond had been constantly subjected during eight weeks. The colouring action is cut off by a thin screen of quartz, whereas the phosphorescing action is kept up by rays which pass through quartz. It is therefore evident that the coloration is due to the α -rays, or atoms of helium, shot from the radium compound (26, 29, 33). The phosphorescence is produced by the β - or γ -rays (19, 26, 43), which readily pass through glass and quartz.

Persistence of Acquired Radio-activity.

11. The acquired radio-activity of diamonds persists for a longer time than I have been enabled to measure, and resists the most violent treatment I have applied to them.

A large brilliant cut diamond of pure water assumed a fine green colour after having been kept for sixteen months (from May, 1904, to Sept., 1905) in a bottle and covered with powdered radium bromide. At the end of that time it was highly radio-active. This diamond has been carried about in my pocket, off and on, since 1905, and has been tested on a sensitive photographic film at intervals of a year or more. No appreciable difference in its radio-activity can be detected from that which it possessed when first removed from the radium bromide in Sept., 1905. Examined at the present time, nine years after its removal from the bottle of radium bromide, it is luminous in the dark, it rapidly discharges a sensitive electroscope when held near it, and produces scintillations on a zinc sulphide screen as if it were a radium compound.

12. A diamond of good water was selected from a stock of inactive stones, and put in a tube of radium bromide for seventy-eight days. At the end of that time it had assumed a greenish colour, and was highly radio-active. It was then heated to 50° C. in a mixture of fuming nitric acid and powdered potassium chlorate for ten days, the acid mixture being daily renewed (3, 9, 32). The only effect of this acid treatment was to remove a slight dull darkening on the surface, and to render the green tint more brilliant.

After well washing and drying it was put on a sensitive film for five hours. On developing it was seen that the stone had made a good impression on the film.

13. The stone was then sent to a diamond cutter to be cut into a brilliant. On receiving it back it was quite white and free from trace of colour. The stone was then placed on a sensitive film and kept in darkness for twenty-two hours. On developing no impression was apparent, although, before cutting, the active diamond had impressed a film in five hours (12, 16).

Effect of Heat on Induced Radio-activity.

14. A New South Wales diamond which had always been kept away from radium salts, was tested in the electroscope (2) and found to give a fall in two seconds, the natural leak being 180 seconds. The stone was then put in a silica crucible and heated in an electric tray furnace until visibly red. No change was observed. After five minutes—at, say, 700°C .—it was allowed to cool. The time of discharge in the electroscope was found to be six seconds.

15. The diamond was then packed in a silica crucible with graphite and heated with a Meker burner to 700°C . several times, cooling and testing the diamond each time when cold.

Heated to 700°C . for 10 mins. and cooled,	time of fall = 6 secs.
„ 15 „ „	= 10 „
„ 20 „ „	= 10 „
„ 20 „ „	= 30 „

It was now left for the night, and measurements resumed next day.

Heated to 700°C . for . . . 60 mins.,	time of fall = 9 secs.
„ in new graphite for . 30 „ „	= 7 „

New graphite again used.

Tested again after 48 hours,	time of fall = 4 secs.
Heated to 700°C . for 30 mins., and allowed to cool	
for 2 hours,	time of fall = 2 „

The diamond after the last measurement (2 secs.) was laid on a sensitive film and kept in the dark for twenty-one days; on development it gave a good image.

This experiment shows a temporary loss of activity by the heat treatment followed in a short time by complete recovery.

16. A Kimberley diamond, rather flat, octahedral shaped, was kept in a bottle of dry radium bromide for some weeks until it was quite green. By means of a steel wheel fed with diamond dust part of one face was cut away, and the surface of an adjoining

face was just removed, the adjoining corner thus being freshly exposed diamond crystal. It was put on a sensitive film in such a position that it could be subsequently examined and compared with the developed image, so that the active portions of the surfaces, natural and ground off, could be seen. It was allowed to act for five days, when the film was developed. The result indicated very decidedly that where the surface of the diamond had been removed it was no longer radio-active (13).

Experiments with a very Active Diamond Crystal.

17. A fine crystal of diamond from Kimberley which had been kept in a bottle of radium bromide for some months was tested on a sensitive film and in the electroscope. It was found to be highly active, and was set aside for further experiments. It was luminous in the dark, quickly discharged the electroscope, and caused an inactive diamond held near it to phosphoresce. It was put on the surface of a screen coated with small crystals of barium platinocyanide, and caused scintillations the same as on a blende screen, but feebler (11, 26, 27, 30, 33).

18. A blende screen was made by coating a glass slip with very sensitive zinc sulphide. This was laid on the diamond, the ZnS side next the diamond; the scintillations were easily seen through it in all their characteristic appearance, with concentration at the edges and corners. A piece of aluminium sheet, 0.06 mm. thick, was moved about between the screen and crystal of diamond, and there was no doubt whatever that the aluminium stopped all the scintillations. It was absolutely dark where the aluminium covered the crystal.

19. The diamond (17) was then held in contact with the card back of a platino-cyanide of barium screen. The luminous patch due to β - or γ -rays, or to both, was quite evident, moving about as the diamond was moved. The sheet of aluminium foil, used in the former experiment (0.06 mm. thick) was put between the crystal and the back of the screen, and there was little, if any, diminution in the luminosity. It is therefore quite certain that the radio-active diamond gives off other rays beside α -rays, and that the rays can penetrate aluminium 0.06 mm. thick and the card back of the screen.

20. The diamond (17) was put into various fluorescent solutions (uranine in water, Silberrard's *p*-nitroso-dimethylanilinenaphthaline compound, and quinine sulphate). It did not occasion the slightest fluorescence, although its own faint luminosity could be seen in the liquid.

21. The active diamond (17) was cemented to a plate of glass, and six small pillars of lead cemented round it the same height as the stone. The whole was inverted on a sensitive film, and kept in the dark for 2.25 hours. Another experiment was then tried with the same apparatus, only altering the position of the crystal so that the lead pillars were opposite the angles of the crystal. The exposure in this case was 6.5 hours. The pictures on development showed a strong

radiation extending some distance round the diamond (six or eight diameters), and the lead pillars showed strong shadows (Plate 9, fig. 1).

22. The same experiment was repeated three times with the interposition of one, two, and three thicknesses of aluminium foil 0.01 mm. thick, each was exposed the same time (two hours), and all were developed together. Each showed strong action near the diamond. One thickness allowed the shadow of the lead pillars to be easily seen. Two thicknesses showed the shadow with difficulty, and three thicknesses only showed the shape of the diamond itself.

23. The crystal (17) was removed from its circle of lead pillars (21). A small cell of brass tube 0.5 inch in diameter had six slots cut in it with a file. The diamond was mounted in such a position that the three corners of the triangular surface that touched the film should come opposite three of the slots. These experiments show that the diamond is giving off β -rays copiously (fig. 2).

γ -Rays from the Radio-active Diamond.

24. The diamond (17) crystal was fixed with its sharpest point upwards in a small thick cell of brass. Exactly over the point was a small hole 0.5 mm. in diameter and 5 mm. long. On the top was a piece of sensitive film enclosed in black paper. The whole was kept in the dark for three hours, when a fair image of the hole was obtained on development. The apparatus, with another sheet of sensitive film in it, was fixed between the pointed poles of a powerful electro-magnet, and a current of 30 amperes was passed through for three hours. The current was then shut off, and the film shifted sideways for half an inch, and the action of the diamond without the magnetism was allowed to go on for another three hours. It was then developed, and both spots appeared about the same intensity.

25. The diamond crystal (17) in its brass box had a thin plate of clear mica put over the hole, and a sheet of lead over that. All was wrapped in sheet lead, and so kept for about six months. There was an extremely faint but hardly appreciable darkening of the mica at the position of the hole.

26. After the above experiment the diamond was kept in its brass box for two years, and then laid on a sensitive film and kept there for three hours. On developing, a spot of action was seen, showing that the diamond was still radio-active. It was removed from its brass cell and examined in the dark on a blende screen. It gave plenty of scintillations easily visible without a lens. Experiments showed that it still gave out, along with α -rays, also β - and γ -rays (10, 19, 42, 43).

27. A small light-tight box was fitted as a camera. The arrangements were such that the image on the sensitive film was 1.5 times the size of the object. A photograph of the radio-active diamond (17) was taken by the light of its own scintillations

on the blende screen (17, 26, 30, 33), giving forty-eight hours' exposure (fig. 3*a*). The three bunches of luminosity come from the corners of the crystal that are in contact with the screen. The photographic impression is certainly discontinuous, appearing granular, corresponding to the granular character of the screen. It appears that the different grains shine and continue to shine by their own residual phosphorescent light under the impact of the α -particles. In a further experiment the diamond was illuminated by the arc lamp, and a photograph was made of the diamond crystal *in situ* as well as of the screen. This gave a good image of the stone, and also of the granular surface of the screen (fig. 3*b*).

28. It is certain that, in addition to α -rays, the diamond (17) gives off a considerable amount of other rays (10, 19, 26, 42, 43). This is shown by the early experiments, where photographic images with the diamond were obtained through one, two, three, and four sheets of aluminium foil—each 0.01 mm. thick.

29. The crystal of active diamond (17), with which most of these experiments have been made, was examined crystallographically by Dr. TUTTON, who reported to me that the crystal is an apparent octahedron—but composed of two supplementary tetrahedra showing on three of the edges the usual grooves where the interpenetration of the tetrahedra is not complete.

Each of the eight sides of the diamond was photographed by its own radio-activity (fig. 4).

30. Experiments show that exposure of a zinc-sulphide screen to the impact of electrons from the negative pole in a vacuum tube does not cause scintillations. It appears that only α -rays (positive atoms) produce this effect (11, 17, 26, 27, 33); it was therefore interesting to see if scintillations could be produced by bombarding a sulphide screen with a stream of positive atoms produced in a "Thompson" tube. An apparatus was fitted up having a zinc-sulphide screen to receive the positive discharge—but no effect of scintillation could be observed.

Action of Radium Emanation on Diamond and on other Bodies.

31. After repeated observations on the action of radium on various substances, it was seen that diamond behaved differently to glass, quartz, and other materials, especially if the action of the radium had been continued for some length of time. The following experiments were made to see if definite light could be thrown upon this point. Some plates and crystals of pure quartz, lead glass, FARADAY'S borosilicate lead glass, and three diamonds were selected—well cleaned, and exposed upon a sensitive film for twenty-four hours. On development it was seen that all were inactive, except in the case of two of the diamonds, but from these the action was so slight as to be only just visible upon the film. These objects were now laid upon a table of clean platinum gauze and enclosed in an air-tight glass vessel, together with an open bottle containing pure radium bromide. They were allowed to remain

exposed to radium emanation (33) for forty-eight hours; they were then removed from the chamber and laid upon a sensitive film for twenty-four hours. Upon development each object was found to be active and there was nothing to show that the three diamonds had acted differently to the quartz or glass (fig. 5). The objects were now all boiled in dilute nitric acid and thoroughly washed in clean water. They were placed on a film and allowed again to remain for twenty-four hours; on development it was seen that practically the whole of the "activity" had been removed—the two diamonds that originally were very slightly active had, perhaps, gained a little, but all the other objects had lost their temporary activity.

32. The experiments were still further extended. Two fragments of lead glass and one diamond crystal that had been buried in radium for over two years and had become deeply coloured, the diamond green and the lead glass brown, were removed from the radium and cleaned as far as possible with a dry brush. They were kept upon a film for five days. On development the photograph showed three equally dense black patches. The glass and diamond were now boiled in dilute nitric acid, thoroughly washed, laid upon a film, and kept there for eighteen hours. Upon development the pieces of glass were seen to have almost lost their activity—the diamond gave a dense black impression. The objects were now placed in a mixture of fuming nitric acid and potassium chlorate and kept just short of boiling point for six hours, frequently removing the weakened solution and adding fresh acid and chlorate (3, 9, 12); they were then thoroughly cleaned, dried, and laid upon a film for eighteen hours. On development the glass blocks gave impressions slightly fainter than before, while the diamond still gave a dense black image almost as strong as before the treatment.

33. An examination with a zinc-sulphide screen showed that the diamond caused brilliant scintillations from its edges and corners and a few erratic scintillations could be detected round each of the fragments of glass (11, 17, 26, 27, 30). The net result of these experiments is to show that although the well-known condensation of emanation (31) upon crystals and objects gives rise to photographic markings of patterns as described in this research and in previous papers by Sir ERNEST RUTHERFORD* and other observers, this superficial activity can be easily and completely removed by washing in dilute acids. The two kinds of acquired activity are illustrated in figs. 6 and 7. Fig. 6 is a radiograph of a crystal of quartz that had been rendered active by exposure to radium emanation, while fig. 7 is a radiograph of a crystal of diamond that had been covered in a bottle with radium bromide crystals for many months. While the former points to a superficial coating of active matter, the latter is more suggestive of a brush discharge from the corner of the diamond crystal. If the action of the emanation is allowed to continue until colour changes are produced both in glass and diamond, the activity is retained even after many

* "Distribution of the Intensity of the Radiation from Radio-active Sources," by Sir ERNEST RUTHERFORD, F.R.S., 'Phil. Mag.,' August, 1906.

hours boiling in powerful acids, and the diamond retains its activity with much greater tenacity than the other materials—nothing short of actually cutting away the surface will remove it. The diamond crystal—that had been rendered active by immersion in dry radium bromide for some months in 1906 (17) and has been treated with acids, heated, and used in hundreds of experiments since—is apparently as active now as when it was removed from the radium, and still pours out streams of α -rays* (Plate 10). Fig. 8 shows two radiographs of this crystal, having an interval of five years between them. Fig. 8*a* was taken in 1909 and fig. 8*b* a few days ago.

34. As in the case of diamond, a quartz glass dish after the action of radium retains its colour and activity, and continues to give off α -, β -, and γ -rays (fig. 9). One-half had a film of aluminium, 0.035 mm. thick, interposed between the dish and the sensitive plate. Vessels of pure fused silica that have been much used for the crystallisation of radium salts become coloured a purplish tint, similar to that assumed by soda glass in similar circumstances. On heating such a coloured vessel with a spirit lamp to a temperature much below a red heat it suddenly phosphoresces a bluish colour and at a red heat becomes colourless.

35. A small equilateral prism was cut from a block of heavy glass (sp. gr. 3.87), polished on its three refracting faces, but left rough ground on the triangular top and bottom. It was put into a bottle and covered with dry crystals of radium bromide for fifteen hours. At the end of this time it was removed, well washed and placed on a sensitive film, base downwards, for twenty-two hours. On development a strong action was apparent with overflow radiation along the sides and the continuation of the faces in triangular lines (fig. 10) as observed by Sir E. RUTHERFORD.

36. One of the rough ground bases of the prism was now polished over the greater part of its surface, leaving a strip of glass along one edge in its original unpolished condition. This end was then laid on a sensitive film for thirty-six hours. On the polished portion of the surface there was absolutely no perceptible action on developing, while the part left unpolished was highly active—as in the first experiment with the prism (fig. 11).

37. The prism was now laid on a sensitive film on one of its polished faces for twenty-four hours. On development slight action was observed where the polished face touched the sensitive surface and a much stronger overflow action with continuation of the edges of the prism face in straight lines as mentioned above (fig. 12).

38. The same prism was taken, and a diagonal scratch made with a writing diamond across the same face that touched the film in the last experiment. It was laid with this face downwards on a sensitive film for twenty-four hours. On

* These experiments are closely connected with those of SWINTON, COLLEY, RAMSAY, and others, who have shown that helium is driven into the glass walls of a vacuum tube even by the slower moving cathode rays. The high speed α -particles evidently are driven into the diamond, below the molecular surface.

development the image showed action as before where the face touched the film, but the line of scratch where the surface of the glass had been abraded by the diamond was blank, showing no action whatever (fig. 13).

39. A triangular plate, about 1 mm. thick, was polished on its three edges, and kept in a bottle of solid radium bromide for sixty-eight hours, then laid on a sensitive film for four hours. There was very little action to be seen on development on the part where the face touched the film, but there was strong action radiating from the edges, with a continuation of the line of edge from each point (fig. 14). The plate was now cut into two equal parts, and the two halves, separated about a centimetre, were laid on a sensitive film and there kept for forty-eight hours. On developing, it was seen that the action of the flat surface was the same as before, but while the overflow action from the original corners was also the same, there was no action at all along the cut surface (fig. 15).

40. A glass tube that had contained 25 mgrm. of pure radium bromide was cut in half to remove the radium, and well washed and boiled in acids. It was of a dark blue colour. After being in a cabinet for many months the two halves were laid on a sensitive film in a line, the cut surfaces opposite each other and separated about 2 mm. After four hours' contact the film was developed when the appearance presented that of a streaming brush discharge from the two ends of each half—the part where the tubes themselves rested having made no impression (fig. 16).

Superficial Action of Radium on Mica.

41. Radium bromide has been imported from the Continent in small ebonite boxes covered with a disc of mica. One of these discs was first well washed to remove any adhering grains of radium salt—it was then split into four flakes. The upper flake, of a strong brown colour, discharged the electroscope in three seconds. The next film, also showing brown discolouration, required 3.5 seconds. The third film, not discoloured at all, required twelve seconds, while the last film, which had been furthest from the radium, required eighteen seconds for discharge, showing that the greater part of the activity was near the surface and corresponded with the coloration.

Action of X-Rays on Diamond.

42. Experiments were instituted to ascertain how X-rays affected the diamond. A tray full of crystals of diamond was exposed to X-rays from a hard tube, covered in card and velvet so as to prevent interference from the luminosity of the glass. Most of the stones became luminous—but in different degrees. Two stones specially were noticed. One large stone shone very brightly, with a blue

tint, the other, smaller, was only just luminous. A sheet of aluminium, 3.2 mm. thick, was now interposed in the path of the X-rays, when the phosphorescence of the large bright stone was considerably reduced, while that of the small faint stone was not diminished at all.

Action of β -Rays on Diamond.

These two diamonds were subjected to the action of solid radium bromide, and the intensity of the phosphorescence thereby induced was seen to be of the same character as that caused by the X-rays. An extension of the experiment with radium, described in the next paragraph, showed that the effect of phosphorescence was mostly due to the β -rays (10, 19, 26).

43. An apparatus (fig. 17) was fitted so that the active rays from a bottle of radium bromide (A) should pass through a tube drilled through a lead block (B), and allowed to pass upwards on a diamond (C) supported on a plate of aluminium, 0.02 mm. thick. This was arranged between the poles of a powerful electro-magnet, so that the active rays from the radium should pass through the hole and act on the diamond when the magnetism was off, and be deflected from it when the magnetism was on.

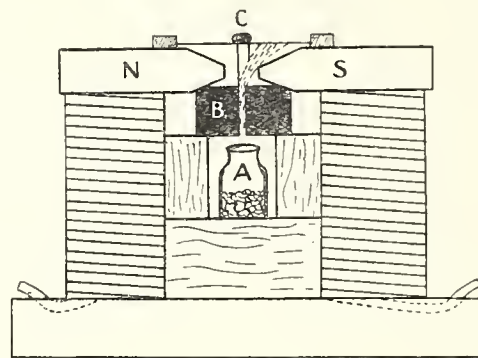


Fig. 17.

A screen of barium platinocyanide put over the hole in the lead block showed a circular spot of phosphorescence. This luminosity vanished when magnetism was turned on, and reappeared when it was turned off, showing that the luminosity on the screen was chiefly due to the β -rays (10, 19, 26). The two large and small diamonds used in the last experiment were put side by side on the aluminium, and the support so arranged that it could be moved sideways to put either stone over or away from the hole in the lead block. It was easy to see by small movements of the support that, as in the case of the X-rays, the large stone was much more phosphorescent than the smaller stone. There was considerable residual luminosity, each of the stones continuing to glow with its relative intensity when effectually removed from the radium. Having found the position of maximum brightness for the large stone when over the radium, the current was turned on, so as to deflect the β -rays. At once the glow declined, and the residual phosphorescence faded. On taking the current off, so as to allow the β -rays again to come into action, the brilliancy increased again. There was no doubt as to the action, although the undeflected γ -rays and the residual luminosity tended to obscure the observations.

44. The results described in this paper may be summarised as follows :—Various objects, diamond, ruby, garnet, quartz, gold, platinum, &c., also the phosphorescent substances yttria, calcium sulphide, zinc blende, and barium platinocyanide, are bombarded in a high vacuum by cathode rays, and in no case can any permanent activity be recognised either by photographic or electrical means (1, 2, 3, 4, 5, 6, 7).

Exposure to radium emanation confers temporary radio-activity on all bodies that have been tried; apparently due to the condensation of the emanation on the surface. This transient activity can be completely removed by washing in dilute acids (31).

Many substances become coloured by direct exposure to radium, the colour depending on the substance. Diamonds take a full sage-green tint, the depth depending on the time of exposure to the radium (33).

In addition to change of colour diamond also becomes persistently radio-active continuously giving off α -, β -, and γ -rays. The acquired colour and activity withstand the action of powerful chemical agents, and continue for years with apparently undiminished activity (12, 31, 32).

Removing the surface by mechanical means removes both colour and radio-activity (13, 16, 35, 37, 41).

The appearance of an auto-radiograph made by placing an active diamond crystal on a sensitive photographic plate, and the visual examination of its "scintillation" luminosity, suggest that there is a special discharge of energy from the corners and points of the crystal (18, 21, 23, 27, 33).



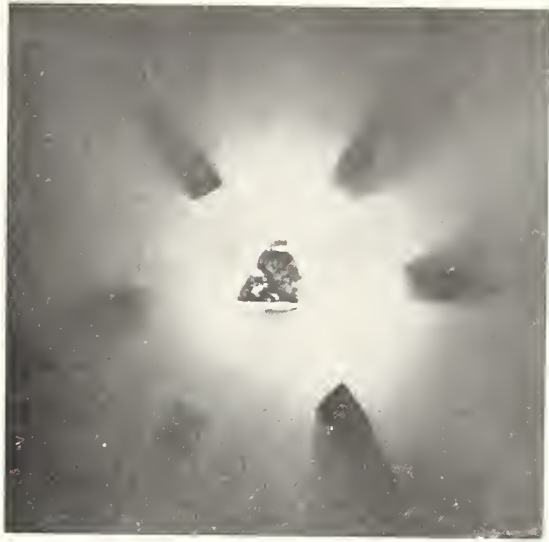
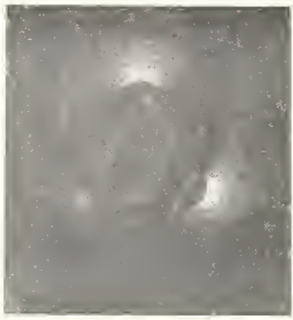


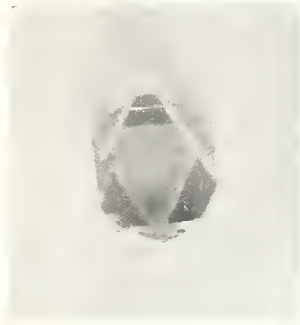
Fig. 1.



Fig. 2.



a



b

Fig. 3.



Fig. 4.

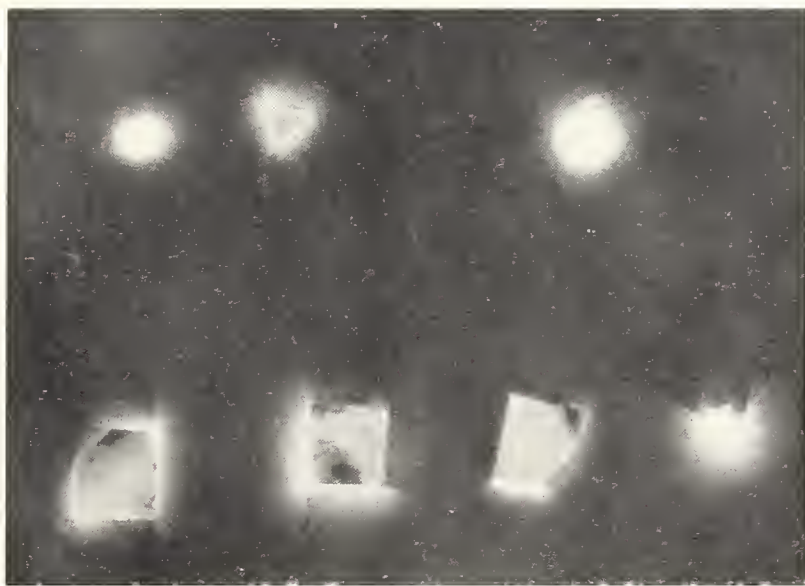


Fig. 5.



Fig. 7.



Fig. 6.





a *b*

Fig. 8.



Fig. 9.

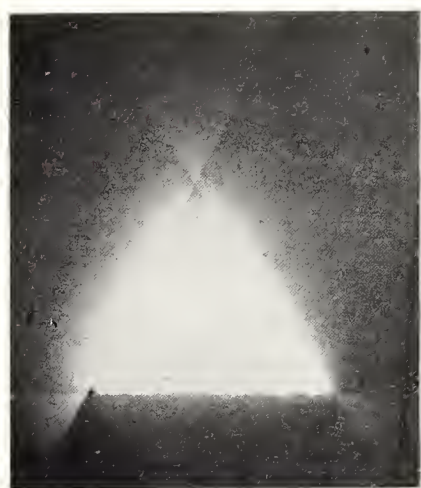


Fig. 10.



Fig. 11.

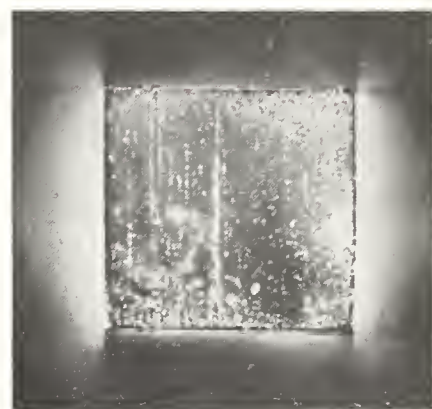


Fig. 12.

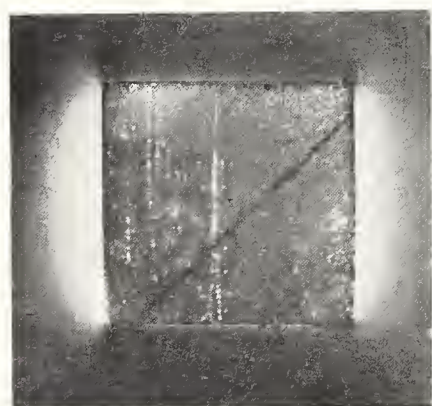


Fig. 13.



Fig. 14.



Fig. 15.



Fig. 16.

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